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## Dissipative processes in tribology

In the paper the dissipative process is considered as the basis of the nature of external friction in solids. It is shown that non-equilibrium processes in the tribosystem can lead to a decrease in the production of entropy and, consequently, the intensity of wear and stably proceed with the formation of dissipative structures during self-organization. It was found that the first term can reach  $f_e = 10^{-3}-10^{-4}$  by using  $X_m$ ,  $\rho_m$  and  $W(X_m)$  in the mode of using special lubricants. In space, the effect of oil filters is absent. It uses solid lubricants which include substances applied to the surface of parts in the form of films that have a shear strength that is significantly less than that of the part material. An expression for the friction coefficient is obtained, which takes into account the surface geometry and its physical properties. It was concluded that homogeneous metal pairs should not be used in friction antifriction pairs. It is shown that a decrease in the dry friction coefficient is possible due to a decrease in surface energy. In this case, friction pairs are used, in which the electron work function differs significantly. This leads to a difference in the contact potential difference and, accordingly, to a decrease in the surface energy. It is shown that the friction coefficient depends on the fractal structure of the surface of many solids and this is their characteristic property.

*Keywords:* dissipative process, friction, wear, surface, entropy, fractal.

### *Introduction*

The origins of tribology — the science of friction, wear and lubrication of machines — go into the depths of human history [1–4]. The first period of history includes the accumulation of human tribological knowledge from prehistoric times to the end of the XVI century. In the XVII–XVIII centuries, the birth of the science of friction, later called «tribology», takes place. In the XIX century, the progress of technology (above all — the development of railways) highlights the problems of a tribological nature. In a word, before the beginning of the 20th century, tribology went a long and glorious way, successfully solving the tasks put forward by practice, and the progress of technology was hardly delayed for a long time because of the impossibility of solving tribological problems [1–4].

The beginning of the XXI century is especially important for friction units operating in extreme conditions (aerospace, atomic energy, high-speed transport, equipment for the development of the Ocean, the deep bowels of the Earth, etc.) [5, 6]. The urgent task is to summarize the experimental data, predict reliability and resource in a wide range of changes in voltages, temperatures, environments, exposure to fields of different physical nature (acoustic, electromagnetic, radiation, etc.) [7].

The beginning of the XXI century laid a comprehensive study of the tribological properties of nanostructures and nanomaterials [8–11]. The use of nano-coatings can significantly reduce the coefficient of friction while maintaining wear resistance, increase the viscosity of coatings while maintaining anti-corrosion properties and increasing the operating temperature by 50 °C. All this is especially important in the field of engineering. Nanostructured materials and suspensions are used to create nanocoatings. Special additives are introduced into the coatings that modify their structure and provide the necessary properties.

Selective transference under friction (the effect of wearlessness) is a natural phenomenon, it is recognized as the scientific discovery of the USSR (D. Garkunov and I. Kragelsky) No. 41 dated September 13, 1966 [12]. In practice, the «wear-free» functioning of friction units is most often achieved by using metal-plating lubricants in real tribo-splicing: oils, grease, self-lubricating materials and coatings [13]. Historically, the first studies of the mechanism of selective transfer [14] were studies of the thickness, roughness, micro-hardness and microstructure formed on rubbing surfaces during friction of a copper alloy on steel in an aqueous or alcoholic solution of glycerol and a visually observed copper servo film. It has been established that the servo-film film spontaneously arising during frictional interaction has a thickness of no more than 1...2 microns, and its mechanical and physicochemical properties are significantly different from the properties of ordinary compact copper.

Considering from the standpoint of green tribology, the effect of wearlessness under friction, the first thing that attracts attention is the conceptual possibility of friction without wear, which has been repeatedly confirmed experimentally and fully corresponds to the principles of green chemistry, engineering and tribology [15]. In other words, the saving of material resources during the implementation of selective transfer is achieved automatically and not only by minimizing wear, but also by increasing the resource life of engineering products.

Secondly, the friction coefficients of  $10^{-3}$  in the implementation of super-anti-friction properties clearly lead to energy savings, which also corresponds to the principles of green science [15].

