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Dynamics of interaction of vortices in shear turbulent flows

The paper analyzes the results of a theoretical study of quasi-two-dimensional turbulence, two-dimensional equations of motion of which contain additional terms. The regularities of the dynamic interaction of vortex structures in shear turbulent flows of a viscous liquid are established. Based on the model of quasi-two-dimensional turbulence, numerical values of the spatial scales of intermittency are determined as an alternation of large-scale and small-scale pulsations of dynamic characteristics. The experimentally observed alternation of vortex structures and the idea of their self-organization form the basis of the assumption of the existence of a geometric parameter determined by the size of the vortex core and the distance between their centers. Therefore, the main attention is paid to the theoretical calculation of the minimum spatial scales of the intermittency of vortex clusters. As a simplification, the vortex pairs are located in a reference frame, relative to which the centers of the vortices are stationary. Thus, the kinematic effect of the transfer of one vortex into the field of another is excluded from consideration. The symmetric and unsymmetric interactions of vortices, taking into account the one-sided and opposite directions of their rotation, are considered as realizable cases. A successful attempt is made to study the influence of the internal structure of vortex clusters on the numerical values of the minimum intermittency scales. The obtained results are satisfactorily confirmed by known theoretical and experimental data. Consequently, they can be used in all practical applications, without exception, where the structure of turbulence is taken into account, as well as for improving and expanding existing semi-empirical theories.

Keywords: turbulence, structures, vortex clusters, intermittency scales, self-organization, quasi-two-dimensional turbulence model, vorticity, fractals.

Introduction

The results of numerous experiments [1, 2] give a clear picture of the vortex structures of shear turbulent flows. The appeared large-scale vortices interact with each other and collapse, resulting in the formation of a cellular structure of developed turbulence downstream. The same pattern can be seen with small scales, which indicates the self-similarity of the process of splitting vortices.

The turbulent flow as a stochastic dynamic system has a special property that has high information content – intermittency, which reflects as an alternation of small-scale pulsations of dynamic characteristics with bursts of large-scale pulsations [3]. The experimentally observed intermittency in turbulent environment is due to the passage through fixed points of spatially separated vortex formations in various forms and phases of their evolution. Therefore it is possible to introduce a quantitative dynamic characteristic – the degree of intermittency [4]

$$æ = l/d_0;\ 0 \le 1/æ \le 1,$$

where l – distance between the centers of vortices, $d_0=2r_0$ – diameter of the vortex core. The idea of the self-organizing nature of intermittent vortex structures points to the possible universality of the internal parameter *æ*, rather than the external geometric scales of the flow. Knowing the realizable values of *æ* allows to specify a certain type of turbulent motion. The formation of vortex clusters-paired vortices with the same and opposite circulations is the most common pattern of shear (gradient) flows. The pairing of vortices with the same rotations is mainly realized in the mixing layer at the free boundary of the jets, and with opposite circulations in the wake of the body in the boundary layer on the axis of the jets in the region of the establishment of the turbulent flow. In the developed turbulent flow, where the role of boundary conditions weakens, both types of vortex clusters are realized [2].

The study of interaction of vortices is one of the central tasks of hydrodynamics, which has both direct application, for example, to the description of geophysical phenomena [5, 6] and general physical significance associated with the knowledge of the regularities of the formation of the energy spectrum of turbulence [7]. A number of publications are devoted to the study of the structure of turbulence and its accounting in various practical applications [4, 8–12]. In contrast to the known studies, this article examines the possibility of a theoretical description of the interaction of vortices of a viscous liquid taking into account the difference in the directions of rotation.

Problem and research method

If we choose the spatial scales so that we can neglect the size of the vortex formations considering them as points, then their movement as a whole is always three-dimensional, turbulence is a chaotic phenomenon in the usual sense of the word. But within the scale of the vortex formations (the physical scale of the self-organization structures) on the basis of main (averaged) flow it is always possible to emphasize a quasi-two-dimensional motion associated with an increase in the rotation of the liquid along the cross-section of the vortex tubes. The predominance of rotation in one direction is due to the stretching of the vortex tubes before they break as a consequence to the tendency to move away from each other (weakening of the correlations of dynamic characteristics) of fixed liquid particles, which is one of the pivotal mechanisms for generating turbulence [3].

Quasi-two-dimensionality of motion means the correlation with rotation on the plane with the pulsational motion caused by a non-stationary perturbation of the surface of any section. In the self-organization of vortex structures, which is a balanced inverse process to their decay, the separation of one direction of rotation also occurs. Thus, the structural properties of turbulence can be described in a quasi-two-dimensional approximation.

