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About late asymptote of transient processes in the magnetic field of the dipole sources

The developed method solutions of the problem of late asymptote of transient processes in the magnetic field, the dipole sources are Innervate layered medium, underlain by an insulator. The method allows to determine the asymptotes of the late signals of the formation of a magnetic field as a power time series, which are present as a prominent member of a number of proportional t^{-4} and carrying information about the total longitudinal conductance of the section and a member of a number of proportional t^{-5} and contains information about capacity and conductivities of the layers. The algorithm of asymptotic solution based on the determination of the transition characteristics of the expanded core in the receiving operating information with one pole on the real axis of the complex plane and its structure is similar to the operating functions of the model a thin conductive layer, excited by the raised sources. The result is an asymptotic temporary solution to a temporary member of a number proportional t^{-5} to the received late asymptote depth equivalent to a conducting plane hτ is the important information of the parameter used in the analysis of experimental pulse field transient for three-layer sections. Analyzed the functional relationship of late asymptote parameter h_τ with a longitudinal conductivity and thickness of layers. The analysis of the possibility of using late asymptotes to determine the total power conductive sediments overlying non-conductive base section. Data in the method of formation field. Theoretical developments are confirmed by the analysis of model.

Keywords: late asymptote, a magnetic field, an insulator, a dual-layer cut, functional factor, geoelectric cut.

In the theory of a method, formation of the field (FF) [1–5] the late asymptote of transition process for a derivative, vertical components of magnetic induction $\partial B_z/\partial t$ in case of the horizontally layered section, spread by the insulator, known for the first term of staid decomposition of temporary function. It does not depend on the type of excitation sources and transients, according to [5, 6], is expressed by:

$$\left.\frac{\partial B_z}{\partial t}\right|_{t\to\infty} = \frac{K_E}{S}\left(\frac{\tau_S}{t}\right)^4; \quad \left.\frac{\partial B_z^*}{\partial t}\right|_{t\to\infty} = \frac{K_M}{S}\left(\frac{\tau_S}{t}\right)^4, \tag{1}$$

$$\text{where } K_E = \frac{3I\cdot AB\cdot\sin\varphi}{2\pi r^3}, \qquad K_M = \frac{3I\cdot Q}{\pi r^4} \text{ —} \tag{2}$$

the coefficients of installs with electric and magnetic dipoles exciting; I — exciting current; AB — the length of the electric dipole; φ — polar angle of installation «dipole-loop»; Q — the area of the magnetic dipole; r — dressing units; S — total conductivity of the longitudinal section; t — the transition process; $\tau_S = r\mu_0 S/2$ — formation parameter of field ($\mu_0 = 4\pi\cdot10^{-7}$H/m).

From the (1), asymptotes are proportional to t^{-4} and in informative plan it is depends only from total longitudinal conductivity of («S asymptote») section. Exact expression of a signal has aspect within asymptotic area, containing the second member of staid decomposition is proportional t^{-5}. It is obvious that in the informative aspect this member has to depend both on electric properties of layers, and on their capacities.

Sidorov V.A. [7] had offered the original heuristic way of the approximate solution of a direct non-stationary task in a near zone of incitement source. The main postulates of this way used by Sidorov V.A., also used, for development of an algorithm of the return — approximate solution that has received wide production. The basis of the offered method is made by two postulates: 1) an each fixed time of transition process corresponds a certain depth distribution of an electromagnetic (pulse) wave of H_τ (influence depth); 2) a transition process in each fixed moment corresponds to the transition process, inspired and located at a depth h_τ ($0 < h_\tau < H_\tau$). Thin conductor layer that has the longitudinal conductivity of S_τ corresponding to total longitudinal conductivity incision to the depth influence H_τ. Depth of the equivalent thin layer h_τ has defined as deep coordinate «center of gravity» depth dependence electric conductivity layer $\gamma(z)$ in the effect of base range H_τ:

$$h_{\tau} = \int_0^{H_\tau} z\gamma(z)dz \Big/ \int_0^{H_\tau} \gamma(z)dz = \int_0^{H_\tau} z\gamma(z)dz \Big/ S_\tau(H_\tau). \quad (3)$$

Eventually, after some period such carrying-out plane plunges, at the same time the parameters H_τ, h_τ, S_τ change. Within the considered approximate way of the solution, the direct assignment established functional connection between time of transition process by t and these parameters:

$$t = \mu_0 S_\tau \left(\frac{4}{3} H_\tau - h_\tau \right), \quad (4)$$

which is used at the solution of the return chore — a observed signal transformation in dependence of $S_\tau(H_\tau)$ [7].