It is noted that the nature of external friction is the dissipative process [16]. Relaxation internal friction is a dissipative process found in the study of the phenomenon of inelasticity. The intensity of relaxation processes found on the temperature-frequency dependences of the dissipation of a part of the energy of an external force field mechanical effect depends on the ratio of speed or frequency from this effect and temperature  $T$ . Depending on this ratio, the reaction of the system under investigation to external influence varies from elastic to viscoelastic.

In this paper, the dissipative process is considered as the basis of the nature of external friction in solids.

#### *Probability of dissipative processes*

Crystal defects (impurity ions, color centers, etc.) will be considered as a system of noninteracting particles immersed in a thermostat (tribological system). Quantum transitions due to the interaction of defects with a thermostat will be dissipative (with probability  $P$ ) as opposed to interaction with an external field (with probability  $F$ ). Dissipative processes lead to the fact that the secondary field (system response) is always smaller than the primary one, which causes the formation of defects [17, 18].

Since the subsystem of defects exchanges only energy with a thermostat, the corresponding ensemble of particles will be canonical. In this case, the expression for statistical entropy is:

$$S = -k \sum_i f_i \ln f_i, \quad (1)$$

where  $f_i$  is the distribution function;  $k$  is the Boltzmann constant.

Differentiating (1) in time and transforming, we get:

$$\frac{dS}{dt} = \frac{k}{2} \sum_{i,j} (hf_i - \ln f_j)(P_{ij}f_i - P_{ji}f_j), \quad (2)$$

where  $P_{ij}$  is the transition probability from the initial  $i$  (with energy  $E_i$ ) to the excited state  $j$  (with energy  $E_j$ ).

For dissipative processes, the principle of detailed equilibrium has the form:

$$\frac{g_i P_{ij}}{g_j P_{ji}} = e^{\frac{E_j - E_i}{kT}}, \quad (3)$$

where  $g_i, g_j$  are statistical weights for levels  $E_i$  and  $E_j$ .

Then (1) takes the form:

$$\frac{dS}{dt} = \frac{k}{2} P_{ij} (\ln f_i - \ln f_j) \left( f_i - \frac{g_i}{g_j} f_j e^{-\frac{E_i - E_j}{kT}} \right), \quad (4)$$

Canonical distribution function:

$$f_{ij} = \frac{1}{Z} e^{-E_{ij}/kT}, \quad (5)$$

where is the statistic sum:

$$Z = e^{-G/kT}, \quad (6)$$

where  $G$  is the Gibbs potential (free energy) of the thermostat + defect system.

We assume that the non-configuration part of the Gibbs potential linearly depends on the concentration of  $N$  defects:

$$e^{-G/kT} = \sum_N h(N), \quad (7)$$

where  $h(N) = \omega(N) \cdot e^{-G/kT}$ ;  $\omega(N)$  — statistical weight.

After simple calculations, it is easy to show that the function  $h(N)$  is the Gaussian distribution near the equilibrium value with a small dispersion, i.e.:

$$h(N) = h(\bar{N})e^{-\Delta\bar{N}^2/N}. \quad (8)$$

Substituting (8) into (7), we have:

$$e^{-G/kT} = h(\bar{N}) \sum_{\Delta N} e^{-\Delta\bar{N}^2/\bar{N}}. \quad (9)$$

To estimate the sum in (9), we replace it with the integral:

$$\sum_{\Delta N} e^{-\Delta\bar{N}^2/\bar{N}} = \int_{-\infty}^{+\infty} e^{-x^2/\bar{N}} dx = \sqrt{\pi\bar{N}}.$$

Then (9) takes the form:

$$e^{-G/kT} = h(\bar{N})(\pi\bar{N})^{1/2}. \quad (10)$$

Using (7) and taking the logarithm from (10), we find:

$$G/kT = -\ln \omega(\bar{N}) + \frac{G(\bar{N})}{kT} + \frac{1}{2} \ln(\pi\bar{N}), \quad (11)$$

where  $G(\bar{N})$  is the part of the total Gibbs potential associated with the concentration of defects. From the evaluation of the first logarithmic term follows:

$$\ln \omega(\bar{N}) = N \ln \left( 1 + \frac{\bar{N}}{N} \right) + \bar{N} \ln \frac{N + \bar{N}}{N}. \quad (12)$$