Figure 1 shows a scheme of a quasi-two-dimensional vortex packet with a free surface having a core with radius r_0 and angular velocity $\vec{\Omega}_0$. The quasi-two-dimensionality is reflected by the possibility of perturbation of the vortex surface level $\eta(x,y,t)$ and the negligible smallness of the change in the average motion characteristics over the equilibrium thickness h_0.

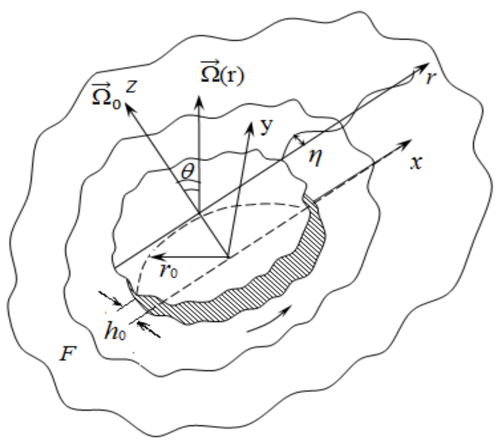

Figure 1. Scheme of a quasi-two-dimensional vortex packet

θ denots the angle between the direction of the angular velocity of the vortex core $\vec{\Omega}_0$ and the direction $\vec{\Omega}(r)$ – the angular velocity of the particle located at a distance r. The angular velocity of the vortex core $\vec{\Omega}_0$ can be estimated through the condition of stationary turbulence generation by an external flow with a velocity U_0 in the form of $\Omega_0 \sim U_0/r_0 \sim U_0^2/\nu$.

In order to find out the dynamic regularities of a quasi-two-dimensional vortex packet in a submerged environment it is necessary to determine the velocity fields $\vec{\upsilon}$ and pressure p formed by the initial circulation

$$\Gamma_0 = 2\pi\Omega_0 r_0^2$$

either on the planes (x, y) in the Cartesian or (r, φ) in the polar coordinate system.

The vortex motion is characterized by the vorticity $rot\vec{\upsilon}$ at each point. Some of its average value (for example, in time t and in angle φ) $< rot\vec{\upsilon} > = 2\vec{\Omega}(r)$ is supported by the stationary generation of turbulence by external influences: the main flow, the inhomogeneity of dynamic, temperature fields and so forth. The correlation of $\vec{\Omega}(r)$ to the circulation is determined by Thomson's theorem:

$$\int_{F_0} < rot\vec{\upsilon} > dF_0 = 2\int_{F_0} \vec{\Omega}(r) dF_0 = \Gamma_0,$$

where F_0 is the contractible simply connected surface of the vortex as a whole with respect to the average dynamic characteristics.

In accordance with the quasi-two-dimensional feature of the task we proceed from the equations of the theory of "shallow water" [13]. Taking into account the viscosity, based on the above the equation of motion can be written in the form [14, 15]:

$$\frac{\partial\vec{\upsilon}}{\partial t} + (\upsilon\nabla)\vec{\upsilon} = -r_0\Omega_0^2\nabla\eta - 2\left[\vec{\Omega}\vec{\upsilon}\right] + \nu\nabla^2\vec{\upsilon} + \frac{\nu}{3}\nabla div\vec{\upsilon}. \quad (1)$$

The continuity equation is obtained from the condition of constancy of the mass of a liquid with a constant density passing through the fractal (depending on $\eta(x, y, t)$) surface of the vortex

$$F(\eta) = F_0 + F' = F_0 + r_{0*}\eta, \quad (2)$$

where r_{0*} is some effective size of the vortex. The area of the fractal surface $F(\eta)$ can be expressed in terms of F_0, η and the fractal dimension of the turbulent vortex. Expression (2) should be considered as a decomposition of $F(\eta)$ over a small argument, and r_{0*} can be expressed in through r_0 due to the possibility of renormalizing r_0 by the condition

$$\upsilon(r_0) = \upsilon_0 = \Omega_0 r_0 = \Gamma_0/2\pi r_0.$$

Fractality, i.e., the perturbation of the vortex surface, corresponds to the turbulent mixing of the incompressible fluid mass. Then finally the continuity equation takes the form

$$div\vec{\upsilon} = -\frac{1}{r_0}\left(\frac{\partial\eta}{\partial t} + (\vec{\upsilon}\nabla)\eta\right). \quad (3)$$

The equations of motion (1) and continuity (3) allow us to determine the velocity and pressure distributions associated with $\eta(x,y\ t)$ in a turbulent vortex. The equation in the form (1) in a differentially rotating coordinate system with an angular velocity $\Omega(r)$ is valid only if there is no dependence of Ω on the other variables, i.e., it corresponds to the simplest implementation of the Taylor hypothesis [3] about the conservativeness of the vortex, or, more generally, to the position of synergetics about the possibility of the existence of a stationary, but nonequilibrium ("standard") state [16].

