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Quaternion Method of Calculating Angles while Measuring via Goniometric Precision Instrument System

The article is devoted to the topical problem: increasing accuracy and performance of angle measurements necessary in various branches of science and technology. One of the ways of increasing accuracy and performance of angle measurements is using modern algorithmic methods and mathematic devices for processing measurement information. Thus, in order to increase accuracy and speed of angle measurements on the example of the well-known goniometric precision instrument system (GPIS), it was offered quaternion calculation of angle values while performing goniometric measurements in the work. The efficiency of quaternion calculation is unquestioned as quaternions unlike other traditional methods (in particular matrix with the use of Euler angles, direction cosines) are presented only with four parameters describing angle positions of the objects and have only one connection equation unlike six equations for matrix methods, in particular for direction cosines. The suggested quaternion calculation is used in GPDS as general theoretic and information basis of contactless precision goniometric measurements in preliminary setting navigation sensitive elements (NSE), plane angles, pyramid prisms etc. The usage of the developed quaternion calculation enabled to increase accuracy by 0,25″ (in 3 times) and measurement performance in 9 times (up to 6.5 sec.) in comparison with the famous ones. Applying quaternion calculation of angle values implies using a smaller RAM capacity of PC that increases performance of system work. Besides, a smaller amount of mathematic operations performed in quaternion way of calculating angles, except increasing performance, enables to decrease a rounding error in calculation results that is accumulated in multiple measurements and may reach great values. Thus, accuracy and performance of measurements increase.

Keywords: quaternion, goniometric system, accuracy, performance, measuring angles, precision.

Introduction

Setting the general problem. Precision angle (goniometric) measurements serve as an important metrological task aimed to provide quality of production. Precision angle measurements are conducted in various branches of machine-building and instrumentation (for example, in producing such precision joints and parts as direction ones like «dovetail» joint, conical seats of precision axes, optical prisms etc.), in preliminary setting navigation sensitive elements (NSE) [1] (accelerometers and gravity meters used in modern systems of orientation and navigation in directing the motion of different moving objects — cars, aircraft systems of various purposes, systems of artillery shells guiding), verification of dividing heads [2], determining straightness tolerance of positioning work benches angles, rounding error of measuring automated-measuring systems [3] etc. At the same time the branches of applying angle measuring means are constantly being expanded and their quality is also dramatically increasing, in particular, their quality; besides, their functional abilities are expanded, the automation of measuring and processing measurement information is provided.

One of the topical ways to increase accuracy and performance of goniometric systems, for example, a famous goniometric precision instrument system (GPIS) [5] is using modern algorithmic methods and mathematical apparatus for processing measurement information, in particular, mathematical apparatus of quaternions.

Famous research and publications analysis. [1–9] showed that mathematical apparatus of quaternions may be successfully applied in tasks of spatial measurement of angles and enables to increase performance and accuracy. It relates to the fact that unlike other classic methods (in particular, matrix ones), quaternions are introduced by four parameters only describing angle positions and have only one constraint equation unlike six equations for classic matrix methods, in particular, for direction cosines.

Thus, in works [1, 2] it was introduced the research results of two algorithms of auto-collimation measurements of object's spatial turn based on the matrix and quaternion models. It was shown the advantage of a quaternion algorithm according to the criterion of reducing measurement rounding error. However, the issue of applying quaternion calculation in angle measurements via modern goniometric systems are fragmentary shown. It was introduced a new quaternion filter for gyroscopes and accelerometers in spatial measurement of angles in the work [3]. So called projected quaternion is calculated by the authors on the basis of angle speed of a gyroscope. It was shown by the authors that application of quaternions increases efficiency of calculations and measurement accuracy. However, unfortunately, the practical aspects of work are only partially shown.

The authors developed quaternion algorithm of determining angular rotation with high calculating efficiency in any positions of measurement objects in the article [4]. It was proved by the authors the opportunity of increasing accuracy and rate of angle measurements. However, the issues of applying quaternions in modern goniometric measurement systems while measuring plane angles of polygonal prisms, pyramidality of prisms and other production objects are not considered.

In the work [5] it was introduced the results of calculations concerning measured angles in spatial turns of the objects. The issues of measuring plane angles of polygonal prisms, pyramidality of prisms and other production objects are not considered by the authors. In the work [6] it was given the results of quaternions application in the formalized description of objects angle motion; it was proved their advantage over other mathematical methods. The issue of applying quaternions in goniometric measurements is not considered by the authors. In the work [7] it was stated that quaternions application is of a special importance in cases when quaternion is used not only for setting object's orientation in three-dimensional space, but also for determining some additional scalar value. Practical aspects of applying quaternions in goniometric measuring plane angles of polygonal prisms, prisms pyramidality and other production objects, except NSE, are not highlighted.

