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Laws of reflection and refraction of TE and TM polarization waves on the border of rhombic crystals

The article analytically solves the problem of reflection and refraction of electromagnetic plane waves of different polarization at the boundary of anisotropic half-spaces of rhombic symmetry. Based on the matrix method, the angles of refraction of electromagnetic waves of different polarization, the amplitudes of the reflected and refracted waves, the angles that determine the direction of the group velocities and vectors of the flow of electromagnetic energy, the magnitudes of the flows of electromagnetic energy and their components depending on the direction of the wave vector of the incident wave are determined. The determination of the angles of total internal reflection and the refractive index of electromagnetic waves of different polarization is considered. A significantly different dependence of the kinematic and energy characteristics of electromagnetic waves of different polarization on the anisotropy of the magnetic and dielectric tensors is shown. For electromagnetic waves, the polarization of which is determined by the component of the electric tension vector perpendicular to the wave propagation plane (TE wave), the characteristics of the reflected and refracted waves, the velocity indicatrix, the propagation angles, etc. determined primarily by the components of the magnetic permeability tensor. In the case of electromagnetic waves, when the polarization is determined by the component of the magnetic field (TM wave), perpendicular to the plane of wave propagation, all characteristics depend mainly on the components of the dielectric constant. The validity of the Fresnel formulas for determining the coefficients of reflected and refracted waves at the boundary of anisotropic media of rhombic symmetry is shown. However, in this case, the components of the wave vectors included in the Fresnel formulas are determined by their indicatrices. In addition, it is necessary to take into account the dependence of these components on the angle of refraction in the second medium.

Key words: anisotropy, rhombic symmetry, electromagnetic waves, reflection, refraction, wave refraction angles, phase and group velocities, density vector.

Introduction

Wave processes in anisotropic media are qualitatively different from the patterns of wave propagation in isotropic media.

The magnitude of the phase velocity of the waves in an anisotropic medium depends on the direction of propagation. The transfer of electromagnetic energy is determined by the group velocity of the wave. The directions of the vectors of phase and group velocities and their values do not coincide [1–3].

The influence of the anisotropy of dielectric media on electromagnetic wave processes is described by the anisotropy of the dielectric and magnetic tensors.

Traditionally, in connection with the study of the optical range and optically transparent media, the physical processes of propagation, scattering of electromagnetic waves in anisotropic media are considered, considering the magnetic permeability equal to unity. Only the anisotropy of the dielectric constant tensor is taken into account [1–5].

Theoretical and experimental work is currently being intensively carried out related to piezomagnetic, magnetostrictive, and magnetoelectric effects and their practical application. The widespread use and importance of materials with piezoelectric effects for acoustoelectronics is well known [6–11].

Moreover, the study of the laws of electromagnetic wave processes taking into account the anisotropy of the dielectric and magnetic permeability tensors shows their significantly different influence on the propagation of TE and TM polarization waves [12–14].

Anisotropy of crystals, the manifestation of the anisotropic properties of media under various physical and mechanical conditions, the creation of artificial materials necessitates a detailed study of the laws of electromagnetic wave processes in media of different symmetries, taking into account the anisotropy of the dielectric and magnetic permeability parameters.

The aim of the work is to construct an analytical solution to the problems of reflection, refraction of TE and TM polarized waves at the boundary of anisotropic half-spaces of rhombic symmetry, determination of

the angle of refraction of waves, directions of group velocities, amplitudes of reflected and refracted waves, calculation of the components of the vectors of electromagnetic energy flux and group velocity depending on the direction wave vector of the incident wave.

1. Matrix equation

The Maxwell initial equations

$$\begin{aligned} \operatorname{rot} \vec{E} &= -\frac{\partial \vec{B}}{\partial t}; \\ \operatorname{rot} \vec{H} &= \frac{\partial \vec{D}}{\partial t}; \\ \operatorname{div} \vec{D} &= 0; \operatorname{div} \vec{B} = 0; \vec{j} = 0, \rho = 0; \\ D_i &= \varepsilon_{ij} E_j; B_i = \mu_{ij} H_j \end{aligned} \quad (1)$$

and tensors of dielectric and magnetic permeability of rhombic anisotropy:

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{pmatrix}; \hat{\mu} = \begin{pmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{pmatrix}. \quad (2)$$

Absolute permeability of ε_0, μ_0 contained (included) $\varepsilon_{ij}, \mu_{ij}$.

Based on the presentation of decisions:

$$f(x, y, z, t) = \bar{f}(z) e^{i\omega t - ik_x x - ik_y y} \quad (3)$$

the system of equations (1) is reduced to the matrix equation [12–18]

$$\frac{d\vec{W}}{dz} = B\vec{W}; \vec{W} = (E_y, H_x, H_y, E_x)^t \quad (4)$$

is the transpose sign of the row vector into the column vector.

