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Modeling the dynamics of charged drop of one liquid in another under the action of an electric field

The work is devoted to the study of the features of the behavior of a group of droplets of one viscous liquid in another under the influence of various physical fields. When considering the dynamics of two drops under the action of an electric field it is assumed that a drop in the form of a sphere with radius a will be placed in an electric field with an intensity \vec{E} , investigates how droplets will react to each other under the influence of an electric field. A mathematical model has been built and a computer program has been developed for the numerical solution of this problem. The behavior of several drops in an electric field is studied for different physical parameters of the material of the drops and the environment, as well as for different initial distributions of drops and the strength of the electric field. It is shown for the first time that emulsion droplets distributed in space, under the action of an electric field, begin to move and after a certain time a new stationary structure of droplets is formed. It was found that the relaxation time depends on the electric field strength, the size of the droplets and their initial distribution.

Keywords: droplet dynamics, electromagnetic field, dispersed system, mathematical model, computer model, computational domain, dynamics of an emulsion system.

Introduction

The theoretical and experimental study of the behavior of individual drops of one viscous liquid in another under the influence of various physical fields (thermal, acoustic, electromagnetic) is of great importance in solving technological problems in various industries. A huge number of scientists from all over the world have been and are engaged in research on various aspects of this problem and have contributed to solving the problem [1–3]. In the case of one drop, the results obtained by the authors of this work are given in [4].

An emulsion is considered, consisting of spherical drops of one viscous incompressible fluid, distributed in the volume of another viscous incompressible fluid, immiscible with the first. Both liquids are considered to be dielectric. When considering the dynamics of two drops under the action of an electric field, it is assumed that a drop in the form of a sphere with radius a will be placed in an electric field with an intensity \vec{E} , investigates how droplets will react to each other under the influence of an electric field.

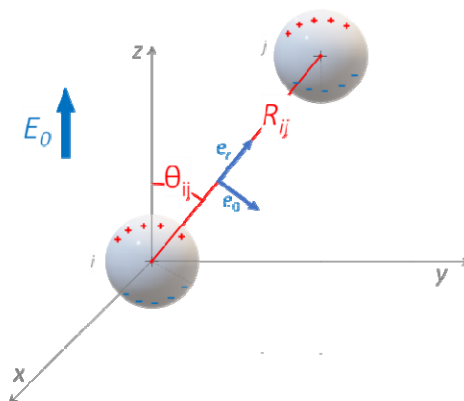


Figure 1. Interaction of two emulsion droplets in an electric field.

Mathematical model of the dynamics of two drops

Consider two spherical emulsion droplets i and j of the same diameter $d = 2a$, the distance between which R_{ij} (Fig. 1). It is assumed that the drops do not have a significant effect on the distribution of polarization charges in them and, using the point-dipole approximation, the electrostatic force acting on i -th drop from the j -th drop can be determined by the formula:

$$\vec{F}_{ij}^e = [(\vec{p}_i \cdot \nabla)\vec{E}_j]_{x=0}, \tag{1}$$

where \vec{p}_i is the equivalent dipole moment of the i -th drop (determined by the formula $\vec{p} = 4\pi\epsilon_0\epsilon_c\beta a^3 E_0 \vec{e}_z$, and \vec{E}_0, \vec{E}_j of the strengths of the electric field of the external environment and the electric field of the j -th drop, which are calculated, respectively, through the electrostatic potentials of the external environment, i.e.: $\varphi_c = -E_0 r \left[1 - \beta \left(\frac{a}{r}\right)^3\right] \cos\theta$ and drop $\varphi_d = -E_0 r \frac{3}{\alpha+2} \cos\theta$. Substituting into formula (1) the expressions $\vec{p}, \varphi_c, \varphi_d$ and $\alpha = \frac{\epsilon_d}{\epsilon_c}, \beta = \frac{\alpha-1}{\alpha+2}$ we get:

$$\vec{F}_{ij}^e = F_0 \left(\frac{d}{R_{ij}}\right)^4 [(3\cos^2\theta_{ij} - 1)\vec{e}_r + \sin 2\theta_{ij}\vec{e}_\theta], \tag{2}$$

$$F_0 = \frac{3}{16} \pi\epsilon_0\epsilon_c d^2 \beta^2 E_0^2.$$

When deriving formula (2), it was assumed that the origin of the spherical coordinate system is at the center of the i -th drop. Strictly speaking, formula (2) is exact only under the following conditions: the dielectric constants of the environment and droplets differ insignificantly, i.e., $\beta \rightarrow 0$; the diameter of the droplets is much less than the distance between them, i.e., $R_{ij}/d \rightarrow \infty$.