In the work [8] it was introduced the results of modelling a quaternion algorithm of determining spatial angles via accelerometer and gyroscope that demonstrate its advantage. However, the possibility of using quaternions in other goniometric systems is not considered.

In the work [9] it was considered the application of quaternions in auto-collimation measurements of the preliminary setting NSE and it was determined its efficiency compared to the matrix method. The given results of computer modelling prove the efficiency of quaternion models for increasing accuracy of angle measurements.

Thus, the perspective of applying quaternions in precision goniometric measurements regarding to the experience obtained, is unquestionable. At the same time the issue of applying quaternions in precision goniometric systems, for example, a famous goniometric precision instrumental system (GPIS) [10], have not been considered yet.

Highlighting the unsolved part of the problem set. Thus, it is possible to claim that despite essential scientific and practical achievements the problem of measuring values of plane angles with high precision and performance has not been solved completely. Rounding errors of the vast majority of modern goniometric systems are unacceptably large and comprise from 1″ till 0,12″. Impossibility of applying such goniometric systems is caused by the fact that according to the international standards of quality, modern production always sets tougher and tougher requirements concerning accuracy and performance of measurements. One of the most effective ways of increasing accuracy and performance of angle measurements is developing new and improving well-known devices and measurement apparatus for processing measurement information including mathematical apparatus of quaternions.

The purpose of the article implies offering quaternion calculation of angle values to provide increasing accuracy and performance in goniometric contactless measurements via famous IPAMS, with preliminary setting NSE, plane angles, prisms pyramidality etc.

Description of the suggested quaternion calculation of angles

Goniometric precision instrumental system (GPIS) [10] (Figure 1) developed on the basis of precision angle-measurement system GS1L [12] (ARSENAL SDP SE (Kyiv, Ukraine)). GPIS may be applied for contactless precision goniometric measurements with preliminary setting NSE, plane angles, prisms pyramidality and other production objects, optical glass index deflection.

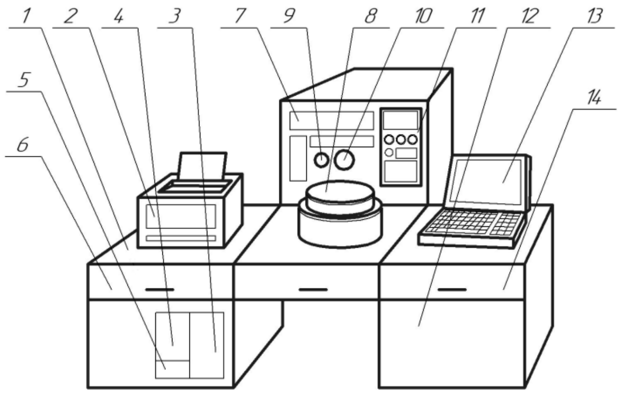


Figure 1. GPIS Scheme: 1 — side bar; 2 — printer; 3 — rectifier; 4 — power supply unit; 5 — voltage stabilizer; 6, 14 — box for implements; 7 — measuring bar; 8 — rotating device with angle converter and subject board; 9 — autocollimator; 10 — spectral (laser) emitter; 11 — panel of switches and control lever; 12 — side bar with the unit of initial processing and managing information; 13 — personal computer (PC)

In order to increase accuracy and performance of GPIS it is offered to apply the previously developed methodology of calculating necessary amount of measurement in multiple observations [13], algorithmic correction of measurement results [14] and the suggested quaternion measurement of angles in goniometric measurements based on the theorems and axioms of quaternions Algebra in a complex and serves as a general theoretic and information basis of defining various angles in preliminary setting NSE, plane angles, prisms pyramidality and other production objects. Applying the stated methodology of calculating the necessary amount of measurements and algorithmic correction of measurement results [13, 14] as for processing measurement results in complex with the suggested quaternion measurement of angles in goniometric measurements will enable to conduct measurements of IPAMS with the increased accuracy and performance. Description of methodology for calculating the necessary amount of measurements in multiple observation and algorithmic correction of measurement results is introduced in sources [13, 14].

In general, quaternion q itself is a structured four of real numbers s, a, b, c, connected in between via four basic elements $1, i, j, k$ (Figure 2), and have the following properties: $i^2 = j^2 = k^2 = -1$; $i \cdot j = k$; $j \cdot k = i$; $k \cdot i = j$; $j \cdot i = -k$; $k \cdot j = -i$; $i \cdot k = -j$.