Calculations and discussion

Symmetric dynamic interaction of vortices. Figure 2 shows graphical images of both types of vortex clusters. The characteristic spatial scales are indicated: r_0 – the radius of the core (the region of quasi-solid rotation) of the vortex, r_ν – the size of the dissipative region of motion, where there is no oncoming directional motion due to the inhibitory effect of viscosity (with the kinematic coefficient ν), l_1, l_2 — the minimum distances between the centers of interacting vortices, respectively, with one-sided and opposite rotations, l_1', l_2' – the size of the localization areas of vortex clusters. Let us consider vortex clusters in a frame of reference with respect to which the centers of the vortices are stationary. In this formulation of the task the dynamic interaction of vortices is described, excluding the kinematic effect of the transfer of one vortex into the field of another. The designations of the main parameters of the task are shown in Figure 3. At the points $(-x_1, x_1)$, $(-x_2, x_2)$ on the x – axis the centers of the nuclei of vortices with radius r_0, rotating with an angular velocity of Ω_0 are located. The values x_1, x_2 are the result of the process of interaction of vortices (self-organization) and are determined from the dynamics of the formation of vortex clusters.

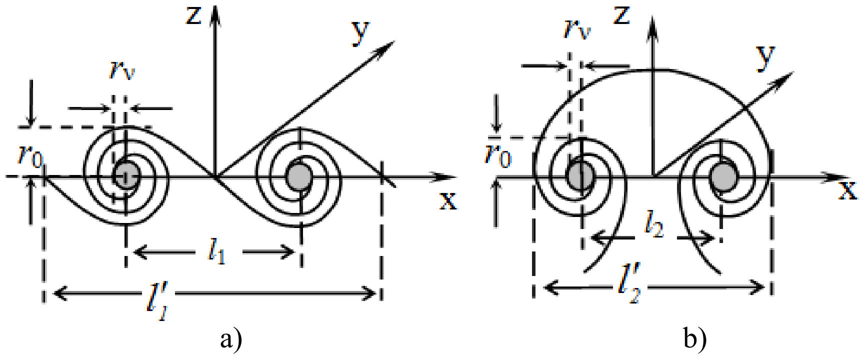

Figure 2. Graphical representation of vortex clusters.
a) one-sided, b) opposite rotation of the elements of vortex clusters

Expressing the distribution of the average vorticity

$$2\Omega(r) = \langle rot\, \vec{\upsilon}(r, \phi) \rangle, \tag{4}$$

where $\vec{\upsilon}$ is the velocity vector, r, φ are the polar coordinates corresponding to the coherent state of the vortex pairs, under certain physical conditions can be searched for throughout the values x_1, x_2, the fluid motion region.

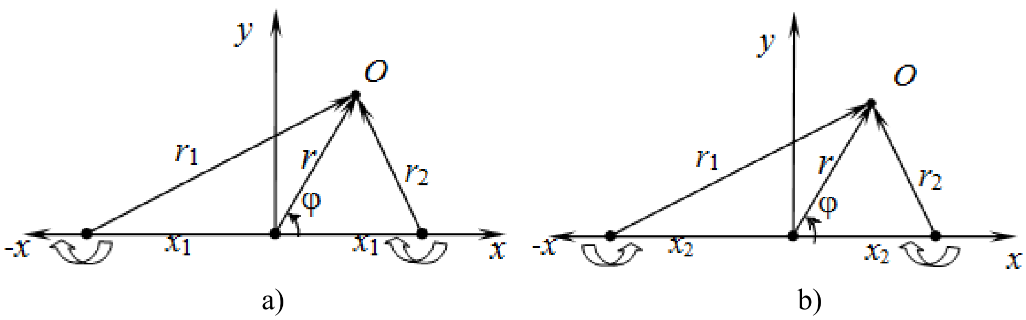

Figure 3. Calculated scheme of interaction of vortices.
a) one-sided, b) opposite rotation of the vortices

Without taking into account the internal structure of the vortices, the distribution of the average vorticity is expressed in terms of Ω_0 using Thomson's theorem:

$$\Omega(r) = \Omega_0\, r_0^2 / r^2. \tag{5}$$

The resulting average vorticity at the point O of the system of two interacting vortices (a scalar value) is determined by the following formulas

$$\Omega_j(r) = \Omega_0 r_0^2 \left(\frac{1}{r_2^2} \pm \frac{1}{r_1^2} \right), \qquad j = 1, 2, \ldots, \tag{6}$$

where Ω_1, Ω_2 and the signs "+", "–" correspond to one-sided ($j = 1$) and opposite ($j = 2$) rotations of the elements of the vortex pairs. Using the cosine theorem, we define r_j from Fig. 3 in terms of r, φ, x_j:

$$r_1^2 = x_j^{\,2} + r^2 + 2x_j r\cos\varphi, \quad r_2^2 = x_j^{\,2} + r^2 - 2x_j r\cos\varphi$$