Inspection of this heuristic approach on the transitional characteristic of two-layer model of incision demonstrates that the parameter of depth equivalences h_τ. On late times directs to the thickness of a conductor layer of H ($h_\tau|_{t\to\infty} = H$), but not to $H/2$, as it would be necessary to expect, proceeding from the considered heuristic conception and a ratio (3). It is confirmed by Figure 1 in which the temporary dependence of $h_\tau(t)$, received by transformation of the valid (not approximate) signal of $\partial B_z/\partial t$, excited electric dipoles in the two-layer cut [5]. Noted discrepancy is the contradictory moment for concerning the applied approximate way of the solution and a direct chore in a method of formation of the field in a near zone. Thus, it should be noted that the analytical late asymptote has temporary dependence of depth of the equivalent carrying-out $h_\tau(t)$ plane in the case that the basis of a section is the insulator, which is unknown.

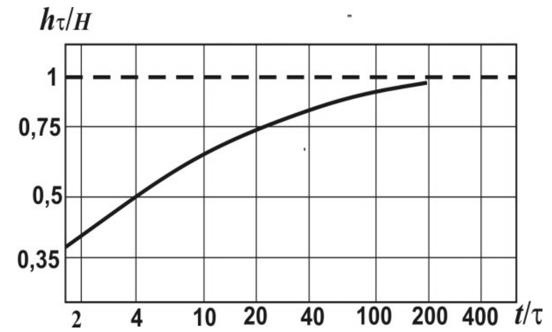

Figure 1. Time dependence h_τ option for a two-layer cut: $\tau = H^2\mu_0 / (4\rho_1)$

At the same time, h_τ represents the main information parameter that determined in the course of the analysis of impulses of the magnetic field formation. Naturally, it is necessary to know a late asymptote of given parameter. The solution of this chore has reduced to exact dissolution of a signal, on late a time row.

On late times, the currents induced in layers are evenly distributed and regularities of transition processes are close to those in the thin conductor layer, upset by the source, raised over it on height h. The character of transition processes, in this case, the asymptotically late times not depend on type of a source of incitement and can be presented in the following form [6–10]:

$$\frac{\partial B_z}{\partial t} = \frac{K_E}{S} \cdot \frac{\varepsilon}{\left(1+\varepsilon^2\right)^{5/2}}; \qquad \frac{\partial B_z^*}{\partial t} = \frac{K_M}{2S} \cdot \frac{\varepsilon\left(2\varepsilon^2 - 3\right)}{\left(1+\varepsilon^2\right)^{7/2}}, \quad (5)$$

where S — longitudinal conductivity of a thin layer, $\varepsilon = 2h/r + t/\tau_S$ — dimensionless parameter ($\tau_S = r\mu_0 S/2$ — parameter field formation in a thin conductive layer).

Late stage transition process in a thin layer, up to the second term of the power series we obtain, letting in (4) $t \to \infty$ and completing the corresponding limit changes. The result is a functionally identical asymptotic relations, which, up to the second row of a term proportional to t^{-5}, are expressed in the following form:

$$\left.\frac{\partial B_z}{\partial t}\right|_{t\to\infty} = K_E \left[\left(\frac{\tau_S}{t}\right)^4 - 8\frac{h}{r}\left(\frac{\tau_S}{t}\right)^5 \right], \quad \left.\frac{\partial B_z^*}{\partial t}\right|_{t\to\infty} = K_M \left[\left(\frac{\tau_s}{t}\right)^4 - 8\frac{h}{r}\left(\frac{\tau_s}{t}\right)^5 \right]. \quad (6)$$

It obviously should be expected that the late asymptote transient, measured on surface of layered media, under lain by an insulator, will have the same type (6), where instead of 8h/r coefficient in the second summand there will be some functional coefficient, which is related to power of h_j layers and their longitudinal conductance $S_j = h_j\gamma_j$ (γ_j — specific conductivity of layers). The purpose of this article is to obtain the late asymptote of transient in (6) type.

According to theoretical calculations, transient in the field of derived by time of vertical component of magnetic induction $\partial B_z/\partial t$ on the surface of horizontal-layered medium is represented as [1–4]:

a) Impulsion by electric dipole

$$\frac{\partial B_z}{\partial t} = \frac{I \cdot AB\mu_0 \mathrm{Sin}\varphi}{4\pi}\frac{\partial}{\partial t}\left[\int_0^\infty mW_1(m,t)J_1(mr)dm\right], \tag{7}$$

b) Impulsion by magnetic field

$$\frac{\partial B_z^*}{\partial t} = \frac{I \cdot Q\mu_0}{4\pi}\frac{\partial}{\partial t}\left[\int_0^\infty m^2W_1(m,\ t)J_0(mr)dm\right], \tag{8}$$

where $W_1(m,\ t)$ is sub-integral transient feature, which contains all information about structure of geoelectrical cut; J_0, J_1 — Bessel functions of zero and the first kind.