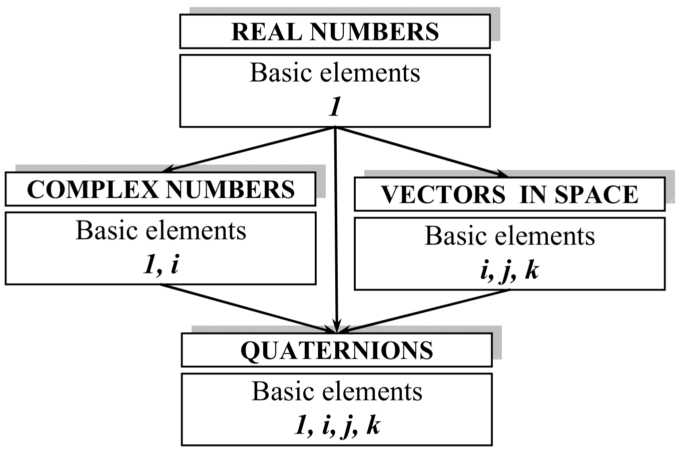


Figure 2. Numeral systems and basic elements of quaternions

Quaternion q is introduced in different ways:

– as a sum of two quaternions: scalar (s) i vector ($a \cdot i + b \cdot j + c \cdot k$), that is $q = s(q) + v(q) = [scalar; (vector)]$;

– as a number and 3D-vector, that is as a hyper-complex number with three imaginative units i, j, k, that is $q = [s, a, b, c] = [scalar, (vektor)] = [s, (a, b, c)] = s \cdot 1 + a \cdot i + b \cdot j + c \cdot k = s + v$;

– as a vector — a quaternion looks like a vector in case its scalar part is equal to zero: $q(vector) = a \cdot i + b \cdot j + c \cdot k$; $scalar = 0$;

– as a sum $\cos\frac{\omega}{2}$ and $\sin\frac{\omega}{2}$ in solving goniometric tasks, that is $q(v,\omega) = \cos\frac{\omega}{2} + v \cdot \sin\frac{\omega}{2}$, where v — a unit vector, co-directed with pivot axis; ω — angular rotation.

An important peculiarity of quaternions lies in the fact that their subset comprises real numbers (s, 0, 0, 0); complex numbers (s, a, 0, 0); vectors in three-dimensional space (0, a, b, c), and the law of commutativity is not obeyed in performing multiplication while multiplying quaternions, that is $q_1 \cdot q_2 \neq q_2 \cdot q_1$.

Besides, three imaginative basic units *i, j, k* of quaternions may be interpreted as basic vectors of Cartesian coordinates system *XYZ* in three-dimensional space (Figure 3).

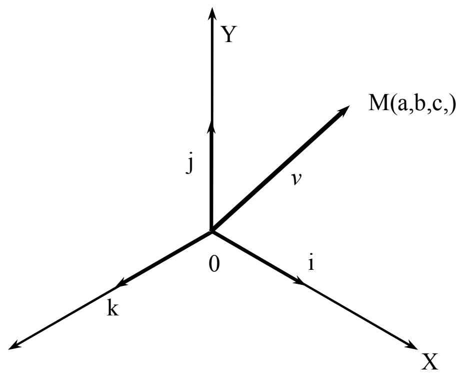
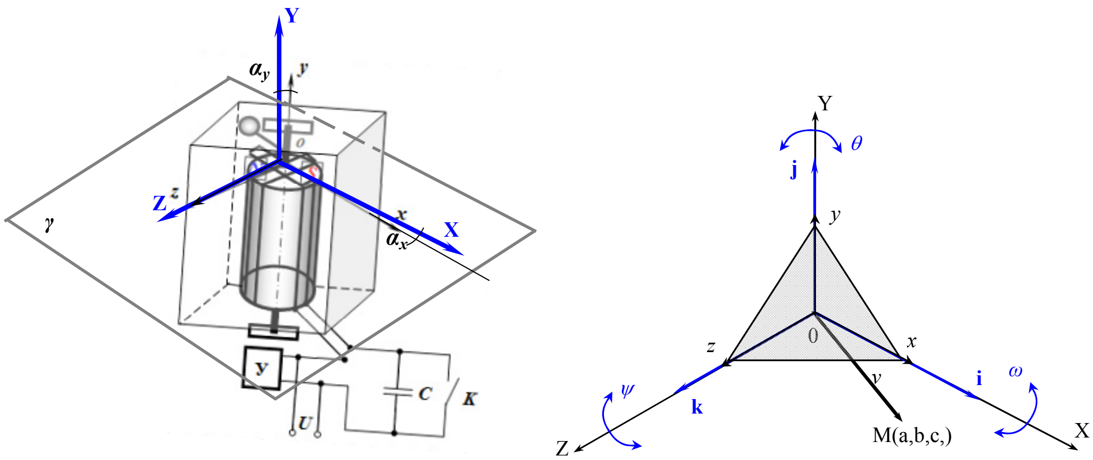


Figure 3.