A qualitative analysis of formula (2) makes it possible to predict the nature of the motion of two drops under the action of an external electric field.

a) If the drops are located along the electric field strength vector coinciding with the direction of the axis z ($\theta_{ij} = 0$), then the vector \vec{F}_{ij}^e will be directed along the vector \vec{e}_r and the drops will attract.

b) If the line connecting the centers of the drops is perpendicular to the intensity vector ($\theta_{ij} = \pi/2$), then the direction of the vector \vec{F}_{ij}^e will be opposite to the direction of the vector \vec{e}_r and drops will repel.

c) In case of an arbitrary arrangement of drops the force \vec{F}_{ij}^e will rotate them, trying to arrange them in the direction of the intensity vector.

If the distance R_{ij} between the emulsion drops, it becomes enough, a short-acting repulsive force appears small (Fig. 2a)

$$\vec{F}_i^r(\vec{R}_i) = -F_0 \exp\left(-\kappa \frac{R_{ij} - d}{d}\right) \vec{e}_r, \tag{3}$$

where κ is characteristic distance at which repulsive forces act.

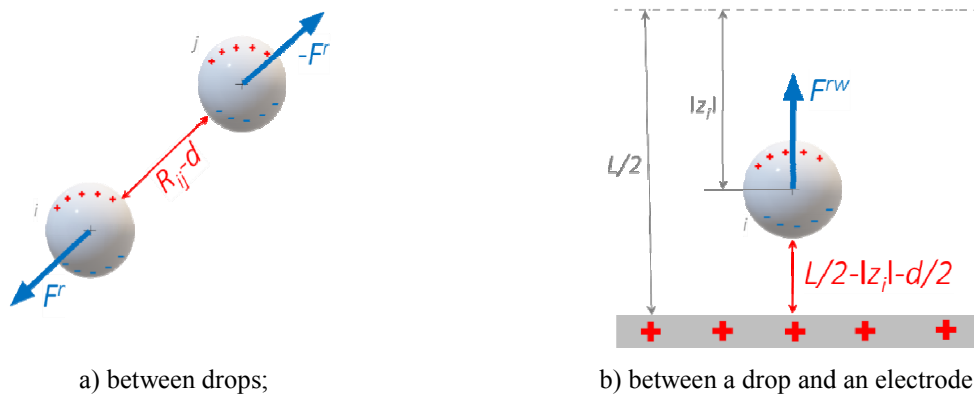


Figure 2. To the definition of repulsive forces

The repulsive force between the drop and the electrode is described similarly (Fig. 2b):

$$\vec{F}_i^{rw}(\vec{R}_i) = -F_0 \exp\left(-\kappa \frac{L/2 - |z_i| - d/2}{d}\right) \vec{n}, \quad (4)$$

where \vec{n} is normal vector to the electrode surface directed inward of the emulsion, z_i is axis coordinate of the drop center Z .

A mathematical model of the dynamics of an emulsion system in an electric field, taking into account the considered forces, can be written by the following equation [5]:

$$m \frac{d^2 \vec{R}_i}{dt^2} = \sum_{\substack{j=1 \\ j \neq i}}^n [\vec{F}_{ij}^e(R_{ij}, \theta_{ij}) + \vec{F}_{ij}^r(R_{ij}, \theta_{ij})] + \sum_{j=1}^n \vec{F}_{ij}^{re}(R'_{ij}, \theta'_{ij}) + \vec{F}_i^h(\vec{R}_i) + \vec{F}_i^{rw}(\vec{R}_i) \quad (5)$$

where \vec{R}_i is the radius vector of the center of the i , th drop, $m = \frac{1}{6} \pi d^3$ is the drop weight, $\vec{F}_i^h(\vec{R}_i)$ is the force of hydrodynamic resistance, according to the Hadamard-Rybczynsky formula. $\sum_{j=1}^n \vec{F}_{ij}^{re}(R'_{ij}, \theta'_{ij})$ is the force of the electrical interaction between the i -th drop and the electrodes.