The transient feature $W_1\,(m,\ t)$ can be determined by the reverse Laplace transform [11] from the relevant operational function of cut $Q_1\,(m,\ p)$:

$$W_1(m,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} Q_1(m,\ p)\frac{e^{pt}}{p}dp. \tag{9}$$

In Riemann-Mellin integral, the (9) $p = -i\omega$ is an operational variable ($i = \sqrt{-1}$, ω — circular frequency).

On the basis of well-known calculations [3, 4], the operational function $Q_1(m,\ p)$ for horizontal-layered cut can be written as:

$$Q_1(m,p) = \frac{mR_1(m,p) - n_1}{mR_1(m,p) + n_1}, \tag{10}$$

where $R_1(m,p) = \mathrm{cth}\left\{n_1h_1 + \mathrm{arcth}\left[\frac{n_1}{n_2}\mathrm{cth}\left(n_2h_2 + ... + \mathrm{arcth}\frac{{}^{n}N-1}{m}\right)\right]\right\}$ — (11)

the recurrent operating inductive function section reflect a non-conductive base, where N — number of layers in the geoelectric cut, $n_j = \sqrt{m^2 + p\mu_0\gamma_j}$, h_j — power layers, γ_j — conductivities of layers (j = 1, 2, …, N-1).

For obtaining the late asymptotes of transients (7) and (8) in type of (6), it is necessary to perform critical transform of operational function (10). Such transform is lead to mathematical compression of over-supporting part of cut into thin multilayered conductive unit (without changing interrelation between power of layers), each layer is featured by longitudinal conductance $S_j = \gamma_jh_j$:

$$\tilde{Q}_1(m,p) = Q_1(m,p)\Big|_{\substack{h_j \to 0 \\ h_j\gamma_j = S_j}} = \frac{mR_1(m,p) - n_1}{mR_1(m,p) + n_1}\Bigg|_{\substack{h_j \to 0 \\ h_j\gamma_j = S_j}} \tag{12}$$

Trying to obtain the late stage of transients (5), excited in thin conductive S-layer, which is uplifted on height h by sources, the proposed concept was used here. This stage was obtained earlier on the basis of direct analysis of specified transients in interval of late time and it is expressed with (6) relation.

Relying on expressions (5), also their integral approximations (7) and (8), Q_S operational function for model of thin layer can be written as:

$$Q_S(m,p) = \frac{-p \cdot e^{-2mh}}{2m/(\mu S) + p}, \tag{13}$$

By compressing model (setting $h \to 0$) and by limiting linear term of power series expansion at h, we will get the critical operational function $\tilde{Q}_S$:

$$\tilde{Q}_S(m,p) = \frac{-(1-2mh)p}{2m/(\mu S) + p}, \tag{14}$$

Apply in operational transform (8) [11] to (13) allows to determine the critical sub-integral transient feature $\tilde{W}_S(m,t)$ as:

$$\tilde{W}_S(m,t) = -(1-2mh)\cdot\exp\left(-m\frac{2t}{\mu_0 S}\right).$$

Performing calculations according to (7) and (8) (integrals are determined on the basis of mathematical references [12]), whereas the critical sub-integral transient feature used its critical variant $\tilde{W}_S(m,t),$ here we will obtain the next relatively h critical transient features of time derivatives of magnetic induction:

$$\frac{\partial\tilde{B}_z}{\partial t} = \frac{K_E}{S}\cdot\left\{\frac{t/\tau_S}{\left[1+(t/\tau_S)^2\right]^{5/2}} - 2\frac{h}{r}\cdot\frac{4(t/\tau_S)^2-1}{\left[1+(t/\tau_S)^2\right]^{7/2}}\right\}, \quad (15)$$

$$\frac{\partial\tilde{B}^*_z}{\partial t} = \frac{K_M}{2S}\cdot\left\{\frac{t/\tau_S[2(t/\tau_S)^2-3]}{\left[1+(t/\tau_S)^2\right]^{7/2}} - 2\frac{h}{r}\cdot\frac{8(t/\tau_S)^4-24(t/\tau_S)^2+3}{\left[1+(t/\tau_S)^2\right]^{9/2}}\right\}. \quad (16)$$

It is obvious that the asymptotically late interval of transient features (15, 16) conform to late asymptotes (6) acquired previously.

The test sample confirms that the late asymptote (6) of transient features of thin conductive layer (5) can be obtained through the critical operational feature (13), determined for condition $h \to 0$, and which have one pole on real axis of complex plane and linear functional presence of geometrical parameter h in it. It proves the correctness of above-mentioned concept for solving issues about late asymptote of transient features in case of horizontal-layered medium, underlain by an insulator.