While conducting GPIS goniometric measurements in three-dimensional space, for example, in preliminary setting NSE angles, at the last stage it is necessary to fix the reflecting element, for instance, a mirror, sensitive to its movements. NSE setting is conducted before navigation system functioning via the method of angle coordination, enabling to reach higher accuracy and comprises comparison of angle position of coordinate system of the local coordinate system *xyz* NSE regarding axes of absolute, in advance adopted Cartesian coordinate system (Figure 1, a). In this case, NSE turns will be defined as altering position of coordinate system *xyz* of reflecting element regarding Cartesian coordinate system XYZ, connected with GPIS autocollimator (Figure 4, b).



a b

Figure. 4. Example of setting NSE (pendulum accelerometer): a — rounding error of setting, b — position of light-reflecting element coordinate system *xyz* regarding Cartesian coordinate system *XYZ*

The position of NSE is described by turns of axes *XYZ* in coordinate system of autocollimator into angles ω, θ, ψ, accordingly. The result of some sequence in NSE movements while its setting is a turn in coordinate system *XYZ* to a certain angle *XYZ* regarding the axis, collinear to the unit vector (Fig.4, b). Correcting movements as for setting initial NSE coordinates and its levelling may be introduced via quaternion $q_k(v;\varphi) = (\cos\frac{\varphi_k}{2} + v\sin\frac{\varphi_k}{2})$, and sequence of movements — via corresponding turn parameters — product of proper quaternions, introduced via expression (1):

$$q_1(v;\omega) = (\cos\frac{\omega_k}{2} + i\sin\frac{\omega_k}{2}),$$
$$q_2(v;\theta) = (\cos\frac{\theta_k}{2} + j\sin\frac{\theta_k}{2}), \quad (1)$$
$$q_3(v;\psi) = (\cos\frac{\psi_k}{2} + k\sin\frac{\psi_k}{2}).$$

Product Q of quaternions $q_1(v;\ \omega)$, $q_2(v;\ \theta)$, $q_3(v;\ \psi)$, is calculated by formulae (2) performing multiplication operations according to the rules of quaternions multiplication:

$$Q(v;\varphi) = (\cos\frac{\omega_k}{2} + i\sin\frac{\omega_k}{2})\cdot(\cos\frac{\theta_k}{2} + j\sin\frac{\omega_k}{2})\cdot(\cos\frac{\psi_k}{2} + k\sin\frac{\psi_k}{2}) = S + iA + jB + kC, \quad (2)$$

where $S = \cos\frac{\omega_k}{2}\cos\frac{\theta_k}{2}\cos\frac{\psi_k}{2} - \sin\frac{\omega_k}{2}\sin\frac{\theta_k}{2}\sin\frac{\psi_k}{2}$;

$$A = \sin\frac{\omega_k}{2}\cos\frac{\theta_k}{2}\cos\frac{\psi_k}{2} + \cos\frac{\omega_k}{2}\sin\frac{\theta_k}{2}\sin\frac{\psi_k}{2};$$
$$B = \cos\frac{\omega_k}{2}\sin\frac{\theta_k}{2}\cos\frac{\psi_k}{2} - \sin\frac{\omega_k}{2}\cos\frac{\theta_k}{2}\sin\frac{\psi_k}{2};$$
$$C = \sin\frac{\omega_k}{2}\sin\frac{\theta_k}{2}\cos\frac{\psi_k}{2} + \cos\frac{\omega_k}{2}\cos\frac{\theta_k}{2}\sin\frac{\psi_k}{2}.$$

For example, while setting NSE it was performed turns $\omega = 10°$ $\theta = 27°$ $\psi = 40°$. Hence the beam of a circular laser IPAMS will be reflected from the reflecting element, fixed on NSE to CMOS-matrix of IPAMS autocollimator at an angle of 50°52′ with coordinates (0.158, 0.189, 0.350) calculated in the following way:

$$q_1(v;10)\cdot q_2(v;27)\cdot q_3(v;40) =$$
$$= (\cos\frac{10}{2} + i\sin\frac{10}{2})\cdot(\cos\frac{27}{2} + j\sin\frac{27}{2})\cdot(\cos\frac{40}{2} + k\sin\frac{40}{2}) =$$
$$= \cos 24.43 + (i0.158 + j0.189 + k0.350)\sin 25.43 =$$
$$= 50.86 + (i0.158 + j0.189 + k0.350) \Rightarrow q(v; 50°52').$$

While conducting goniometric measurement of plane angle values, for example, polygonal prisms with *n*-number of facets, the lengths of which comprise $l_0, l_1, \dots, l_n$ and proper angles φ_k, where k∈(0, 1, …, *n*); if each vertex is connected with right Cartesian coordinates system with unit basis *i*, *j*, *k* (Figure 5), it is possible to use the normalized quaternion as:

$$q_k(v;\varphi) = (\cos\frac{\varphi_k}{2} + v\sin\frac{\varphi_k}{2}), \quad (3)$$

where: v — unit vector, regarding which the value of angle φ, $v \in (i, j, k)$ is figured out

φ — flat angle value;

k — ordinary number of angle (facet), $k \in (0, 1, \dots, n)$.