The matrix of coefficients B has the structure:

$$B = \begin{pmatrix} 0 & b_{12} & b_{13} & 0 \\ b_{21} & 0 & 0 & b_{24} \\ -b_{24} & 0 & 0 & b_{34} \\ 0 & -b_{13} & b_{43} & 0 \end{pmatrix} \quad (5)$$

with elements b_{ij} :

$$\begin{aligned} b_{12} &= i\omega\mu_x - i\frac{k_y^2}{\omega\varepsilon_z}, \quad b_{13} = i\frac{k_x k_y}{\omega\varepsilon_z}; \\ b_{21} &= i\omega\varepsilon_y - i\frac{k_x^2}{\omega\mu_z}, \quad b_{24} = i\frac{k_x k_y}{\omega\mu_z}; \\ b_{34} &= -i\omega\varepsilon_x + i\frac{k_y^2}{\omega\mu_z}, \quad b_{42} = -i\omega\mu_y + i\frac{k_x^2}{\omega\varepsilon_z}. \end{aligned} \quad (6)$$

In the framework of the matrix method in the case of homogeneous media, the equations of the surface of wave vectors can be determined from the condition [12–14, 16].

$$\det[B^2 + k_z^2 I] = 0.$$

Here I is the identity matrix.

2. Waves of TE polarization.

2.1 For waves of TE polarization in the coordinate plane, matrix equation follows from (4–6)

$$\begin{aligned} \frac{d\vec{W}}{dz} &= B\vec{W}; \\ B &= \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix}; \end{aligned}$$

$$\begin{aligned}\vec{W} &= (E_y, H_x)^t; \\ b_{12} &= iw\mu_x; \\ b_{21} &= iw\left(\epsilon_y - \frac{k_x^2}{w^2\mu_z}\right).\end{aligned}\quad (7)$$

Equation (7) has an analytical solution. In the form of a matrix, it has the form of:

$$T = I \cos k_z z + \frac{B}{k_z} \sin k_z z; \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (8)$$

To solve the problems of reflection refraction, it is converted to the form:

$$T = \frac{1}{2} \left(I + \frac{B}{ik_z} e^{ik_0 z} \right) + \frac{1}{2} \left(I - \frac{B}{ik_z} \right) e^{-ik_z z}. \quad (9)$$

The first term describes the backward waves, the second: the direct waves in (8) and (9) omit the factor e^{iwt} .

The condition of continuity of the incident wave fields (\vec{W}_0), reflected (\vec{W}_R) and refracted W_T waves at the interface of ($z = 0$), based (9) has the form of:

$$\left(\frac{1}{2}I - R_0\right)\vec{W}_0 + \left(\frac{1}{2}I + R_0\right)\vec{W}_R = \left(\frac{1}{2}I - R\right)\vec{W}_T; \quad (10)$$

Given the continuity of decisions:

$$\vec{W}_0 + \vec{W}_R = \vec{W}_T \text{ at } z = 0 \quad (11)$$

from (10) it can be taken:

$$(R_0 + R)\vec{W}_R = (R_0 - R)\vec{W}_0. \quad (12)$$

Boundary Matrixes of R_0 and R have the form of:

$$R_0 = \frac{1}{2ik_{0z}} B_0; \quad R = \frac{1}{2ik_z} B. \quad (13)$$

The field of reflected waves is determined based on (12):

$$\vec{W}_R = (R_0 + R)^{-1} (R_0 - R)\vec{W}_0; \quad (14)$$

field of refracted waves \vec{W}_T follows from (11)

$$\vec{W}_T = [I + (R_0 + R)^{-1} (R_0 - R)]\vec{W}_0. \quad (15)$$

By introducing the following notation:

$$(R_0 \pm R) = \frac{1}{2i} \begin{pmatrix} 0 & r_{12}^\pm \\ r_{21}^\pm & 0 \end{pmatrix}; \quad (16)$$

$$r_{12}^\pm = \frac{b_{12}^0}{k_{0z}} \pm \frac{b_{12}}{k_z}; \quad r_{21}^\pm = \frac{b_{21}^0}{k_{0z}} \pm \frac{b_{21}}{k_z}$$

after calculations:

$$(R_0 + R)^{-1} (R_0 - R) = \begin{pmatrix} \frac{r_{21}^-}{r_{21}^+} & 0 \\ 0 & \frac{r_{12}^-}{r_{12}^+} \end{pmatrix}. \quad (17)$$

2.2 Amplitudes of reflected and refracted waves of TE polarization.

From (17) it follows:

$$\frac{r_{21}^-}{r_{21}^+} = \frac{b_{21}^0 k_z - b_{21} k_{0z}}{b_{21}^0 k_z + b_{21} k_{0z}} = \frac{\mu_x k_{0z} - k_z \mu_{x0}}{\mu_x k_{0z} + k_z \mu_{x0}}, \quad (18)$$

it is taken into account that:

$$b_{12} b_{21} + k_z^2 = 0. \quad (19)$$

It is followed from (17).