By approximating the logarithm in the first term of the right-hand side of (12), the first term of its expansion in a series, and the second term, expressing through the Gibbs potential of the system of defects  $G'$ , we get:

$$\ln \omega(\bar{N}) = \bar{N} + \bar{N}G'/kT. \quad (13)$$

Substituting (13) into (11) and neglecting the term  $1/2 \ln(\pi\bar{N})$  as compared with  $\bar{N}$ , we get:

$$G = G(\bar{N}) - \bar{N}G' - \bar{N}kT. \quad (14)$$

As above, assuming that the thermodynamic potential  $G(\bar{N})$  depends on the equilibrium number of defects  $C'$  in a linear fashion, that is:

$$G = G^0 + \bar{N}G', \quad (15)$$

where  $G^0$  is the thermodynamic potential of the thermostat, we find:

$$G = G^0 - \bar{N}kT. \quad (16)$$

Using (8), expression (6) is converted to the form:

$$Z = e^{-G^0/kT} e^{\bar{N}}. \quad (17)$$

Substituting (16) into (4), we find

$$\frac{dS}{dt} = \frac{k}{2} \sum_{i,j} P_{ij} e^{G^0/kT} e^{-\bar{N}} e^{-E_i/kT} \left( \frac{E_j - E_i}{kT} \right) \left( 1 - \frac{g_i}{g_j} e^{2\frac{E_i - E_j}{kT}} \right). \quad (18)$$

Neglecting the small terms and replacing in (18) the sum with the integral we get:

$$P = \frac{2\Delta S}{k\tau} \exp \left\{ -\frac{\Delta E_m - G^0/N}{kT} \right\}, \quad (19)$$

where  $\Delta S$  is the change in entropy in the dissipative process;  $\Delta E_m$  is the average value of the energy of the ground state of defects;  $\tau$  is the relaxation time.

### Friction and entropy

In the dissipative process the friction  $f_m$  changes completely. Then follows from (19):

$$f_m = \frac{2}{k} \left( \frac{dS}{dt} \right) \exp \left\{ -\frac{\Delta E_m - G^0/N}{kT} \right\} = \frac{2\Delta S}{k\tau} \exp \left\{ -\frac{\Delta E_m - G^0/N}{kT} \right\}. \quad (20)$$

Nonequilibrium processes in the tribosystem can lead to a decrease in the production of entropy and, consequently, the intensity of wear and stably proceed with the formation of dissipative structures during

self-organization [19–23]. The process of self-organization can begin only after the system has passed through instability [19–23]. The system can lose stability with a negative excess production of entropy (according to the Lyapunov function [21]). For the tribosystem to self-organize, more than one independent process should take place in it [19–23]. The paper [24] gives the formula:

$$f_e = \left( \frac{dS}{dt} \right)_e = X_m \cdot \rho_m \cdot W(X_m), \quad (21)$$

where  $X_m$  is the thermodynamic force that causes mass transfer (stress gradients or chemical potentials, respectively, for deformation or diffusion),  $\rho_m$  is the average density of the substance involved in mass transfer,  $W(X_m)$  is the average mass transfer rate, depending on  $X_m$ , will increase with  $X_m$ .

From (21) follows:

$$X_m \cdot \rho_m \cdot W(X_m) \rightarrow 0. \quad (22)$$

From (22), it follows that the first term in (20) can reach  $f_e = 10^{-3} - 10^{-4}$  by using  $X_m$ ,  $\rho_m$  and  $W(X_m)$  in the mode of using special lubricants.

According to the method of incorporation into the lubrication system, the filters are divided into series and parallel, according to the dispersity of the filtrate filtered out — into filters of coarse and fine cleaning. Detailed classification of oil filters is shown in Figure 1.