Substitute the obtained values in (5) and (6):

$$\Omega_1(r,\varphi) = 2\Omega_0 r_0^2 \left(r^2 + x_1^{\ 2}\right) \Big/ f_1(r,\varphi), \tag{7}$$

$$\Omega_2(r,\varphi) = 4\Omega_0 r_0^2 x_2 r cos\varphi \Big/ f_2(r,\varphi), \tag{8}$$

where $f_j(r,\phi) = \left(r^2 + x_j^{\ 2}\right)^2 - 4x_j^{\ 2} r^2 \cos^2\phi, \quad j = 1,2.$

Characteristics of homogeneous turbulence are considered on the basis of the model of a quasi-two-dimensional vortex packet [14, 17, 18]. It follows that the simplest vortex clusters formed from elements of homogeneous, isotropic turbulence should be described by the regularity (4) in the regions of motion, where their internal structure can be ignored. This statement is taken as a physical condition that determines the minimum spatial scale of localization of vortex clusters, beyond which they can be considered point structures.

Thus, two conditions are formulated for determining the desired scales of vortex clusters x_1, x_2: the implementation of Thomson's theorem for the average vorticity outside this scale and the implementation of an extreme coherent state with minimal scales x_1, x_2. These accepted conditions will be taken into account when solving the problem under consideration to simplify the conservation law (instead of the equations of motion), the form of which will be set below.

The energy of the bound state of the vortices is less than the additive sum of the energies of the individual vortices. Therefore, in the task under consideration energy is not an invariant: when the vortices are clustered, energy is dissipated – the energy is transferred to a smaller-scale motion. The momentum of the vortex motion is not true, but a quasi-pulse, which is also not preserved during the interaction of vortices. The universal mechanism for maintaining turbulence is the presence of an energy flow caused by the influx of energy from outside and its dissipation in the medium. Therefore, to establish the most general patterns of interaction of vortices it is necessary to link the main characteristics of the problem with the value of the energy flow.

In the presence of vortices of different signs the invariant characteristic is the square of the vorticity $|rot\vec{\upsilon}|^2$, or, enstrophy. The following connection of enstrophy with the energy dissipation of turbulent vortices is generally accepted [3]:

$$\varepsilon = \frac{d\left(\upsilon^2/2\right)}{dt} = 2\nu \int\limits_0^\infty k^2 \mathrm{E}\left(\vec{k}\right) d\vec{k} = \nu |rot\vec{\upsilon}|^2,$$

where $\mathrm{E}\left(\vec{k}\right)$ is the spectral energy density of the turbulence. In the question under consideration about the energy distribution with intermittency on the scale $1/k \sim r$, $r_{\min} = x_1, x_2$, obviously, the invariant value will be:

$$r \cdot \nu |rot\vec{\upsilon}|^2 = const. \tag{9}$$

Or by using angle velocity:

$$r \cdot \Omega^2(r) = const. \tag{10}$$

From the conservation law (10) we determine the desired interleavability scales, using the conditions formulated above for the minimality of their values and the fulfillment of Thomson's theorem outside of these scales. Equating the derivative of expression (10) with respect to r for $r = x_1, x_2$ to zero we have:

$$\left(\Omega_1(r) + 2r\Omega_1'(r)\right)\Big|_{r=x_1} = 0, \quad \left(\Omega_2(r) + 2r\Omega_2'(r)\right)\Big|_{r=x_2} = 0. \tag{11}$$

In order to exclude the derivatives of $\Omega(r)$ we write the expressions (11) in the form:

$$\frac{d\Omega_1(r)}{\Omega_1(r)} = -\frac{dr}{2r}\bigg|_{r=x_1}, \qquad \frac{d\Omega_2(r)}{\Omega_2(r)} = -\frac{dr}{2r}\bigg|_{r=x_2},$$

from which it follows

$$\Omega_1(x_1)=\frac{C_1}{\sqrt{x_1}}, \qquad \Omega_2(x_2)=\frac{C_2}{\sqrt{x_2}},$$

where C_1, C_2 – constant integrations.