Similarly:

$$\frac{r_{12}^-}{r_{12}^+} = \frac{b_{12}^0 k_z - k_{0z} b_{12}}{b_{12}^0 k_z + k_{0z} b_{12}} = \frac{\mu_{x0} k_z - \mu_x k_{0z}}{\mu_{x0} k_z + \mu_x k_{0z}}. \quad (20)$$

Amplitudes of reflected waves:

$$E_{yR} = \frac{r_{21}^-}{r_{21}^+} E_{y0}; \quad \frac{H_{xR}}{H_{x0}} = -\frac{E_{yR}}{E_{y0}}; \quad (21)$$

$$H_{x0} = -\frac{k_{0z}}{\omega \mu_{x0}} E_{y0}; \quad H_{xR} = \frac{k_{z0}}{\omega \mu_{x0}} E_{yR};$$

$$H_{z0} = \frac{k_{x0}}{\omega \mu_{z0}} E_{y0}; \quad H_{zR} = \frac{k_{z0}}{\omega \mu_{x0}} E_{yR}. \quad (22)$$

For a refracted wave:

$$E_{yT} = E_{y0} + E_{yR} = \frac{2\mu_x k_{0z}}{\mu_x k_{0z} + \mu_{x0} k_z} E_{y0}; \quad (23)$$

$$H_{xT} = -\frac{k_z}{\omega \mu_x} E_{yT}; \quad H_z = \frac{k_x}{\omega \mu_z} E_{yT}.$$

2.3 The angle of refraction of the TE wave. Components of wave vectors.

Formulas (18)–(23) have the form of Fresnel formulas [1–3]. The difference lies in obtaining values k_{0z}, k_z, k_x, k_{x0} based on indicatrices of wave vectors in an anisotropic medium [12–14].

From (19) follows the equation of the indicatrix k

$$k^2 = \frac{\omega^2 \varepsilon_y \mu_x \mu_z}{\mu_z \cos^2 \theta + \mu_x \sin^2 \theta}. \quad (24)$$

At the media boundary, the following conditions must be met:

$$k_{0x} = k_x \Rightarrow k_0 \sin \theta_0 = k \sin \theta, \quad (25)$$

θ_0 — determines the direction of the \vec{k}_0 incident wave vector; θ — wave refraction angle (\vec{k}).

Condition (25) based on (24) gives:

$$\frac{\varepsilon_{y0} \mu_{x0} \mu_{z0} \sin^2 \theta_0}{\mu_{z0} \cos^2 \theta_0 + \mu_{x0} \sin^2 \theta_0} = \frac{\varepsilon_y \mu_x \mu_z \sin^2 \theta}{\mu_z \cos^2 \theta + \mu_x \sin^2 \theta}. \quad (26)$$

From (26) follows:

$$\sin^2 \theta = \frac{\varepsilon_{y0} \mu_{x0} \mu_{z0} \mu_z \sin^2 \theta_0}{\varepsilon_y \mu_x \mu_z \mu_{z0} - [\varepsilon_y \mu_x \mu_z (\mu_{x0} - \mu_{z0}) + \varepsilon_{y0} \mu_{x0} \mu_{z0} (\mu_x - \mu_z)] \sin^2 \theta_0}. \quad (27)$$

Formula (27) determines the angle θ of the refracted wave θ_0 as a function of rhombic symmetry at the crystal boundary. If the first medium is isotropic, then: ($\mu_{x0} = \mu_{z0}$).

$$\sin^2 \theta = \frac{\mu_z \varepsilon_{y0} \mu_{x0} \sin^2 \theta_0}{\mu_z \mu_x \varepsilon_y - \varepsilon_{y0} \mu_{x0} (\mu_x - \mu_z) \sin^2 \theta_0} \quad (28)$$

With the isotropy of both media:

$$\sin^2 \theta = \frac{\varepsilon_{y0} \mu_{x0}}{\mu_x \varepsilon_y} \sin^2 \theta_0. \quad (29)$$

The angle of total internal reflection can be determined by setting either

$$\sin^2 \theta = 1, \text{ or } \sin^2 \theta_0 = 1.$$

If it is $\sin^2 \theta = 1$, then the full reflection of the wave propagating in the first medium will be subject to:

$$\sin^2 \theta_0 = \frac{\varepsilon_y \mu_z \mu_{z0}}{\varepsilon_{y0} \mu_{x0} \mu_{z0} + \varepsilon_y \mu_z (\mu_{x0} - \mu_{z0})}. \quad (30)$$