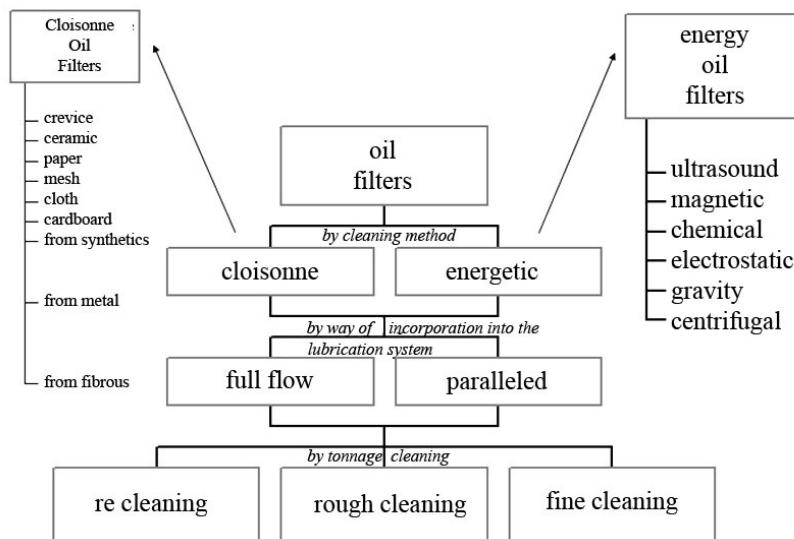


Figure 1. Classification of oil filters [25]

In space, the effect of oil filters is absent. It uses solid lubricants (TSP) which include substances applied to the surface of parts in the form of films that have a shear strength that is significantly lower than that of the part material. Films of sulfides ( $\text{MoS}_2$ ,  $\text{WS}_2$ ,  $\text{PbS}$ ,  $\text{NbS}_2$ , etc.), selenides ( $\text{MoS}_2$ ,  $\text{WSe}_2$ ,  $\text{NaSe}_2$ , etc.), tellurides ( $\text{MoTe}_2$ ,  $\text{WTe}_2$ ,  $\text{NbTe}_2$ ,  $\text{TaTe}_2$ ), chlorides ( $\text{CoCl}$ ,  $\text{PbCl}_2$ ,  $\text{CdCl}_2$ ,  $\text{CuCl}$ ), fluorides ( $\text{AlF}_3$ ,  $\text{CaF}_2$ ,  $\text{BaF}_2$ ,  $\text{MgF}_3$ ), iodides ( $\text{CaI}_2$ ,  $\text{PbI}_2$ ,  $\text{BiI}_3$ ), metal oxides ( $\text{PbO}$ ,  $\text{Bi}_2\text{O}_3$ ,  $\text{PbO} + \text{SiO}_2$ ), graphite, boron nitride, etc. In the Table 1 the density, coefficient of friction and the operating temperature of some sulfides, selenides and metal tellurides are shown.

Table 1  
Characteristics of TSP [25]

Connections	Density, g/sm <sup>3</sup>	Coefficient friction	Maximum permissible operating temperature, °C	
			vacuum	air
1	2	3	4	5
$\text{Bi}_2\text{S}_3$	7.40	0.07–0.14	500	—
$\text{CuS}$	4.28	0.17–0.41	520	—
$\text{Cu}_2\text{S}$	5.60	0.18	450	—
$\text{MoS}_2$	4.8–5.16	0.02–0.3	1100	400
$\text{NbS}$	5.9–6.0	0.04	1300	400

Continuation of Table 1

1	2	3	4	5
PbS	7.5–7.59	0.37	410	500
TaS <sub>2</sub>	7.16	0.06	1500	—
WS <sub>2</sub>	7.5–7.63	0.03–1.6	1400	500
NbSe <sub>2</sub>	6.25	0.06–0.17	1350	350
MoSe <sub>2</sub>	6.90	0.03–0.22	1350	400
WS <sub>2</sub>	9.0	0.02–0.18	1350	350
MoTe <sub>2</sub>	7.8	0.10–0.34	1240	400
WTe <sub>2</sub>	9.44	0.27–0.49	1020	—
VTe <sub>2</sub>	—	0.22	450	500

*Friction and energies of the ground state of defects*

We estimate the contribution of the energy of the ground state of defects:

$$f_a \approx \exp\left\{-\frac{\Delta E_m}{kT}\right\}, \quad (23)$$

Based on the concepts of the kinetic thermofluctuation concept of solid strength developed by S.N. Zhurkov and co-workers [26, 27], in [28] proposed physical and computational models of wear of friction surfaces.