Transform (12) of operational function of layered cut Q_1 into critical operational function $\tilde{Q}_1$ should be performed in a such way that the last one should have similar structure to critical operational function of thin layer $\tilde{Q}_s$ (13) — it should have one pole on the real axis of complex plane and linear functional presence of powers of layers h_j. It provides an opportunity for analytical definition of integrals in (6) and (7), consequently asymptotical analysis of transients.

The performed critical transform of operational function of cut has rather cumbersome nature. It is based on expansion of numerator and denominator of ratio (10) into series by powers of vector of layer thickness $h = \{h_1, h_2,\ldots,h_{N-1}\}$ limiting with number of infinitesimals $O(h)$. It provides linear functional relationship of numerator and denominator of relative vector of layer power h. Corresponding critical transforms are performed with use of recurrent bond of inductive operational function (11) on surface of j-layer (R_j) with same operational function on surface of $(j+1)$-layer (R_{j+1}):

$$R_j(m,p) = \mathrm{cth}\left[n_j h_j + \mathrm{arcth}\left(\frac{n_j}{n_{j+1}}R_{j+1}\right)\right] = \frac{n_{j+1} + n_j R_{j+1}\mathrm{cth} n_j h_j}{n_j R_{j+1} + n_{j+1}\mathrm{cth} n_j h_j}. \quad (17)$$

The regularities of series formation are defined during critical transforms.

By dropping the cumbersome critical transforms, which perform (12) algorithm and satisfy the mentioned requirements regarding type of critical operational function, we write the obtained calculation results of this function for horizontal-layered cut, underlain by an insulator, as:

$$\tilde{Q}_1(m,p) = -(1-B)\cdot\frac{p}{\left(\frac{2m}{\mu S}\right)\left[1+m\left(A-B\right)\right]+p}, \quad (18)$$

where S — the total longitudinal conductivity layers: $S=\sum_{j=1}^{N-1} h_j\gamma_j$; function coefficients A and B — coefficients of the order of smallness $O(h)$, which have a functional relationship with the vector of longitudinal conductivities of layers $S = \{S_1, S_2, \ldots, S_{N-1}\}$ and a linear functional relationship with the power vector layers $h = \{h_1, h_2,\ldots,h_{N-1}\}$:

$$A(h,S) = H + \sum_{i=2}^{N-1}\left(\bar{S}_i\sum_{j=1}^{i-1}h_j - \bar{S}_{i-1}\sum_{j=1}^{N-1}h_j\right), \quad (19)$$

$$B(h,S) = \frac{2}{3}\left[\sum_{i=1}^{N-1}\bar{S}_i h_i - \sum_{i=2}^{N-1}\left(\bar{S}_i\sum_{j=1}^{i-1}\bar{S}_j h_j + \bar{S}_{i-1}\sum_{j=i}^{N-1}\bar{S}_j h_j\right)\right] +$$

$$+2\sum_{i=2}^{N-1}\left[\overline{S}_i\sum_{j=1}^{i-1}h_j-\overline{S}_{i-1}\sum_{j=i}^{N-2}\left(h_j\sum_{m=j+1}^{N-1}\overline{S}_m\right)\right], \tag{20}$$

where $H=\sum_{j=1}^{N-1}h_j$ — the total capacity of the layers overlying the non-conductive base cut, $\overline{S}_i=S_i/S$ — relative longitudinal conduction layers.

As an example, coefficients A and B for some geoelectric cuts:

а) for a two-layer cut:

$$A=h, \qquad B=\frac{2}{3}h; \tag{21}$$

б) for a three layered cut:

$$\left.\begin{aligned}A&=(h_1+h_2)+\frac{1}{S}(S_2h_1-S_1h_2),\\ B&=\frac{1}{S}\left\{\frac{2}{3}\left[S_1h_1+S_2h_2-\frac{S_1S_2}{S}(h_1+h_2)\right]+2S_2h_1\right\};\end{aligned}\right\} \tag{22}$$

в)for a four layered cut:

$$\left.\begin{aligned}A&=H+\frac{1}{S}\left[S_3(h_1+h_2)+S_2(h_1-h_3)-S_1(h_2+h_3)\right],\\ B&=\frac{2}{3S}\left\{S_1h_1+S_2h_2+S_3h_3-\frac{1}{S}\left[S_1S_2(h_1+h_2)+S_1S_3(h_1+h_3)+S_2S_3(h_2+h_3)\right]\right\}+\\ &+\frac{2}{S}\left[S_2h_1+S_3(h_1+h_2)-\frac{S_1S_3}{S}h_2\right],\end{aligned}\right\} \tag{23}$$