The quantities k_{0z} and k_z included in the reflection and refraction formulas (18)–(23) are determined on the basis of (24):

$$k_{0z}^2 = \frac{\omega^2 \varepsilon_{y0} \mu_{x0} \mu_{z0} \cos^2 \theta_0}{\mu_{z0} \cos^2 \theta_0 + \mu_{x0} \sin^2 \theta_0} = k_0^2 \cos^2 \theta_0; \quad (31)$$

$$k_z^2 = \frac{\omega^2 \varepsilon_y \mu_x \mu_z \cos^2 \theta}{\mu_z \cos^2 \theta + \mu_x \sin^2 \theta} = k^2 \cos^2 \theta. \quad (32)$$

In (32), it must be borne in mind that the angle θ is determined by expressions (26), (27).

2.4 Energy ratios.

When TE waves propagate in the xoz plane the electromagnetic field in the medium has non-zero components:

$$E_y, H_x, H_z. \quad (33)$$

The difference from zero H_z is a consequence of electromagnetic induction and is determined from the equation:

$$\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t}. \quad (34)$$

Components H_x and H_z related E_y ratios:

$$H_x = -\frac{k_z}{\omega \mu_x} E_y, \quad H_z = -\frac{k_x}{\omega \mu_z} E_y. \quad (35)$$

The electromagnetic energy flux vector has components:

$$\vec{S} = [\vec{E} \times \vec{H}] = \vec{i} S_x + \vec{k} S_z = \vec{i} E_y H_z - \vec{k} E_y H_x,$$

with (35), it must be taken:

$$S_x = \frac{k_x}{\omega \mu_x \varepsilon_y} \varepsilon_y E_y^2; \quad S_z = \frac{k_z}{\omega \mu_x \varepsilon_y} \varepsilon_y E_y^2. \quad (36)$$

The components S_x , S_z for the incident, reflected and refracted waves are:

$$S_{z0} = \frac{k_{z0}}{\omega \mu_{x0}} E_{y0}^2; \quad S_{x0} = \frac{k_{x0}}{\omega \mu_{z0}} E_{y0}^2; \quad (37)$$

$$S_{zR} = \frac{k_{z0}}{\omega \mu_{x0}} E_{yR}^2; \quad S_{xR} = \frac{k_{x0}}{\omega \mu_{z0}} E_{yR}^2; \quad (38)$$

$$S_{zT} = \frac{k_z}{\omega \mu_x} E_{yT}^2; \quad S_{xT} = \frac{k_x}{\omega \mu_z} E_{yT}^2. \quad (39)$$

The direction of the vector \vec{S} coinciding with the direction of the group velocity [4, 5] is determined by the ratio:

$$\frac{S_x}{S_z} = \text{tg } \beta = \frac{\mu_x}{\mu_z} \text{tg } \theta. \quad (40)$$

Angle θ determines the direction of the wave vector \vec{k} , angle β the direction of the group velocity vector.

The full stream is:

$$S = \sqrt{S_x^2 + S_z^2} = V_g \varepsilon_y E_y^2. \quad (41)$$

Here is the group velocity V_g :

$$V_g^2 = \left(\frac{k_x}{\omega \mu_x \varepsilon_y} \right)^2 + \left(\frac{k_z}{\omega \mu_x \varepsilon_y} \right)^2 = \frac{1}{\varepsilon_y \mu_x \mu_z} \frac{\mu_z^2 + \mu_x^2 \text{tg}^2 \theta}{\mu_z + \mu_x \text{tg}^2 \theta}. \quad (42)$$

The phase velocity of the TE polarization wave is determined from (24)

$$V_f^2 = \frac{1}{\varepsilon_y \mu_x \mu_z} (\mu_z \cos^2 \theta + \mu_x \sin^2 \theta). \quad (43)$$

For the refractive index:

$$n^2 = \frac{c^2}{V_f^2} = \frac{c^2 \epsilon_y \mu_x \mu_z}{\mu_z \cos^2 \theta + \mu_x \sin^2 \theta}, \quad (44)$$

c — speed of electromagnetic waves in vacuum.

In an anisotropic medium $n = n(\theta)$.

From (37)–(39) it follows that for S_{zR} , S_{z0} , S_{zT} , the relation:

$$\frac{S_{zR}}{S_{z0}} + \frac{S_{zT}}{S_{z0}} = 1. \quad (45)$$

Insofar as:

$$\frac{S_{zR}}{S_{z0}} = \frac{E_{yR}^2}{E_{y0}^2} = \left(\frac{\mu_x k_{0z} - k_z \mu_{x0}}{\mu_x k_{0z} + k_z \mu_{x0}} \right)^2; \quad (46)$$

$$\frac{S_{zT}}{S_{z0}} = \frac{k_z \mu_{x0}}{\mu_x k_{z0}} \left(\frac{2\mu_x k_{0z}}{\mu_x k_{0z} + k_z \mu_{x0}} \right)^2. \quad (47)$$

The sum of the right-hand sides of equalities (46), (47) is equal to one.