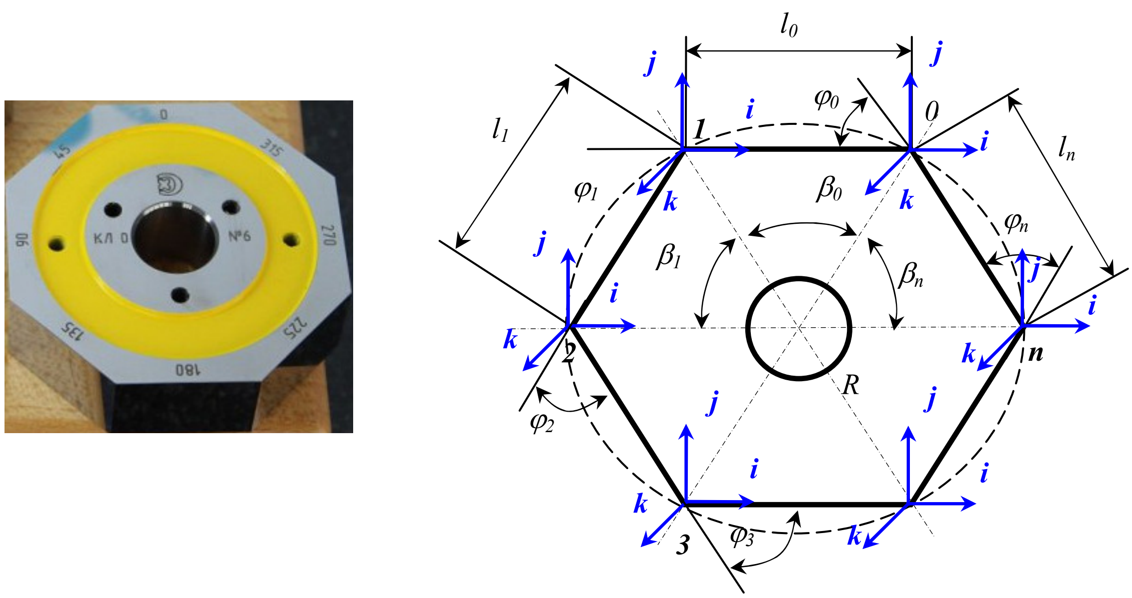


Figure 5.

Each n-vertex of polygonal prism is connected with Cartesian coordinates system with unit basis i, j, k (Fig. 5). Resulting this, n of coordinates system is formed, each system is supplied quaternion in accordance (3). Then, for each vertex (and angle accordingly $\varphi_k \mid k \in (0, 1, \ldots, n)$) of polygonal prism, it is possible to form a proper quaternion $q_1(v; \varphi_1)$, $q_2(v; \varphi_2)$, …, $q_n(v; \varphi_n)$.

Quaternions of plane angles in polygonal prism φ_k in three-dimensional space of coordinates system with basis i, j, k is convenient to introduce in the form of a vector in the following way:

$$q_k(v; \varphi) = (\cos\frac{\varphi_k}{2}; i \sin\frac{\varphi_k}{2}; j \sin\frac{\varphi_k}{2}; k \sin\frac{\varphi_k}{2}) . \qquad (4)$$

At the same time, if proper quaternion (4) constituents equate zero, angle φ_k will be defined regarding one or two coordinates axes of right Cartesian coordinates system with unit basis i, j, k. Thus, for example, if angle φ_k is defined regarding axis X, then quaternion (4) will have the following simple expression:

$$q_k(v; \varphi) = (\cos\frac{\varphi_k}{2}; i \sin\frac{\varphi_k}{2}; 0; 0) .$$

Quaternions of so-called exponential angles β (Fig. 5) in three-dimensional space of coordinates system with basis i, j, k are convenient to introduce in vector form in the following way:

$$q_k(v; \beta) = (\cos\frac{\beta_k}{2}; i \sin\frac{\beta_k}{2}; j \sin\frac{\beta_k}{2}; k \sin\frac{\beta_k}{2}) , \qquad (5)$$

where $\beta_k = \arccos(1 + \frac{l_k^2}{2R})$, figured out via cosines theorem, where R — radius;

k — ordinary angle number, $k \in (0, 1, \ldots, n)$.

In a similar way, if angle β is defined regarding certain coordinates axis of right Cartesian coordinates system, collinear to one of the vectors of basic elements $i, j, k,$ then corresponding constituents of quaternion (5), connected with other basic elements, will equate zero. In this case quaternion (5) will have a simple form.