Applying the reverse Laplace transform (9) to (18) allows to obtain the critical transient feature of sub-integral function $\tilde{W}_1(m,t)$ as:

$$\tilde{W}_1(m,t)=-\left[1-mB(h,\ S)\right]\cdot\exp\left\{-\frac{2m^2t}{\mu S}\left[A(h,\ S)-B(h,\ S)\right]\right\}\cdot\exp\left(-\frac{2mt}{\mu S}\right). \tag{24}$$

It is obvious that late stage of transients in the field of derivative by time of vertical component of magnetic induction may be obtained after analysis of analytical relations, received during calculations performed according to (7) and (8) algorithms [11]. However, unfortunately, integrals in (7) and (8) cannot be defined for $\tilde{W}_1(m,t)$ function. Then, bearing in mind, that the determination of late asymptote is performed through compression of sub-surface into layered overburden by tending vector of layer powers to zero ($h\to 0$), there is a reason to apply power-series expansion to (24) relatively A and B functional coefficients, because they have order of smallness O(h), and limit this expansion with linear members. As a result, we have:

$$\tilde{W}_1(m,t)=-\left\{1-mB(h,\ S)+m^2\frac{2t}{\mu S}\left[B(h,\ S)-A(h,\ S)\right]\right\}\exp\left(-\frac{2mt}{\mu S}\right). \tag{25}$$

Now integrals in (7) and (8) can be analytically defined [12] and the result of calculating the critical transients can be written as:

а) Impulsion by electric dipole

$$\frac{\partial B_z}{\partial t}=\frac{K_E}{S}\left[\frac{\overline{t}}{\left(1+\overline{t}^2\right)^{5/2}}-\left(\frac{2B(h,\ S)-A(h,\ S)}{r}\right)\frac{4\overline{t}^2-1}{\left(1+\overline{t}^2\right)^{7/2}}+5\left(\frac{B(h,\ S)-A(h,\ S)}{r}\right)\frac{\overline{t}^2\left(4\overline{t}^2-3\right)}{\left(1+\overline{t}^2\right)^{9/2}}\right]; \tag{26}$$

б) Impulsion by magnetic dipole

$$\frac{\partial B_z^*}{\partial t}=\frac{K_M}{2S}\left[\frac{\overline{t}\left(2\overline{t}^2-3\right)}{\left(1+\overline{t}^2\right)^{7/2}}-\left(\frac{2B(h,\ S)-A(h,\ S)}{r}\right)\cdot\frac{8\overline{t}^4-24\overline{t}^2+3}{\left(1+\overline{t}^2\right)^{9/2}}+\right.$$

$$+5\left(\frac{B(h,\ S)-A(h,\ S)}{r}\right)\cdot\frac{\overline{t}^{2}\left(8\overline{t}^{4}-40\overline{t}^{2}+15\right)}{\left(1+\overline{t}^{2}\right)^{11/2}}\Bigg], \qquad (27)$$

where $\bar{t} = t/\tau_S$ — normalized transient time ($\tau_S = r\mu_0 S/2$ — field formation parameter, S — the total longitudinal conductivity of the layered strata overlying the base nonconductive cut).

The correctness of relations (26, 27) is confirmed with that they can provide a special case, which is a model of thin conductive layer, excited with uplifted impulsive source. For this purpose, the first layer in the three-layer model of cut should be converted into non-conductive one, its longitudinal conductance should be equal to zero($S_1 = 0$), and the second layer should be transformed into thin conductive plane with longitudinal conductance $S_2 = \lim(\gamma_2 h_2)|_{h_2\to 0}$. Then, according to (22), functional coefficients will be identical

($A = B = 2h_1$) and relatively relations (26) and (27) will become the specified relations (15, 16) for model of thin conductive plane.

Asymptotical analysis of approximate temporary solutions (26) and (27) by time power series expansion with an accuracy of t^{-5} leads to result, which does not depend on field's excitation type, and it can be expressed as:

$$\left.\frac{\partial B_z}{\partial t}\right|_{t\to\infty} = K_E,\left[\left(\frac{\tau_S}{t}\right)^4 - 8\cdot\frac{2A(h,S)-1,5B(h,S)}{r}\left(\frac{\tau_S}{t}\right)^5\right], \qquad (28)$$

$$\left.\frac{\partial B_z^*}{\partial t}\right|_{t\to\infty} = K_M\left[\left(\frac{\tau_S}{t}\right)^4 - 8\cdot\frac{2A(h,S)-1,5B(h,S)}{r}\left(\frac{\tau_S}{t}\right)^5\right], \qquad (29)$$

where K_E, K_M — setting the coefficients of electric and magnetic stimulation (Fig. 2).