According to fig. 3, the desired scales x_1, x_2 are determined for the direction $\varphi=0$, and in accordance with formulas (7), (8) we have

$$\mathrm{C}_1=2\Omega_0\, r_0^{1/2}, \qquad \mathrm{C}_2=4\Omega_0\, r_0^{1/2}, \tag{12}$$

$$\left(\bar{x}_1^2+1\right)^2-4\bar{x}_1^2-\left(\bar{x}_1^2+1\right)\bar{x}_1^{1/2}=0, \quad \bar{x}_1=\frac{x_1}{r_0}, \tag{13}$$

$$\left(\bar{x}_2^2+1\right)^2-4\bar{x}_2^2-\bar{x}_2^{3/2}=0; \qquad \bar{x}_2=\frac{x_2}{r_0}, \tag{14}$$

where the indexes 1, 2 refer respectively to vortex clusters with the same and opposite rotations of their elements, the C_1, C_2 constants are determined for the case $r=r_0$. Numerical solutions of equations (13), (14) give the desired results in the form of

$$x_1=1.86\cdot r_0, \qquad x_2=1.54\cdot r_0. \tag{15}$$

The results (15) can be obtained directly from the equation of motion of a quasi-two-dimensional vortex of a viscous liquid, taking into account the inhomogeneity of the turbulent motion [18].

The existence of a limiting degree of intermittency was shown earlier [4]. The minimum value of the degree of intermittency is determined from the recurrent formula for the energy of vortex structures:

$$\chi=D_\infty/2L_\infty=\sqrt{3}=1.73, \tag{16}$$

where D_∞ is the distance between the centers of the two vortices, L_∞ is the root-mean-square radius of the vortex after an infinite repetition of the process of merging the vortices.

Taking into the account that given work is considering the statistical theory of structureless vortices, a satisfactory coincidence of the results (16) with (15), the average value of which is 1.70, can be noted. The proposed model of the interaction of vortices allows us to analyze the known visual patterns of flows [2], one of which is shown in figure 4 as an example.

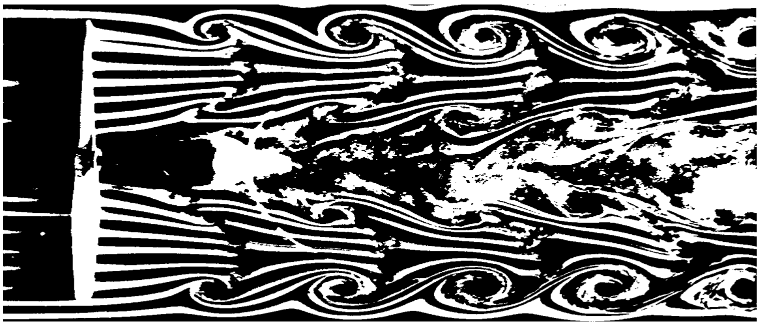

Figure 4. Vortices behind the rotating screw

Taking into account the quality of the photo, possible distortions, and the absence of some experimental parameters, the obtained values $x_1 \approx (1.9 \pm 0.1)r_0$ and $x_2 \approx (1.6 \pm 0.1)r_0$ confirm the results of (15).

Asymmetric interaction of vortices. Let us consider a system of two vortices (fig. 5), when they may differ in their evolution in size (radii of the nuclei r_{0i}) or angular velocities Ω_{0i}. Since the comparison of theoretical results with experimental data is conveniently (possibly) carried out as the ratio of the radii of the vortex cores, the asymmetry of the system of two vortices is represented as the ratio of circulations [18]. This is all the more justified for the case of homogeneous turbulence or free flows (without streamlined solid surfaces), when due to pressure equalization the angular velocities are equal to $\Omega_1=\Omega_2$ and the change in the size of the vortices is equivalent to a change in the circulations. So, if we enter the unbalance parameter in the form

$$\alpha = \frac{\Omega_{02} r_{02}^2}{\Omega_{01} r_{01}^2}, \tag{17}$$

then from the expressions (6) we obtain, respectively, for a system of equally rotating vortices (sign "+") and oppositely rotating vortices (sign "–")

$$\Omega_j(r) = \Omega_{01} r_{01}^2 \left(\frac{\alpha}{r_2^2} \pm \frac{1}{r_1^2} \right). \tag{18}$$

The amount of α lies within $0 \le \alpha \le 1$; it cannot be less than zero for physical reasons, $\alpha > 1$ would only mean a change in the numbering of the vortices. Then, using the cosine theorem for the transformation (18) and the condition for the invariance of the enstrophy moment in the dynamic interaction of vortices (9), we obtain two equations similar to (13) and (14):