For example, presenting quaternion (5) in the form $q_2(v; \beta) = (\cos\frac{45}{2}; 0; j \sin\frac{45}{2}; 0)$, means that exponential angle β_2 between 2nd and 3rd vertices of polygonal prisms (Fig.5) equals 45° and it was determined regarding axis Y of right Cartesian coordinates system, collinear to unit vector j and basis i, j, k.

Taking into account properties of quaternions, their application for calculating angles in goniometric measurements of GPIS in preliminary setting NSE, plane angles, prisms pyramidality etc., enables to figure out directly angle values, and also axis of rotation and coordinates of laser radiation reflection onto CMOS-matrix of GPIS auto-collimator.

Random examples of quaternion introduction of angles in absolute right Cartesian coordinates system with basis *i, j, k,* are introduced in Table 1.

Table 1

Random examples of quaternion introduction of angle values in absolute right Cartesian coordinates system with basis, *i, j, k*

Values of turning angles OB regarding absolute coordinates system *X, Y, Z*			Results of turns. Position of vector v regarding basis of absolute coordinates system *X, Y, Z*			
ω°	***θ°***	***ψ°***	***φ***	***i***	***j***	***k***
5	5	5	9	0.45	0.45	0.42
7	7	7	$12^0 47'$	0.64	0.64	0.57
10	27	40	$50^0 52'$	0.158	0.189	0.350
60	25	70	$82^0 5'$	0.5070	- 0.1269	0.3967
60	25	70	$101^0 52'$	0.5070	-0.1269	0.5736
60	25	70	$101^0 52'$	0.2923	0.4333	0.5736
60	25	70	$82^0 5'$	0.2923	-0.1269	0.5736
60	25	70	$101^0 52'$	0.5070	0.4333	0.3967
90	90	–	120	0.5	0.5	0.5
–	90	90	120	-0.5	0.5	0.5
50	40	–	$63^0 17'$	0.397	0.31	0.145
–	50	30	$57^0 52'$	0.11	0.409	0.236
53	53	–	74	0.399	0.399	0.199

In general, the algorithm of quaternion angle calculation in IPAMS may be introduced in the following sequence of steps:

1. Angle description via product of proper quaternions in expressions (1), (2);
2. Resulting quaternion calculation in accordance with quaternions characteristics:

– commutativity and associativity in addition: $q_1 + q_2 = q_2 + q_1$, $(q_1 + q_2) + q_3 = q_2 + (q_1 + q_3)$,

– non-commutativity in multiplication: $q_1 \cdot q_2 \neq q_2 \cdot q_1$,

– associativity in multiplication: $(q_1 \cdot q_2) \cdot q_3 = q_1 \cdot (q_2 \cdot q_3)$,

– distributivity: $q_1 \cdot (q_2 + q_3) = q_1 \cdot q_2 + q_1 \cdot q_3$;

3. Definition of angle, direction and coordinates of light beam reflection to CMOS-matrix of IPAMS.

Application of the quaternion method of calculating angles in goniometric measurements IPAMS implies performing 29 mathematic operations, in particular 16 multiplication operations, 12 addition operations and 1 operation to figure out angle arccosine. While applying classic matrix method engaging Euler's angles, described in sources [15], it is necessary to fulfil 45 mathematic operations, in particular 30 multiplication operations, 15 addition operations and 3 operations to define arccosines of angles in the similar case.

It is obvious that applying quaternion calculation method enables to decrease the number of mathematic operations in 1.55 times compared with classic matrix methods of angle calculation. At the same time, taking into account the fact that multiple observations are applied in measurement, the number of results *N* may be quite large, so it obviously leads to decreasing the time spent and RAM capacity of computer system of IPAMS. Measurement performance increases due to this fact. In general, a complex approach to applying quaternion methods of calculating angles and other methods and means of automated processing of measurement information in GPIS, described in sources [6, 10, 13, 14] enabled to increase performance of goniometric measurements in 9 times (up to 6.5 sec) in comparison with well-known goniometric means [12, 16].

The developed algorithm of GPIS functioning and software allowing application of a quaternion method and measuring GPIS angles in the automatic mode with the increased accuracy and performance (Figure 6).