Comparing (28), (29) and (6), we obtain the asymptotic limit $\left(h_\tau^a\right)$, which tends to average depth of the equivalent of a thin conductive layer $h_\tau(t)$ at $t \to \infty$:

$$h_\tau^a = h_\tau(t)\big|_{t\to\infty} = 2A(h,S)-1,5B(h,S). \qquad (30)$$

This parameter was mentioned in the beginning of article as an important informational parameter, which is often used at transformation of impulses of near-field transient by Sidorov-Tikshaev method [8]. Particularly, two-layered cut with critical asymptote of this parameter is a power of conductive layer $\left(h_\tau^a = H\right)$, it confirms above-mentioned result, obtained by numerical analysis of real transient feature (Fig. 1).

For three-layered cut the asymptote of h_τ parameter is:

$$h_\tau^a = \left(H + \frac{S_1\left(h_1S_2 - h_2S_1\right)}{S^2}\right) = H\left[1+\frac{\nu/\mu-\nu}{(1+\nu)(1+\nu/\mu)^2}\right], \qquad (31)$$

where $H = h_1 + h_2$ — the total capacity of the conductive layers (h_1, h_2 — power of the first and second layers); $S = S_1 + S_2$ — total conductivity of the longitudinal cut: $S_1 = h_1/\rho_1$, $S_2 = h_2/\rho_2$ — longitudinal conduction layers (ρ_1, ρ_2 — resistivities of the layers); $\mu = \rho_2/\rho_1$; $\nu = h_2/h_1$. As it follows from (31) this asymptote carries information about the total power cut H adjusted for the effect of the parameters of layers – relationship longitudinal conductivities $S_2/S_1 = \nu/\mu$ and capacity $h_2/h_1 = \nu$.

Research depending on the module h_τ^a cut leads to a completely logical result:

$$h_\tau^a\big|_{\mu\to 0} = h_1 + h_2 = H; \qquad h_\tau^a\big|_{\mu\to\infty} = \frac{h_1+h_2}{1+\nu} = h_1. \qquad (32)$$

At the «resonant» power relationships and longitudinal conductivities of layers, which is expressed in the form

$$S_2/S_1 = 1 + 2h_2/h_1 \text{ или } \mu = \nu/(1+2\nu), \qquad (33)$$

Asymptote h_τ^a has an extreme (maximum), depending on the ratio of capacity layers — module $\nu = h_2/h_1$ cut:

$$h_\tau^a, \max = \left(h_1 + h_2\right)\left[1 + \frac{1}{4}\frac{h_1^2}{\left(h_1 + h_2\right)^2}\right] = H\left[1 + \frac{1}{4\left(1+\nu\right)^2}\right]. \quad (34)$$

As follows from (34), the maximum value of this asymptote is limited: $h_{\tau,\max}^a \leq 1{,}25H.$

Figure 2 shows charts of time relations of depth of equivalent conductive layer h_τ for two-layered and three-layered cut types *H* and *A*, which were received due to numerical analysis of theoretically calculated transient features [5]. Also we cited the theoretically determined late asymptotes h_τ^a, which were calculated by using (31) formula. As shown in illustrations, everywhere curves $h_\tau(t)$ in the interval of late times match their theoretically determined asymptotes (dashed lines).

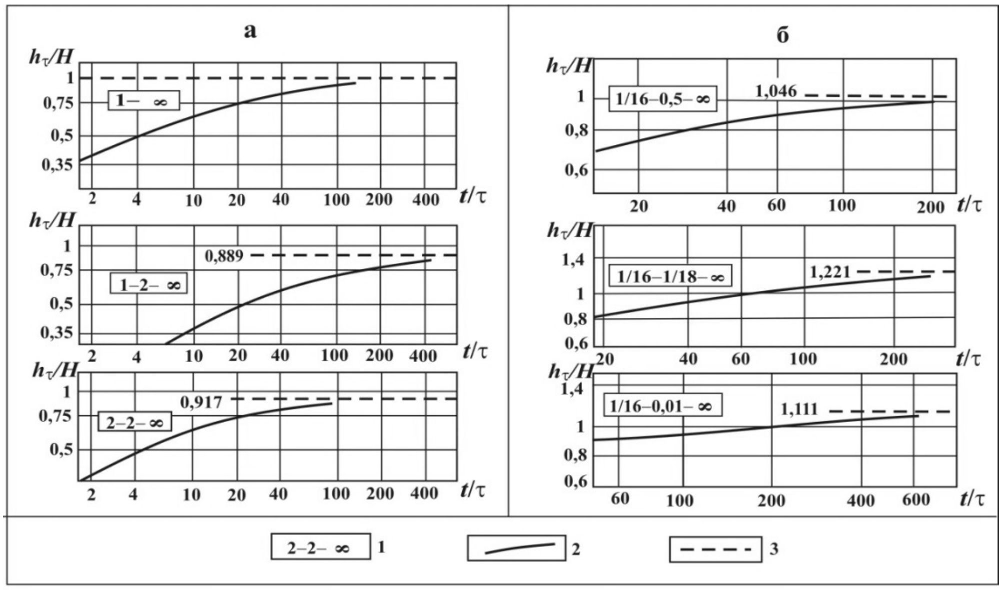

Figure 2. Time relations of parameter h_τ/H for two-layered cut (a) and three-layered cut of *A* (*a*) and *H* (*b*) types: 1 — cut parameter: $h_2/h_1 – \rho_2/\rho_1 – \rho_3$; 2 — curve h_τ/H; 3-asymptotes