Thus: formulas (18)–(23), (31), (32) determine the amplitudes of the reflected and refracted waves; based on (24)–(27), the angles of refraction are determined at the boundary of anisotropic media; formulas (36)–(39) determine the flows of electromagnetic energy; (42), (43) give the values of the group and phase velocities; (40) determines the direction of the group velocity vector and the flow of electromagnetic energy (41).

3. Waves of TM polarization

The propagation of TM polarized waves in the plane, as follows (4)–(6), $k_y = 0$ is described by the matrix equation:

$$\begin{aligned} \frac{d}{dz} \vec{W} &= B \vec{W}; \quad \vec{W} = (H_y, E_x)^t; \\ B &= \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix}; \\ b_{12} &= -i\omega \epsilon_x; \\ b_{21} &= -i\omega \left(\mu_y - \frac{k_x^2}{\omega^2 \epsilon_z} \right). \end{aligned} \quad (48)$$

3.1 The amplitudes of the reflected and refracted waves of the TM polarization are determined by the expressions (18)–(20), with b_{ij} ; from (48):

$$\frac{r_{21}^-}{r_{21}^+} = \frac{\epsilon_x k_{0z} - \epsilon_{x0} k_z}{\epsilon_x k_{0z} + \epsilon_{x0} k_z}; \quad (49)$$

$$\frac{r_{12}^-}{r_{12}^+} = -\frac{r_{21}^-}{r_{21}^+}. \quad (50)$$

From (14) and (17) follows:

$$H_{yR} = \frac{r_{21}^-}{r_{21}^+} H_{y0}; \quad \frac{H_{yT}}{H_{y0}} = \frac{2\epsilon_x k_{0z}}{\epsilon_x k_{0z} + k_z \epsilon_{x0}}. \quad (51)$$

The electromagnetic field of a TM polarized wave in a medium has non-zero components:

$$H_y, E_x, E_z. \quad (52)$$

The components E_x and E_z and H_y are related by the relations:

$$E_x = \frac{k_z}{\omega \epsilon_x} H_y; \quad E_z = -\frac{k_x}{\omega \epsilon_z} H_y. \quad (53)$$

$$E_{x0} = \frac{k_{z0}}{\omega \epsilon_{x0}} H_{y0}; \quad E_{xR} = -\frac{k_{z0}}{\omega \epsilon_{z0}} H_{yR}.$$

3.2 The angle of refraction of the TM wave. Components of wave vectors.

The indicatrix of the wave vector of the TM wave is determined by condition (7), taking into account with b_{ij} (48)

$$k^2 = \frac{\omega^2 \mu_y \epsilon_x \epsilon_z}{\epsilon_z \cos^2 \theta + \epsilon_x \sin^2 \theta}. \quad (54)$$

From equality: $k_{x0} = k_x$ at the media boundary, the law of refraction of TM polarization waves follows

$$\frac{\mu_{y0} \epsilon_{x0} \epsilon_{z0} \sin^2 \theta_0}{\epsilon_{z0} \cos^2 \theta_0 + \epsilon_x \sin^2 \theta_0} = \frac{\mu_y \epsilon_x \epsilon_z \sin^2 \theta}{\epsilon_z \cos^2 \theta + \epsilon_x \sin^2 \theta}. \quad (55)$$

From (55) it can be taken:

$$\sin^2 \theta = \frac{\mu_{y0} \epsilon_{x0} \epsilon_{z0} \epsilon_z \sin^2 \theta_0}{\mu_y \epsilon_x \epsilon_z \epsilon_{z0} - [\mu_y \epsilon_x \epsilon_z (\epsilon_{x0} - \epsilon_{z0}) + \mu_{y0} \epsilon_{x0} \epsilon_{z0} (\epsilon_x - \epsilon_z)] \sin^2 \theta_0}. \quad (56)$$

The angle θ_0 determines the direction of the vector \vec{k}_0 of the wave incident on the boundary of the TM wave θ ; the angle determines the direction of the vector \vec{k} of the refracted wave. Similarly to formulas (28), (29), special cases can be obtained.

The angles of total internal reflection can also be obtained on the basis of (56) by setting either $\sin^2 \theta_0 = 1$, or $\sin^2 \theta = 0$.