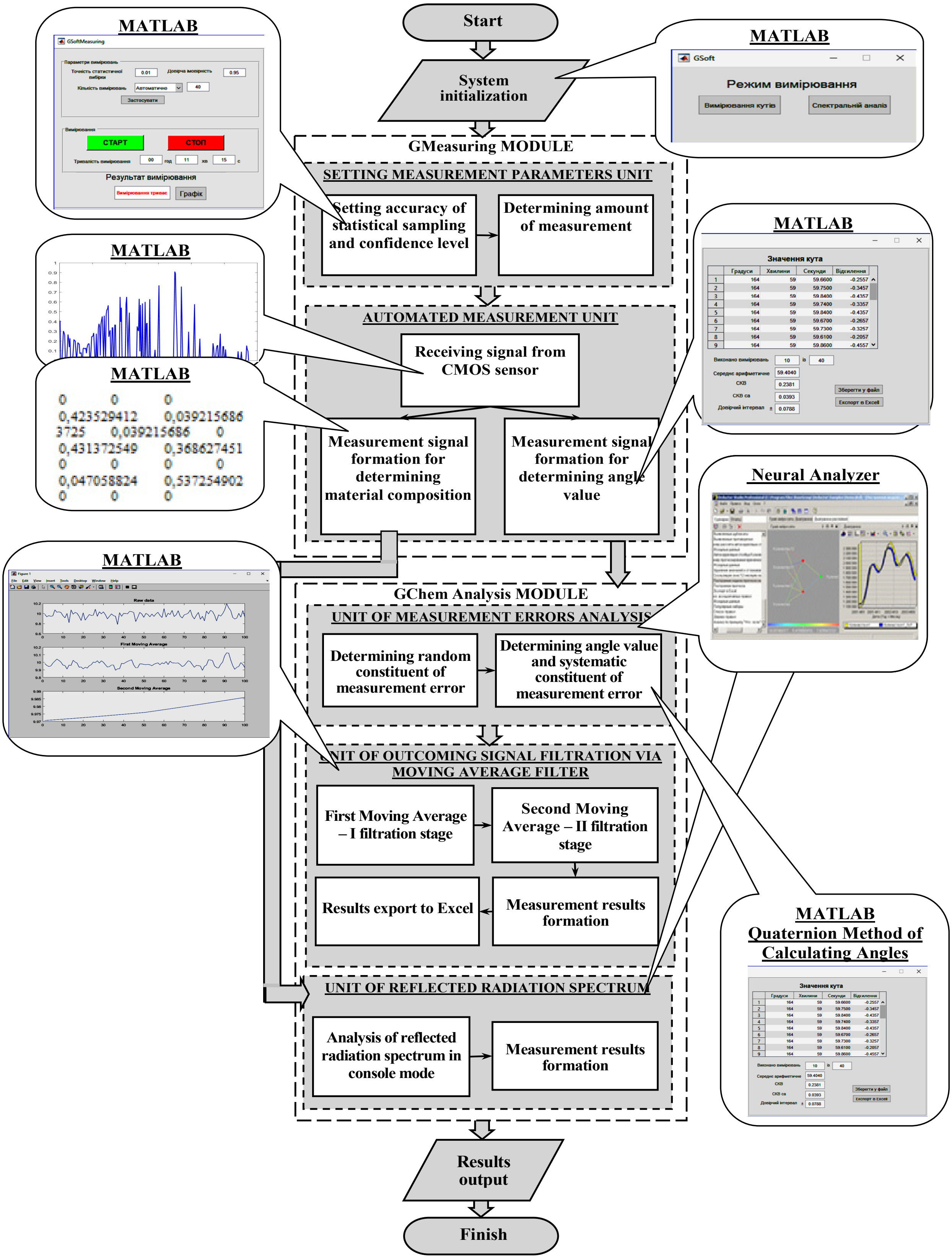


Figure. 6. Structure scheme of GPIS software functioning

Increasing accuracy of goniometric measurements GPIS in applying the suggested quaternion method of angles calculation is reached due to a smaller number of mathematical operations. It allows reducing rounding errors for both end and intermediate results of calculation that are accumulated in multiple measurements and may reach rather large values. In general, application of a quaternion method of calculating angles in IPAMS in complex with other methods and means of automated processing measurement information, described in sources [5, 10, 13, 14], enabled to increase accuracy in 3 times (by 0.2″), compared to famous goniometric means [12, 14].

Conclusion

1. It was introduced a quaternion method of calculation in goniometric measurements that serves as a basic theoretical and information ground for contactless precision goniometric measurements GPIS, with preliminary setting NSE, plane angles prisms pyramidality etc.
2. Application of a quaternion method of calculation in GPIS enables to expand its functional abilities, providing measurements of angle values in three-dimensional space with preliminary setting NSE, and also plane angles of polygonal prisms, and may be used in determining prisms pyramidality and other production objects, index of optical glass deflection.
3. It was defined that quaternion models enable calculation of angle values with preliminary setting NSE, plane angles of polygonal prisms and may be applied in determining prisms pyramidality and other production objects, index of optical glass deflection. Applying quaternion models of calculating angle values requires a smaller RAM capacity of PC and increases measurement performance.
4. A comparatively smaller amount of mathematical operations performed via quaternion method of angles calculation enables to reduce rounding errors in calculation results accumulated in multiple measurements and may reach larger values. Thus, measurement accuracy increases.
5. Application of quaternion method of angles calculation in GPIS enabled to increase accuracy by 0.2″ (3 times) and measurement performance from 6.5 s (9 times) compared to the well-known goniometric means.