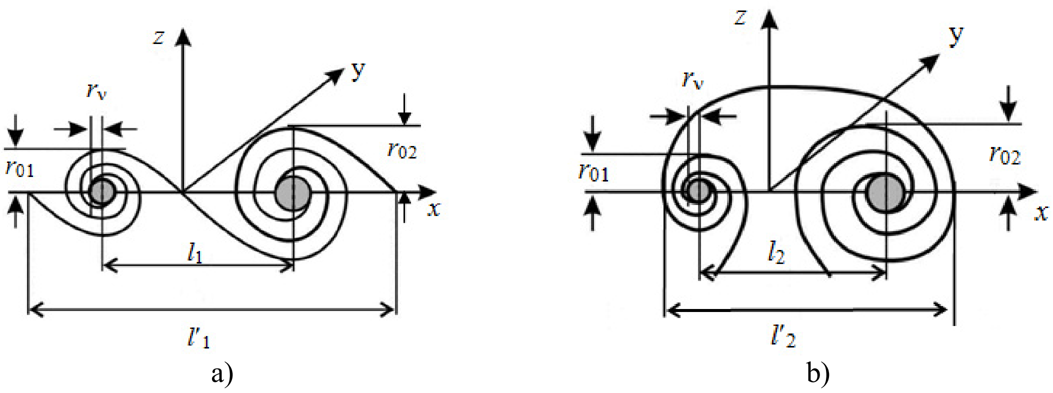

Figure 5. Graphical representation of the asymmetric interaction of vortices.
a) one-sided, b) opposite rotation of the elements of vortex clusters

$$\frac{\sqrt{\bar{x}_1}}{2}\left\{\left(\bar{x}_1^2+1\right)(1+\alpha)-2\bar{x}_1(1-\alpha)\right\}=\left(\bar{x}_1^2-1\right)^2, \tag{19}$$

$$\frac{\sqrt{\bar{x}_2}}{4}\left\{2\bar{x}_2(1+\alpha)-\left(\bar{x}_2^2+1\right)(1-\alpha)\right\}=\left(\bar{x}_2^2-1\right)^2, \tag{20}$$

where α is defined by the formula (17). The numerical solutions (19), (20) are presented in the form of graphs in Fig.6. The constants C_i analogous to (12) are equal to $C_1 = 2\Omega_{01} r_{01}^{1/2}, C_2 = 4\Omega_{01} r_{01}^{1/2}$.

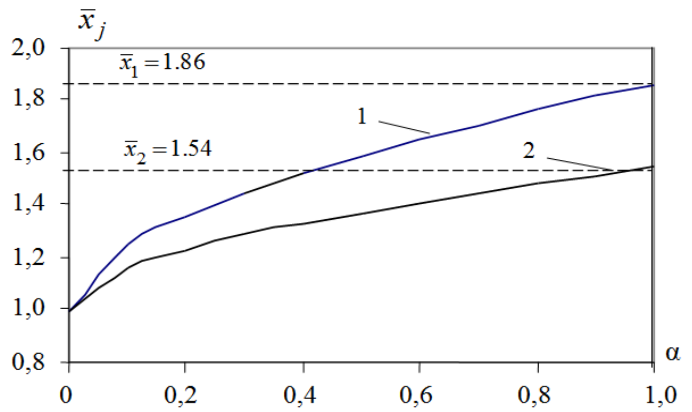

Figure 6. Change in the minimum scale of intermittency in the case of asymmetric interaction of vortices: 1 – one-sided, 2 – opposite rotation of the vortices.

As can be seen from the figure, the solutions of equations (19), (20) at $\alpha = 1$ coincide with the solutions of (15), and at $\alpha = 0$, as can be expected, $\bar{x}_1 = \bar{x}_2 = 1.0$. The value $\alpha = 0$ corresponds either to the absence of the second vortex, or to the condition $r_{oi} >> r_{oj}$, and x_i must coincide with the boundary of the core: $\bar{x}_1 = \bar{x}_2 = r_0$.

Interaction of vortices with the internal structure. The fixed values (15) of the minimum spatial scales of intermittency, as well as their dependence on the asymmetry parameter (Fig. 6), are obtained without taking into account the structure for a homogeneous-isotropic developed turbulence. To account for the structure of vortex clusters it is necessary to express the average vorticity in terms of the current function ψ, which satisfies the continuity equation (3) and expresses the dynamic characteristics of the vortex. Because of that the expression (4) will be written in the following form:

$$\Omega(r) = -\frac{1}{2}\Delta\psi,$$

From (1) and (3), the amount of $\Delta\psi$ will be determined by the following expression [18]:

$$\Delta\psi(r_j) = \psi_{0j}\left[\frac{\sqrt{\lambda_0} sin(\sqrt{\lambda_0} ln\, r_j / r_0 - \lambda_0 cos(\sqrt{\lambda_0} ln\, r_j / r_0)}{r_j^2}\right], \; j = 1, 2;$$

where $\psi_{0j} = \Omega_0 r_{0j}^2$.