When $\sin^2 \theta = 1$ has

$$\sin^2 \theta_0 = \frac{\mu_y \epsilon_z \epsilon_{z0}}{\mu_{y0} \epsilon_{z0} \epsilon_{x0} + \mu_y \epsilon_z (\epsilon_{x0} - \epsilon_{z0})}. \quad (57)$$

The components of the wave vectors k_{0z} and k_z included in the refraction reflection formulas (49)–(51) are determined based on the indicatrix equation for both media (54)

$$k_{0z}^2 = \frac{\omega^2 \mu_{y0} \epsilon_{x0} \epsilon_{z0}}{\epsilon_{z0} \cos^2 \theta_0 + \epsilon_{x0} \sin^2 \theta_0}; \quad (58)$$

$$k_z^2 = \frac{\omega^2 \mu_y \epsilon_x \epsilon_z}{\epsilon_z \cos^2 \theta + \epsilon_x \sin^2 \theta}. \quad (59)$$

As in the case of TE waves of polarization (32), the angle θ is determined by relations (55), (56) (in the case of TE waves (26), (27)).

Energy relations for TM polarized waves.

The vector of electromagnetic energy flow \vec{S} has the form:

$$\vec{S} = [\vec{E} \times \vec{H}] = iS_x + kS_z = -i\vec{H}_y E_z + \vec{k} H_y E_x. \quad (60)$$

$$S_x = \frac{k_x}{\omega \epsilon_z} H_y^2; \quad S_z = \frac{k_z}{\omega \epsilon_x} H_y^2. \quad (61)$$

In (61), dependences (53) were taken into account

The direction of the group velocity vector and the electromagnetic energy flux vector is determined by the formula.

$$\frac{S_x}{S_z} = \text{tg} \beta = \frac{\epsilon_x}{\epsilon_z} \text{tg} \theta. \quad (62)$$

The angle β determines the direction of the group velocity vector \vec{V}_g .

The components of the electromagnetic energy flows for a TM wave have the following meanings.

– incident wave

$$S_{x0} = \frac{k_{x0}}{\omega \epsilon_{z0}} H_{y0}^2; \quad S_{z0} = \frac{k_{z0}}{\omega \epsilon_{x0}} H_{y0}^2; \quad (63)$$

– reflected wave

$$S_{xR} = \frac{k_{x0}}{\omega \epsilon_{z0}} H_{yR}^2; \quad S_{zR} = \frac{k_{z0}}{\omega \epsilon_{x0}} H_{yR}^2; \quad (64)$$

– refracted wave

$$S_{xT} = \frac{k_x}{\omega \epsilon_z} H_{yT}^2; \quad S_{zT} = \frac{k_z}{\omega \epsilon_x} H_{yT}^2. \quad (65)$$

The quantities H_{yR} , H_{yT} are determined by relations (49)–(51), and are related by relationships (53).

Calculation of the total flow gives:

$$S = \sqrt{S_x^2 + S_z^2} = V_g \mu_y H_y^2. \quad (66)$$

The value of the group velocity V_g is equal to:

$$V_g^2 = \frac{1}{\mu_y \epsilon_x \epsilon_z} \frac{\epsilon_z^2 + \epsilon_x^2 \operatorname{tg}^2 \theta}{\epsilon_z + \epsilon_x \operatorname{tg}^2 \theta}. \quad (67)$$

Phase speed:

$$V_f^2 = \frac{\epsilon_z \cos^2 \theta + \epsilon_x \sin^2 \theta}{\mu_y \epsilon_x \epsilon_z} = \frac{\omega^2}{k^2}. \quad (68)$$

Refractive index:

$$n^2 = \frac{c^2}{V_f^2} = \frac{c^2 \mu_y \epsilon_x \epsilon_z}{\epsilon_z \cos^2 \theta + \epsilon_x \sin^2 \theta}, \quad (69)$$

c — speed of electromagnetic waves in vacuum.

For the components of the flows S_{z0}, S_{zR}, S_{zT} , the equality is:

$$\frac{S_{zR}}{S_{z0}} + \frac{S_{zT}}{S_{z0}} = 1. \quad (70)$$

The proof is similar. (46), (47).

Settlement part

1. Indicatrixes of wave vectors of TE and TM polarization waves in an anisotropic medium of rhombic symmetry:

$$k_{TE}^2 = \frac{\omega^2 \epsilon_y \mu_x \mu_z}{\mu_z \cos^2 \theta + \mu_x \sin^2 \theta}; \quad (71)$$

$$k_{TM}^2 = \frac{\omega^2 \mu_y \epsilon_x \epsilon_z}{\epsilon_z \cos^2 \theta + \epsilon_x \sin^2 \theta}. \quad (72)$$

2. Phase velocities:

$$V_{fTE}^2 = \frac{1}{\epsilon_y \mu_x \mu_z} (\mu_z \cos^2 \theta + \mu_x \sin^2 \theta); \quad (73)$$

$$V_{fTM}^2 = \frac{1}{\mu_y \epsilon_x \epsilon_z} (\epsilon_z \cos^2 \theta + \epsilon_x \sin^2 \theta). \quad (74)$$