The proposed models are based on a fundamental pattern [26], which relates stress, absolute temperature, and durability:

$$t = \tau_0 \exp[(U_0 - \gamma\sigma)/kT], \quad (24)$$

where  $\sigma$  is the breaking stress:

$$\sigma = (1/\gamma)(U_0 - kT \ln(t/\tau_0)), \quad (25)$$

where  $t$  is the time to failure (durability) of the sample under load, s;  $\tau_0 = (10^{-12} \dots 10^{-13})$ , c — the oscillation period of atoms in a solid;  $U_0$  is the interatomic bond energy, J;  $T$  is the thermodynamic temperature, K;  $k = 1.38 \cdot 10^{-23}$  J/K is the Boltzmann constant;  $\gamma$  — parameter (fitting), having the dimension of volume, m<sup>3</sup>; activation energy of destruction  $\Delta U = (U_0 - \gamma\sigma)$ .

For the energy of the ground state of defects in [28], it was found that for typical defects on metal surfaces it is  $\Delta E \approx 2–5$  J. Then for  $kT \approx 0.3$  eV, we get  $\exp(-33) \approx 10^{-15} \approx 1$  and  $f_a = 1$ .

*Friction, roughness, fracture energy of defects and Gibbs energy*

Remain the third term in equation (19), which can be brought to mind:

$$f_G = \frac{A_r}{G^0} \cdot N, \quad (26)$$

where  $A_r$  is the destruction energy of defects;  $N$  — contact roughness;  $G^0$  — Gibbs energy.

Nowadays, in mechanical engineering, high-alloyed, including stainless steels are often used for critical machine parts, the processing of which is complicated by traditional mechanical methods. Particularly problematic is obtaining accurate small holes. For this case, the most effective is the use of the EDM method of piercing a hole [29].

The mathematical model describing the dependence when using water as a working fluid has the form:

$$N = 0,11 \cdot E^{0,66} \cdot F^{-0,26}, \quad (27)$$

where  $E$  is the pulse energy, mJ;  $F$  is the pulse frequency, kHz.

GOST defined six types of directions of irregularities that are selected when recording a profilogram: parallel, perpendicular, intersecting, arbitrary, circular and radial. When removing profilograms from friction surfaces, the first two types are most often used [30]. The concepts parallel and perpendicular are related to the direction of friction during the operation of the tribo-conjugation.

The arithmetic average of the  $R_a$  profile is determined by the arithmetic average of the absolute values of the profile deviations within the base length:

$$R_a = \frac{1}{\ell} \int_0^\ell |y| dx \quad \text{or} \quad R_a = \frac{1}{n} \sum_{i=1}^n |y_i|, \quad (28)$$

where:  $\ell$  is the base length,  $n$  is the number of selected profile points on the base length.

The height of the profile of irregularities at ten points Rz is defined as the sum of the average absolute values of the heights of the five largest profile protrusions and the depths of the five largest cavities of the profile within the base length:

$$R_z = \frac{\sum_{i=1}^5 |ypmi| + \sum_{i=1}^5 |yvmi|}{5}, \quad (29)$$

where  $ypmi$  is the height of the  $i$ -th largest profile protrusion;  $yvmi$  is the depth of the  $i$ -th largest depression of the profile.

The greatest height  $R_{\max}$  of the profile is the distance between the line of the protrusions of the profile and the line of the cavities of the profile within the base length. The average step of the irregularities of the profile  $S_m$  is the average step of the local projections of the profile within the base length. The average pitch of the local projections of the profile S is the average pitch of the local projections of the profile within the base length. The relative profile reference length  $t_p$  is the ratio of the profile length to the base length and is defined as

$$t_p = \frac{\sum_{i=1}^n b_i}{\ell}. \quad (30)$$

The cross section level of the profile p is the distance between the profile protrusion line and the line intersecting the profile equidistantly of the profile protrusion line. When studying profilograms, one should take into account that the vertical and horizontal scales of increasing the surface profile are different and real irregularities have a much smaller angle of inclination than it looks on profilograms.