The resulting average vorticity of the system of two interacting vortices (formula (6)) in this case is written as:

$$\Omega_j(r) = -\frac{1}{2}\left[\Delta\psi(r_2) \pm \Delta\psi(r_1)\right]. \tag{21}$$

Expressing, as before r_j, through r, φ, x_j (fig. 3), after simple transformations from (21) we get the following expression

$$\Omega_j = \frac{\Omega_0}{2}\left[\frac{\lambda_0 A_j - \sqrt{\lambda_0} B_j}{\left(\bar{x}_j - 1\right)^2} \pm \frac{\lambda_0 A'_j - \sqrt{\lambda_0} B'_j}{\left(\bar{x}_j + 1\right)^2}\right], \tag{22}$$

where $A_j = cos\left[\sqrt{\lambda_0} ln\left(\bar{x}_j - 1\right)\right]$, $B_j = sin\left[\sqrt{\lambda_0} ln\left(\bar{x}_j - 1\right)\right]$,

$A'_j = cos\left[\sqrt{\lambda_0} ln\left(\bar{x}_j + 1\right)\right]$, $B'_j = sin\left[\sqrt{\lambda_0} ln\left(\bar{x}_j + 1\right)\right]$, $\bar{x}_j = \frac{x_j}{r_0}$.

In the output (22) it is taken into account that the desired scales $\bar{x}_j$ are determined for the direction $\varphi = 0$ at $r = r_0$. Next, we use the conservation law for this task in the form of the invariance of the enstrophy moment (see formula (9)). For the common solution of (9) and (22) we apply the condition of extremality of the intermittency scales. Equating to zero the derivative of expression (9) with respect to r at $r = x_j$ we have:

$$\left[\Omega_j(r) + 2r\Omega'_j(r)\right]\Big|_{r = x_j} = 0,$$

from which it follows

$$\Omega_j\left(x_j\right) = \frac{C_j}{\sqrt{r_0}\sqrt{\bar{x}_j}}, \tag{23}$$

where C_1, C_2 are the constant integrations. Equating the right-hand sides (22) and (23), we obtain the following equations with respect to $\bar{x}_j$

$$\sqrt{\bar{x}_1}\left[(\bar{x}_1+1)^2\left(\lambda_0 A_1-\sqrt{\lambda_0}B_1\right)+(\bar{x}_1-1)^2\left(\lambda_0 A_1'-\sqrt{\lambda_0}B_1'\right)\right]=4\left(\bar{x}_1^2-1\right)^2, \qquad (24)$$

$$\sqrt{\bar{x}_2}\left[(\bar{x}_2+1)^2\left(\lambda_0 A_2-\sqrt{\lambda_0}B_2\right)-(\bar{x}_2-1)^2\left(\lambda_0 A_2'-\sqrt{\lambda_0}B_2'\right)\right]=8\left(\bar{x}_2^2-1\right)^2, \qquad (25)$$

$$C_1=2\Omega_0 r_0^{1/2}, \quad C_2=4\Omega_0 r_0^{1/2},$$

where the indexes 1, 2 refer, respectively, to vortex clusters with one-sided and opposite rotations of their elements. The results of the numerical solution of equations (24) and (25) are shown in figure 7.

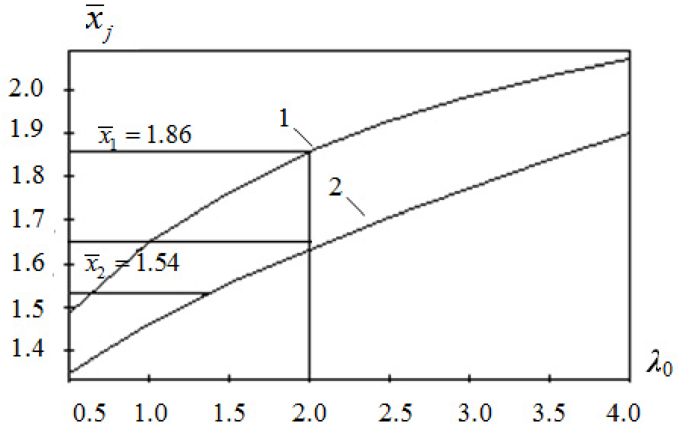

Figure 7. Minimum spatial scales of intermittency of vortex clusters with the internal structure of their elements: 1 – one-way, 2 – opposite rotation of the elements. $\bar{x}_j = x_j / r_0$

As can be seen from the figure, at $\lambda_0 = 2$, corresponding to the developed turbulence [18], the values of $\bar{x}_j$ approach their values for the structureless interaction $\bar{x}_1 = 1.86$ and $\bar{x}_2 = 1.54$.

Conclusion

Based on the quasi-two-dimensional turbulence model, using fairly simple methods, the numerical values of the minimum intermittency scales are calculated. It is shown that the intermittency, in this case, the alternation in space of large-scale pulsations of the dynamic characteristics of a turbulent medium with small-scale one, is explained and described by the result of the interaction of vortices, the structure of the resulting vortex clusters with one-sided and opposite rotations of their elements. The results obtained are confirmed by well-known thermoanemometric measurements, experiments on flow visualization, and the statistical theory of structureless vortices.