Pay special attention to Figure 2, b. The graphics of three-layered cut of H type with identical values of $\nu = h_2/h_1 = 1/16$ modulus and different values of $\mu = \rho_2/\rho_1 = 0{,}01$; 1/18; 0,5 modulus. At $\mu = 1/18$ there will be specified condition of «resonance» relation of μandν modules of cut ($\mu = \nu/(1 + 2\nu)$), where h_τ^a has a maximum. As shown at graphics, time relation $h_\tau(t)$ at $\mu = 1/18$ provides the value of asymptote $h_\tau^a/H = 1{,}221$, while at μ=0,5 asymptote is equal to $h_\tau^a / H = 1{,}046$, and at $\mu = 0{,}01$ it will be $h_\tau^a / H = 1{,}111$. These confirms the presence of determined extreme peculiarity of asymptote h_τ^a.

Figure 3a shows a class of graphics of h_τ^a parameter dependence, which is expressed in unit power of over-supporting overburden H, from $\mu = \rho_2/\rho_1$ modulus of three-layered cut, while Fig. 3b shows the dependence of this parameter in the same relative concept from relation of longitudinal conductance of layers. A class of graphics were calculated for different values of modulus $\nu = h_2/h_1$: $1/64 \leq \nu \leq 32$. In fact, the both of curve classes are the alignment chart of coherence of h_τ^a parameter with power *H* of over-supporting part of three-layered cut.

As shown in the illustrations and above-reviewed analysis, asymptotic value of depth of equivalent conductive plane h_τ^a has extreme peculiarity, which becomes apparent well, when the power of intermediate layer is lower than the power of the first one, and the longitudinal conductance is commensurable. Meanwhile the less the power of intermediate layer is, the more the h_τ^a parameter exceeds the total power of conductive layers in the S_2/S_1 variation interval. So at $h_2 / h_1 = 1/64$ $h_\tau^a > H$ in the interval of $0{,}1 < S_2/S_1 < 10$.

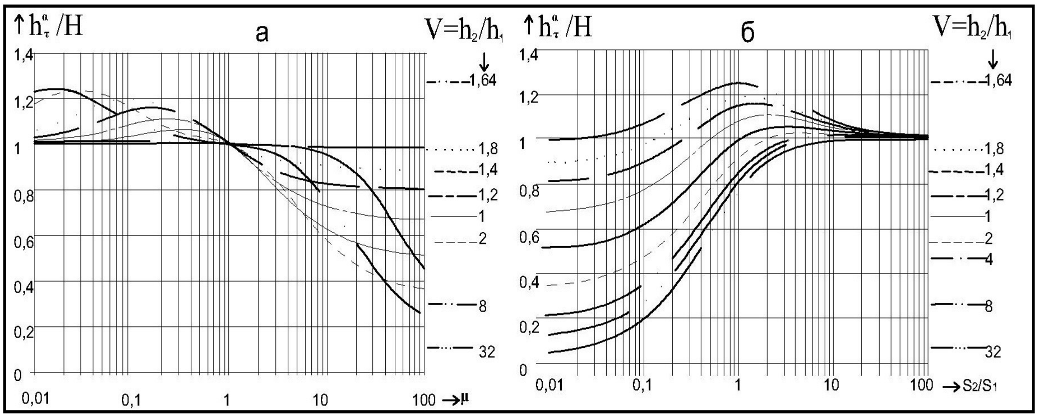

Figure 3. Dependence of h_τ^a/H from geoelectric features of three-layered cuts of H and A type: (on Fig. 3b values of h_1/H are marked with dotted lines)

At the same time, it was noted above, the extreme surpassing the total power of conductive layers is limited: $h_{\tau,\max}^a \leq 1{,}25H$. Analysis of calculation results testifies that at $1/2 < h_2/h_1 < 16$ and $S_2/S_1 > 2$ the parameter of h_τ^a can be used for determining the total power H of cut formation, which covers nonconductive basis, providing determination accuracy no less than 10%. At $h_2/h_1 > 1/2$ and $S_2/S_1 < 0{,}3$ this determination is linked with the significant inaccuracies. The dramatic decrease of longitudinal conductance of intermediate layer, in comparing with longitudinal conductance of layer, which covers it (such situation is typical for A type cuts), leads to distribution of the induced current mainly in the first layer. As a result, the asymptotic depth of equivalent thin layer in the time interval of field formation will be close to power of the first layer, which is shown on Fig. 3b (dotted lines on fig. 3b mark power of the first layer).