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Гониометриялық дәл аспаптық жүйемен өлшеу кезінде бұрыштарды есептеудің кватерниялық әдісі

Мақала ғылым мен техниканың әр түрлі салаларында қажетті бұрыштық өлшеулердің дәлдігі мен жылдамдығын арттырудың өзекті мәселесіне арналған. Бұрыштық өлшеулердің дәлдігі мен жылдамдығын арттырудың жолы — өлшеу ақпаратын өңдеу үшін соңғы алгоритмдік әдістер мен математикалық құрылғыларды қолдану. Сондықтан, бұрыштық өлшеулердің дәлдігі мен әсер етуін арттыру мақсатында жұмыста белгілі гониометриялық дәл аспаптық жүйенің (ГДАЖ) мысалында гониометриялық өлшеулердегі бұрыштардың шамаларын сандық есептеу ұсынылды. Кватернионды есептеудің тиімділігі күмән тудырмайды, өйткені басқа дәстүрлі әдістерден айырмашылығы, кватерниондар (атап айтқанда Эйлер бұрыштарын қолданатын матрицалар, косинус бағыттаушылары) объектілердің бұрыштық позицияларын сипаттайтын төрт параметрмен ғана ұсынылған және алтыдан айырмашылығы бір ғана байланыс теңдеуі бар, матрицалық әдістер үшін, атап айтқанда, косинус бағыттаушыларына арналған. Ұсынылған кватернионды есептеу ГДАЖ бағыттағыш сезімтал элементтердің (БСЭ), жазық бұрыштардың, призмалардың пирамидалығының және т.б. алдын ала көрсету үшін жанаспайтын дәл гониометриялық өлшемдердің жалпы теориялық-ақпараттық негізі ретінде пайдаланылды. Әзірленген кватернионды есептеуді қолдану белгілі есептермен салыстырғанда дәлдікті 0,25 (3 есе) және өлшеу жылдамдығын 9 есе (6,5 С дейін) арттыруға мүмкіндік береді. Бұрыштардың шамаларын кватернионды есептеуді қолдану дербес компьютердің жедел жадының аз көлемін пайдалануды көздейді, бұл жүйенің жұмысын жақсартады. Сонымен қатар, бұрыштарды есептеудің квадрат әдісін қолдана отырып орындалатын математикалық операциялардың аз саны, өнімділікті арттырумен қатар, бірнеше өлшеулерде жиналатын және үлкен мәндерге жететін есептеу нәтижелерінің жуықтау қатесін азайтуға мүмкіндік береді. Осылайша өлшеулердің дәлдігі мен жылдамдығы артады.

Кілт сөздер: кватернион, гониометриялық жүйе, дәлдік, жылдамдық, бұрыштарды өлшеу, дәл жүйе.

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Кватернионный метод расчета углов при измерениях гониометрической прецизионной приборной системой

Статья посвящена актуальной проблеме — повышению точности и быстродействия угловых измерений, необходимых в различных областях науки и техники. Одним из путей повышения точности и быстродействия угловых измерений является применение новейших алгоритмических методов и математических аппаратов для обработки измерительной информации. Поэтому в работе с целью повышения точности и быстройдействия угловых измерений на примере известной гониометрической прецизионной приборной системы (ГППС) предложен кватернионный расчет величин углов при гониометрических измерениях. Эффективность кватернионного расчета не вызывает сомнения, поскольку, в отличие от других традиционных методов (в частности, матричных с применением углов Эйлера, направляющих косинусов), представляется только четырьмя параметрами, описывающими угловые положения объектов, и имеет лишь одно уравнение связи, в отличие от шести, для матричных методов, в частности, для направляющих косинусов. Предложенный кватернионный расчет используется в ГППС как общая теоретико-информационная основа бесконтактных прецизионных гониометрических измерений при предварительной выставке навигационных чувствительных элементов, плоских углов, пирамидальности призм и т.п. Применение разработанного кватернионнного расчета позволило повысить точность на 0,25″ (в 3 раза) и быстродействие измерения в 9 раз (до 6,5 с) по сравнению с известными. Применение кватернионного расчета величин углов предусматривает использование меньшего объема оперативной памяти персонального компьютера, что повышает быстродействие работы системы. Кроме того, меньшее количество математических операций, выполняемых с использованием кватернионного способа расчетов углов, кроме повышения быстродействия, позволяет уменьшить погрешность округления результатов вычислений, которая накапливается при многократных измерениях и может достигать больших значений. Таким образом, повышаются точность и быстродействие измерений.

Ключевые слова: кватернион, гониометрическая система, точность, быстродействие, измерение углов, прецизионная система.

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