The established dynamic regularities of intermittency can be useful for improving the existing semi-empirical theories of turbulence by reducing the number of empirical constants, clarifying and expanding the types of hydrodynamic tasks. The agreement with the experiment of the conclusions of the given dynamic approach gives grounds to use analytical expressions of the structural elements of turbulence to describe the correlation and spectral patterns of inhomogeneous turbulence.

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Турбулентті ағыстардағы құйындық құрылымдардың динамикалық өзара әрекеттері

Мақалада құрамында қосымша мүшелері бар екі өлшемді қозғалыс теңдеулерімен өрнектелетін квазиекіөлшемді турбуленттілікті теориялық әдістермен зерттеудің нәтижелері сарапталды. Тұтқыр сұйықтықтың ығысу турбулентті ағысындағы құйындық құрылымдардың динамикалық өзара әрекетінің заңдылықтары тұжырымдалған. Квазиекіөлшемді турбуленттілік моделі негізінде өзара алмасудың кеңістіктік масштабтарының сандық мәндері динамикалық сипаттамалардың үлкен- және кішкенемасштабты лүпілдерінің алма-кезек ауысулары түрінде анықталған. Тәжірибе жүзінде бақыланатын құйындық құрылымдардың алма-кезек ауысуы мен олардың өзіндік қауымдасуы туралы көзқарастың құйындар ядросының өлшемдері мен олардың арасындағы қашықтықпен анықталатын кейбір геометриялық параметрдің бар болуы туралы болжамның негізін құрайтыны белгілі. Сондықтан негізгі көңіл құйындық кластерлердің кезек ауысуының кіші кеңістіктік масштабын теориялық әдіспен есептеуге бөлінген. Есептеуді ықшамдау үшін қос құйындар, олардың центрлері салыстырмалы түрде қозғалмайтын, санақ жүйесінде орналастырылған. Осыған байланысты бір құйынды екінші құйынның өрісіне апарған кезде мүмкін болатын кинематикалық эффект ескерілмейді. Нақты мүмкін болатын жағдайлар ретінде құйындардың бір бағыттағы және қарсы бағыттағы айналуларын ескере отырып, олардың симметриялы және симметриялы емес өзара әрекеттері қарастырылған. Кезек алмасудың кіші масштабтарының сандық мәндеріне құйындық кластерлердің ішкі құрылымының әсері анықталған. Алынған нәтижелер белгілі тәжірибелік және теориялық деректермен салыстырылып, олардың сәйкестігі дәлелденген. Осыған байланысты зерттеу нәтижелері ортаның турбуленттілік құрылымдары ескерілетін барлық техникалық және технологиялық қолданыстарда жүзеге асырылып, белгілі жартылайэмпирикалық теорияларды кеңейтіп, жетілдіруге қолданылуы мүмкін.

Кілт сөздер: турбуленттілік, құрылымдар, құйындық кластерлер, кезек ауысудың масштабы, өзіндік қауымдасу, квазиекіөлшемді турбуленттілік моделі, құйындалу, фракталдар.

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Динамика взаимодействия вихрей в сдвиговых турбулентных течениях

В статье проанализированы результаты теоретического исследования квазидвумерной турбулентности, двумерные уравнения движения которых содержат дополнительные слагаемые. Установлены закономерности динамического взаимодействия вихревых структур в сдвиговых турбулентных течениях вязкой жидкости. На основе модели квазидвумерной турбулентности определены численные значения пространственных масштабов перемежаемости как чередование крупно- и мелкомасштабных пульсаций динамических характеристик. Экспериментально наблюдаемое чередование вихревых структур и идея об их самоорганизации составляют основу предположения о существовании геометрического параметра, определяемого размерами ядра вихрей и расстоянием между их центрами. Поэтому основное внимание уделено теоретическому расчету минимальных пространственных масштабов перемежаемости вихревых кластеров. В качестве упрощения вихревые пары расположены в системе отсчета, относительно которой центры вихрей неподвижны. Тем самым из рассмотрения исключается кинематический эффект переноса одного вихря в поле другого. В качестве конкретно реализуемых случаев рассмотрены симметричное и несимметричное взаимодействия вихрей с учетом одностороннего и противоположного направлений их вращения. Авторами предпринята попытка изучить влияние внутренней структуры вихревых кластеров на численные значения минимальных масштабов перемежаемости. Полученные результаты подтверждаются известными теоретическими и экспериментальными данными. Следовательно, они могут быть использованы во всех, без исключения, практических приложениях, где имеет место учет структуры турбулентности, а также для совершенствования и расширения существующих полуэмпирических теорий.

Ключевые слова: турбулентность, структуры, вихревые кластеры, масштабы перемежаемости, самоорганизация, модель квазидвумерной турбулентности, завихренность, фракталы.

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