Conclusion. The performed calculations and their analysis completely confirm the correctness of obtained asymptotic approximation (28, 29) for late stage of transients in magnetic field of dipole sources, exciting layered media, underlain by an insulator. There was defined an analytical link between late stage of transient and the total longitudinal conductance of cut, moreover its link with the powers and conductance of separate layers. For the first time, the late asymptote of depth of equivalent conductive plane for layered media, underlain by an insulator, was determined analytically. The analysis of performed studies testifies about opportunity to use the late asymptote of transient for determining powers of conductive sediments, which cover the nonconductive basis of cut.

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Диполь көздерінің магнит өрісіндегі өтпелі үрдістердің кейінгі асимптотасы туралы

Диполь көздерінің магнит өрісіндегі өтпелі үрдістердің кейінгі асимптотасы туралы тапсырмаларды шешу әдісі шығарылды. Бұл әдіс қуатты уақыт қатары түріндегі магнит өрісіндегі сигналдардың кейінгі асимптотасын анықтауға мүмкіндік береді, онда пропорционалды t^{-4} және қиманың суммалық көлденең өткізгіштігі туралы ақпарат тасушы, сонымен қатар пропорционалды t^{-5} қабаттардың өткізгіштігі мен қалыңдықтары туралы ақпарат құрайтын қатар мүшелері бар. Асимптотикалық шешімдердің ұсынылған алгоритмі өтпелі интеграл өзегінің ақпараттық функциясын алу арқылы анықталуымен негізделген, кешенді жазықтықтағы нақты осінде бір полюсқа ие және құрылымдық ұқсас операциялық функциясы моделі жұқа өткізгіш қабаты қозғалған жоғары көздерімен жүзеге асады. Тепе-тең, уақытша бірқатар мүшесі дәлдікпен асимптотикалық уақытша шешім пропорционалды нәтижесінде t^{-5} өткізгіш жазықтықтағы тереңдігі h_τ кейінгі асимптота алынды — ақпараттың маңызды параметрі, ол кен орнын игеру әдісі эксперименттік мәліметтерді талдау кезінде пайдаланылды. Теориялық талдау үш қабатты учаскелерін импульстік саласындағы қалыптастыру үшін дамыту моделін растады. Бойлық өткізгіштік және электр қабаттарымен кейінгі асимптота h_τ параметрінің функционалдық байланысы сараланған. Кейінгі асимптотаны негізгі қиманың жабылатын ток өткізбейтін өткізгіш қабаттардың сомалық қалыңдығын анықтау үшін қолдану мүмкіндігіне сараптама жасалды.

Кілт сөздер: кейінгі асимптота, магнит өрісі, изолятор, қос қабатты қима, функционалдық коэффициент, геоэлектрлік қима.

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О поздней асимптоте переходных процессов в магнитном поле дипольных источников

Разработан способ решения задачи о поздней асимптоте переходных процессов в магнитном поле, дипольных источников, возбуждающих слоистые среды, подстилаемые изолятором. Способ позволяет определять поздние асимптоты сигналов становления магнитного поля в виде степенного временного ряда, в котором присутствуют как известный член ряда, пропорциональный t^{-4} и несущий информацию о суммарной продольной проводимости разреза, так и член ряда пропорциональный t^{-5}, содержащий информацию о мощностях и проводимостях слоев. Предложенный алгоритм асимптотического решения основан на определении переходной характеристики подынтегрального ядра через получение операционной информативной функции, имеющей один полюс на действительной оси комплексной плоскости и по своей структуре аналогичной операционной функции модели тонкого проводящего слоя, возбуждаемого приподнятыми источниками. В результате асимптотического временного решения с точностью до члена временного ряда, пропорционального t^{-5}, получена поздняя асимптота глубины эквивалентной проводящей плоскости h_τ-важного информационного параметра, используемого в процессе анализа экспериментальных данных в методе становления поля. Теоретические разработки подтверждены анализом модельных импульсов становления поля для трехслойных разрезов. Проанализированы особенности функциональной связи поздней асимптоты параметра h_τ с продольными проводимостями и мощностями слоев. Выполнен анализ возможности использования поздней асимптоты для определения суммарной мощности проводящих отложений, перекрывающих непроводящее основание разреза.

Ключевые слова: поздняя асимптота, магнитное поле, изолятор, двухслойный разрез, геоэлектрический разрез.

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