Consider in (26) —  $A_r$  is the damage energy of defects [31, 32]. Work  $A(J)$ , spent on the destruction of contacts is proportional to the newly formed surface of the particles of the destroyed product:

$$A_r = \gamma \Delta S = K_R D^2, \quad (31)$$

where  $\gamma$  is the temporary compressive resistance ( $\text{Nm/m}^2$ ),  $\Delta S$  is the area of the newly formed surface ( $\text{m}^2$ ),  $K_R$  is the proportionality coefficient ( $\text{Nm/m}^2$ );  $D$  is the characteristic size of the contact (m).

Equation (31) corresponds to the Rittinger hypothesis [33]. If during the destruction of a cubic-shaped contact, energy is expended mainly on volume deformation, then in this case the work performed is directly proportional to the change in its initial volume and is determined by the Kirpichev-Kik formula [34, 35]:

$$A_r = K \Delta V = K_K D^3, \quad (32)$$

where  $K$  and  $K_K$  are the proportionality coefficients ( $\text{Nm/m}^3$ );  $\Delta V$  is the deformed volume ( $\text{m}^3$ ).

Rebinder [36] combined both hypotheses and in this case the total work of destruction is equal to:

$$A = K_R D^2 + K_K D^3. \quad (33)$$

According to Bond's hypothesis [37], the total work of failure is proportional to the geometric mean between the volume and the contact surface area:

$$A = K_B \sqrt{D^2 D^3} = K_B D^{2.5}. \quad (34)$$

All formulas (31)–(34) differ in proportionality coefficients and indicators of the degree of contact diameter. This indicator is related to the dimension of the contact surface, which in the modern sense has a fractal nature [38] (Fig. 2).

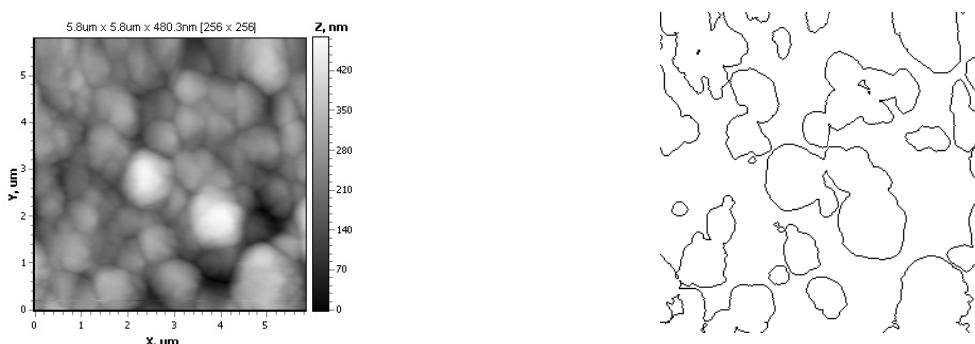


Figure 2. AFM image and the fractal structure of the Zn-Al coating ( $D = 1.81$ )

For pure metals, equation (26) is converted to:

$$f_G = \frac{A_r}{\mu} \cdot N, \quad (35)$$

where  $\mu$  is the chemical potential of the metal.

For pure metals, the chemical potential coincides with the Fermi energy. In Table 2 these values are given for some metals [39].

Table 2

**Fermi energy of some metals [39]**

Metal	Fermi energy, eV	Metal	Fermi energy, eV
Cu	7,00	Zn	9,39
Ag	5,48	Al	11,63
Au	5,51	Pb	9,37
Mg	7,13	Sn	10,03
Sr	3,95	Ca	4,68

From equation (35) it follows that the coefficient of dry friction is the smaller, the greater the chemical potential (Fermi energy). Table 3 shows the dry friction coefficients for pairs of the most common materials [40]. In general, the correlation between the Table 2 and 3 is observed.

Table 3

**Dry friction coefficients for homogeneous pairs of the most common materials [40]**

Material combinations		Coefficient dry friction
Al	Al	1.05–1.35
Cu	Cu	1.0
Steel	Steel	0.8
Fe	Fe	1.0
Cd	Cd	0.5
Cr	Cr	0.41
Graphite	Graphite	0.5–0.8
Mg	Mg	0.6
Ni	Ni	0.7–1.1
Pt	Pt	1.2
Ag	Ag	1.4
Zn	Zn	0.6

If now we bring two dissimilar metals into contact, a contact potential difference  $V_K = \mu$  will arise. Table 4 shows the values of dry friction coefficients for dissimilar pairs of the most common materials [41].