3. Group speeds:

$$V_{gTE}^2 = \frac{1}{\epsilon_y \mu_x \mu_z} \frac{\mu_z^2 + \mu_x^2 \operatorname{tg}^2 \theta}{\mu_z + \mu_x \operatorname{tg}^2 \theta}; \quad (75)$$

$$V_{gTM}^2 = \frac{1}{\mu_y \epsilon_x \epsilon_z} \frac{\epsilon_z^2 + \epsilon_x^2 \operatorname{tg}^2 \theta}{\epsilon_z + \epsilon_x \operatorname{tg}^2 \theta}. \quad (76)$$

4. The refraction angles of TE and TM waves at the boundary of rhombic symmetry media:
 – for TE polarization waves:

$$\sin^2 \theta = \frac{\varepsilon_{y0} \mu_{x0} \mu_{z0} \mu_z \sin^2 \theta_0}{\varepsilon_y \mu_x \mu_z \mu_{z0} - [\varepsilon_y \mu_x \mu_z (\mu_{x0} - \mu_{z0}) + \varepsilon_{y0} \mu_{x0} \mu_{z0} (\mu_x - \mu_z)] \sin^2 \theta_0}; \quad (77)$$

– for TM waves:

$$\sin^2 \theta = \frac{\varepsilon_{y0} \mu_{x0} \mu_{z0} \mu_z \sin^2 \theta_0}{\varepsilon_y \mu_x \mu_z \mu_{z0} - [\varepsilon_y \mu_x \mu_z (\mu_{x0} - \mu_{z0}) + \varepsilon_{y0} \mu_{x0} \mu_{z0} (\mu_x - \mu_z)] \sin^2 \theta_0}. \quad (78)$$

5. Directions of group velocity vectors in incident TE and TM waves

$$\operatorname{tg} \beta_{TE} = \frac{\mu_{x0}}{\mu_{z0}} \operatorname{tg} \theta_0; \quad \operatorname{tg} \beta_{TM} = \frac{\varepsilon_{x0}}{\varepsilon_{z0}} \operatorname{tg} \theta_0 \quad (79)$$

are different even for the same angle of incidence. The directions of the group velocity vectors in the reflected waves are determined by formulas (79), taking into account the change in the direction of propagation.

6. The directions of the group velocity vectors in the refracted TE and TM polarized waves have a similar form:

$$\operatorname{tg} \beta_{TE} = \frac{\mu_x}{\mu_z} \operatorname{tg} \theta; \quad \operatorname{tg} \beta_{TM} = \frac{\varepsilon_{x0}}{\varepsilon_{z0}} \operatorname{tg} \theta_0, \quad (80)$$

however, it must be borne in mind that the angle θ in (80) is determined by formulas (77) and (78).

7. For the reflection and refraction coefficients of TE and TM waves at the boundary of anisotropic media of rhombic symmetry, the Fresnel formulas [1–3] are valid. Moreover, the components k_z and k_{z0} are determined on the basis of the indicatrix equations (71), (72).

$$k_{zTE}^2 = \frac{\omega^2 \varepsilon_y \mu_x \mu_z}{\mu_z + \mu_x \operatorname{tg}^2 \theta}; \quad k_{zTM}^2 = \frac{\omega^2 \mu_y \varepsilon_x \varepsilon_z}{\varepsilon_z + \varepsilon_x \operatorname{tg}^2 \theta}. \quad (81)$$

In refracted waves, the angle θ is determined by relations (77), (78).

Based on comparisons of the characteristics of TE and TM polarization waves (71)–(81), it can be concluded that the components of the magnetic permeability tensor have a determining, predominant effect on the characteristics of TE waves.

For the characteristics of TM waves, the anisotropy of the dielectric constant tensor is decisive.

A study of the anisotropic properties of locally isotropic inhomogeneous elastic media showed that averaging of one-dimensionally inhomogeneous media (the wavelength is much larger than the characteristic dimensions of the inhomogeneity) leads to the manifestation of the anisotropic properties of hexagonal symmetry by the medium.

Two and three-dimensionally inhomogeneous locally isotropic media, when averaged, have properties and are described by the parameters of rhombic anisotropy media [18].

Similar results were obtained and are valid for inhomogeneous, locally isotropic dielectric media.

Results and discussion

An analytical solution to the problems of reflection and refraction of plane electromagnetic waves TE and TM polarization at the boundary of anisotropic media of rhombic symmetry is constructed, the dependences on the direction of the wave vector k_0 (angle θ_0) of the incident wave are determined:

- refraction angles of TE and TM polarization waves;
- amplitudes of reflected and refracted waves (reflection and refraction coefficients);
- directions of vectors of phase and group velocities in refracted waves;
- component of the vectors of the flow of electromagnetic energy of incident reflected and refracted waves;
- values of phase and group velocities in refracted TE and TM polarized waves.