Table 4

**The work function of electrons from some metals [41]**

Metal	$W$ , eV
Al	4,25
W	4,54
Fe	4,31
Cu	4,4
Ni	4,5
Sn	4,39
Pt	5,32
Ag	4,25
Zn	4,54

Table 5 shows the values of dry friction coefficients for dissimilar pairs of the most common materials [42]. Comparison of Tables 4 and 5 shows a significant decrease in the coefficient of dry friction in the case of dissimilar metals in friction pairs.

Table 5

**Dry friction coefficients for dissimilar pairs of the most common materials [42]**

Material combinations		Coefficient dry friction
Al	Low carbon steel	0.61
Brass	Steel	0.35
Cd	Cr	0.41
Cu	Low carbon steel	0.53
Ni	Low carbon steel	0.64
Wolfram carbide	Cu	0.35

The practical use of knowledge about the fractal structure of a solid surface has not yet been studied in detail and it is not yet known in what phenomena it will be decisive (Fig. 2).

Let us consider the well-known phenomenon of cold emission of electrons from a metal under the action of an external electric field, mainly due to the quantum tunneling effect. As is known, the current of cold emission is described by the expression:

$$j = j_0 \exp(E_0 / E), \quad (36)$$

where  $E$  is the external field strength,  $E_0$  is the electric field of the metal surface:

$$E_0 = \frac{4\sqrt{2m}}{3e\hbar} (A - T_e)^{3/2}, \quad (37)$$

$A$  is the electron work function,  $T_e$  is its kinetic energy.

Above, for the dependence of the electrical conductivity of the film on its thickness  $h$ , we obtained the following expression:

$$\sigma = \sigma_0 \left(1 - \frac{d}{h}\right), \quad (38)$$

where  $d$  is some critical size of the film, starting from which the bulk properties «disappear». After the transformations, we get:

$$\begin{aligned} j &= \sigma E, \quad j_0 = \sigma_0 E_0, \\ \sigma E &= \sigma_0 E_0 \exp(E / E_0), \\ \sigma_0 \left(1 - \frac{d}{h}\right) E &= \sigma_0 E_0 \exp(E_0 / E), \\ E &= E_0 \exp(E_0 / E) \left(1 - \frac{d}{h}\right) \approx E_0 \left(1 - \frac{E_0}{E}\right) \left(1 - \frac{d}{h}\right). \end{aligned} \quad (39)$$

Denoting  $E_0/E = z$ ,  $1 - d/h = k$ , where  $z$  are complex numbers, and  $k$  is real, we get the famous iterative Mandelbrot equation:

$$z = z^2 + k.$$

The solution algorithm is quite simple. Iterations are performed for each starting point from a rectangular or square area — a subset of the complex plane. The iteration process continues until  $z$  goes beyond the circumference of radius 2 whose center lies at (0,0) (this means that the attractor of the dynamical system is infinite) or after a sufficiently large number of iterations (for example, 200–500)  $z$  converge to some point of the circle. Depending on the number of iterations during which  $z$  remained inside the circle, you can set the color of point C (if  $z$  remains inside the circle for a sufficiently large number of iterations, the iteration process stops and this raster point turns black).

The following is important for us: the given example shows the fractal structure of the metal surface. Similar patterns are manifested in thermal emission, exoemission of electrons from metals and semiconductors. All this indicates that the fractal structure of the surface of many solids is a characteristic property of them.

### Conclusion

Summarizing the study, we can draw the following main conclusions:

- non-equilibrium processes in the tribosystem can lead to a decrease in the production of entropy and, consequently, the intensity of wear and stably proceed with the formation of dissipative structures during self-organization;
- it is obtained that the first term from (20) can reach  $f_e = 10^{-3}-10^{-4}$  due to the use of  $X_m$ ,  $\rho m$  and  $W(X_m)$  in the mode of using special lubricants;
- in space, the effect of oil filters is absent. It uses solid lubricants (TSP) which include substances applied to the surface of parts in the form of films, having a shear strength much lower than that of the part material;
- an expression for the friction coefficient is obtained, taking into account the surface geometry and its physical properties;
- it was concluded that homogeneous vapors of metals should not be used in friction antifriction pairs;
- it is shown that a decrease in the dry friction coefficient is possible due to a decrease in surface energy. In this case, friction pairs are used, in which the electron work function differs significantly. This leads to a difference in the contact potential difference and, accordingly, to a decrease in the surface energy;
- it is shown that the friction coefficient depends on the fractal structure of the surface of many solids and this is their characteristic property.

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## Трибологиядағы диссипативтің үрдістер

Макалада диссипативті үрдіс қатты денелердегі сыртқы үйкелістің табиги негізі ретінде қарастырылды. Трибожүйенің тепе-тен емес үрдістері энтропия өндірісінің төмендеуіне әкеліп соғуы мүмкін, демек, тозу қарқындылығы және өзін-өзі үйімдастыру кезінде диссипативті құрылымдардың қалыптасуымен тұрақты түрде журуі мүмкіндігі көрсетілген. Арнайы майлауларды пайдалану режимінде  $X_m$ ,  $\rho_m$  және  $W(X_m)$  арқылы алғашкы мүшесі  $f_e = 10^{-3} - 10^{-4}$  жетуі мүмкін екендігі аныкталды. Farышта май сүзгілерінің асері жок. Мұнда, бөлшек материалдарына қарағанда, ығыспалы беріктілігі анағұрлым төмөнірек бөлшектердің бетіне қабықша түрінде жағылған заттар түрінде катты майлау материалдары пайдаланды. Беттің геометриясын және оның физикалық қасиеттерін ескеретін үйкеліс коэффициентіне арналған өрнек алынды. Антифрикционды жұптарда біртекті металл жұптардың үйкеліс колданылмауы керек деген қорытынды жасалды. Беттік энергияның төмендеуіне байланысты құрғақ үйкеліс коэффициентінің төмендеуі мүмкін екенін көрсетті. Бұл жағдайда электрондардың шығу жұмысы айтарлықтай ерекшеленетін үйкеліс жұбы колданылды. Бұл контактілік потенциалдар айрымының айырмашылығына және тиісінше беттік энергияның төмендеуіне әкеледі. Үйкеліс коэффициенті қатты заттардың беттік фракталдық құрылымына тәуелді және бұл оларға тән қасиет екенін көрсетті.

*Kielt сездер:* диссипативті үрдіс, үйкеліс, тозу, бет, энтропия, фрактал.

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**Диссиликативные процессы в трибологии**

В статье диссиликативный процесс рассмотрен как основа природы внешнего трения в твердых телах. Показано, что неравновесные процессы в трибосистеме могут приводить к снижению производства энтропии и, следовательно, интенсивности изнашивания и устойчиво протекать с образованием диссиликативных структур при самоорганизации. Получено, что первый член может достигать  $f_e = 10^{-3} - 10^{-4}$  за счет использования  $X_m$ ,  $\rho_m$  и  $W(X_m)$  в режиме использования специальных смазок. В космосе эффект масляных фильтров отсутствует. Здесь используют твердосмазочные материалы, к которым относятся вещества, наносимые на поверхность деталей в виде пленок, имеющие сдвиговую прочность значительно меньшую, чем у материала детали. Получено выражение для коэффициента трения, учитывающее геометрию поверхности и ее физические свойства. Сделан вывод о том, что нельзя использовать в антифрикционных парах трения однородные пары металлов. Показано, что уменьшение коэффициента сухого трения возможно за счет уменьшения поверхностной энергии. При этом используются пары трения, у которых значительно различается работа выхода электронов. Это приводит к различию в контактной разности потенциалов и, соответственно, к уменьшению поверхностной энергии. Показано, что коэффициент трения зависит от фрактальной структуры поверхности многих твердых тел и это является их характерным свойством.

*Ключевые слова:* диссиликативный процесс, трение, износ, поверхность, энтропия, фрактал.

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