The results obtained are applicable to the analysis of the processes of reflection and refraction of electromagnetic waves at the boundary of anisotropic media with higher symmetry.

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Ромбылық кристалдардың шекарасында ТЭ және ТМ поляризациялы толқындардың шағылу және сыну заңдылықтары

Мақалада ромбылық симметрияның анизотропты жартылай кеңістігінің шекарасындағы әртүрлі поляризациялы электромагниттік жазық толқындардың шағылу және сыну есебінің аналитикалық шешілуі берілген. Матрица әдісі негізінде поляризациялануы әртүрлі электромагниттік толқындардың сыну бұрыштары, шағылған және сынған толқындардың амплитудалары, топтық жылдамдықтар бағыты және электромагниттік энергия ағынының векторларын анықтаушы бұрыштары, түсетін толқынның толқындық векторына тәуелді болатын электромагниттік энергия ағындарының шамалары және олардың құраушылары анықталған. Поляризациясы әртүрлі электромагниттік толқындардың сыну көрсеткіші және толық ішкі шағылу бұрыштарын анықтау қарастырылған. Магниттік және диэлектрлік өтімділіктері тензорлар анизотропиясына поляризациялануы әртүрлі болып келетін электромагниттік толқындардың кинематикалық және энергетикалық сипаттамаларының тәуелділігі әртүрлі болатындығы көрсетілген. Поляризациясы толқынның таралу жазықтығына перпендикуляр (ТЭ толқындар) болып келетін электр кернеулігі векторының құраушысымен анықталатын электромагниттік толқындар үшін, шағылған және сынған толқындар сипаттамалары, жылдамдықтар индикатрисалары, таралу бұрыштары және т.б. басым жағдайда магниттік өтімділік тензоры құраушыларымен анықталған. Ал поляризациясы толқынның таралу бағытына перпендикуляр болып келетін магнит өрісі (ТМ толқындар) кернеулігінің құраушысымен анықталатын электромагниттік толқын жағдайында, барлық сипаттамалар көбіне диэлектрлік өтімділік тензоры құраушысына

тәуелді. Ромбылық симметриялы анизотропты органың шекарасында шағылған және сынған толқындардың коэффициенттерін анықтауға Френель формуласының дұрыстығы көрсетілген. Бірақта, бұл жағдайда Френель формуласына енетін толқын векторларының құраушылары олардың индикатрисасымен анықталған. Сонымен қатар, осы құраушылардың екінші ортадағы сыну бұрышына тәуелділігі ескерілуі керек.

Кілт сөздер: анизотропия, ромбылық симметрия, электромагниттік толқындар, шағылу, сыну, толқындардың сыну бұрыштары, фазалық және топтық жылдамдықтар, тығыздық векторы.

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Закономерности отражения и преломления волн ТЕ и ТМ поляризации на границе ромбических кристаллов

В статье аналитически решена проблема отражения и преломления электромагнитных плоских волн различной поляризации на границе анизотропных полупространств ромбической симметрии. На основе матричного метода найдены углы преломления электромагнитных волн различной поляризации, амплитуды отраженных и преломленных волн; углы, определяющие направление групповых скоростей и векторы потоков электромагнитной энергии; потоки электромагнитной энергии и их составляющие на направление волнового вектора падающей волны. Рассмотрено определение углов полного внутреннего отражения и показателя преломления электромагнитных волн различной поляризации. Показана принципиально разная зависимость кинематических и энергетических характеристик электромагнитных волн различной поляризации от анизотропии тензоров магнитной и диэлектрической проницаемости. Для электромагнитных волн, поляризация которых определяется компонентой вектора электрической напряженности, перпендикулярной плоскости распространения волны (ТЕ-волна), характеристиками отраженных и преломленных волн, показательными для скоростей, углов распространения и т.д., определяется в первую очередь компонентами тензора магнитной проницаемости. В случае электромагнитных волн, когда поляризация определяется компонентом напряженности магнитного поля (волна ТМ), перпендикулярной плоскости распространения волны, все характеристики зависят главным образом от компонент тензора диэлектрической проницаемости. Показана обоснованность формул Френеля для обозначения коэффициентов отраженной и преломленной волн на границе анизотропных сред орторомбической симметрии. Однако в этом случае компоненты волновых векторов, входящих в формулы Френеля, определяются их индикатрисами. Кроме того, необходимо учитывать зависимость этих составляющих от угла преломления во второй среде.

Ключевые слова: анизотропия, ромбическая симметрия, электромагнитные волны, отражение, преломление, углы преломления волн, фазовые и групповые скорости, вектор плотности.

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