The solution of the ordinary differential equation (5) with closing relations (2)-(4) and a given initial distribution of drops will allow us to determine the dynamics of each drop in the emulsion and, thus, to simulate the dynamics of the emulsion as a whole under the action of an electric field.

Computer model of the dynamics of two drops

To build a computer model and carry out numerical modeling of the dynamics of emulsion drops in an electric field, we first write expressions for the forces entering the equations of the mathematical model in the two-dimensional case, using the global Cartesian coordinate system.

The strength of the electrical interaction between the i -th and j -th drops \vec{F}_{ij}^e depends on distance R_{ij} between drops and angle θ_{ij} between the vector of the electric field strength and the vector connecting the centers of the drops. Suppose that all emulsion droplets are in the plane (y, z) , the coordinates of the center of the i -th drop in the global Cartesian coordinate system we denote (y_i, z_i) . Then we can write that:

$$R_{ij} = \sqrt{(y_i - y_j)^2 + (z_i - z_j)^2}, \quad \cos \theta_{ij} = \frac{z_j - z_i}{R_{ij}}, \quad \sin \theta_{ij} = \frac{y_j - y_i}{R_{ij}}.$$

Vector \vec{F}_{ij}^e in the local polar coordinate system associated with the i -th drop, it is decomposed in the vectors of the local basis \vec{e}_r and \vec{e}_θ : $\vec{F}_{ij}^e = (\vec{F}_{ij}^e)_r \cdot \vec{e}_r + (\vec{F}_{ij}^e)_\theta \cdot \vec{e}_\theta$, where

$$(\vec{F}_{ij}^e)_r = F_0 \left(\frac{d}{R_{ij}}\right)^4 (3 \cos^2 \theta_{ij} - 1), \quad (\vec{F}_{ij}^e)_\theta = 2F_0 \left(\frac{d}{R_{ij}}\right)^4 \sin \theta_{ij} \cos \theta_{ij}.$$

Because the relationship between the reference vectors of the local and global coordinate systems are given by $\vec{e}_r = \sin \theta \cdot \vec{e}_y + \cos \theta \cdot \vec{e}_z$, $\vec{e}_\theta = \cos \theta \cdot \vec{e}_y - \sin \theta \cdot \vec{e}_z$, then the projections of the vector \vec{F}_{ij}^e on the axis of the global cartesian coordinate system (y, z) will be written in the following form:

$$(\vec{F}_{ij}^e)_y = (\vec{F}_{ij}^e)_r \sin \theta_{ij} + (\vec{F}_{ij}^e)_\theta \cos \theta_{ij} = F_0 \left(\frac{d}{R_{ij}}\right)^4 \sin \theta_{ij} (5 \cos^2 \theta_{ij} - 1),$$

$$(\vec{F}_{ij}^e)_z = (\vec{F}_{ij}^e)_r \cos \theta_{ij} - (\vec{F}_{ij}^e)_\theta \sin \theta_{ij} = F_0 \left(\frac{d}{R_{ij}}\right)^4 \cos \theta_{ij} (5 \cos^2 \theta_{ij} - 3).$$

The components of the vector of the repulsive force and the force of hydrodynamic resistance are written in a similar way in the global Cartesian coordinate system:

$$\begin{aligned} (\vec{F}_{ij}^r)_y &= -F_0 \exp\left(-\kappa \frac{R_{ij} - d}{d}\right) \sin \theta_{ij}, \quad (\vec{F}_{ij}^r)_z = -F_0 \exp\left(-\kappa \frac{R_{ij} - d}{d}\right) \cos \theta_{ij}, \\ (\vec{F}_i^h)_y &= -\pi \mu_c d \frac{2\mu_c + 3\mu_d}{\mu_c + \mu_d} \frac{dy_i}{dt}, \quad (\vec{F}_i^h)_z = -\pi \mu_c d \frac{2\mu_c + 3\mu_d}{\mu_c + \mu_d} \frac{dz_i}{dt}. \end{aligned}$$

Thus, we finally write the second-order differential equation (5) describing the dynamics of drops in an electric field in the form of a system of first-order differential equations (6):

$$\begin{cases} \frac{dy_i}{dt} = u_i, \\ \frac{dz_i}{dt} = v_i, \\ \frac{du_i}{dt} = \sum_{\substack{j=1 \\ j \neq i}}^N (\vec{F}_{ij}^e)_y + \sum_{\substack{j=1 \\ j \neq i}}^N (\vec{F}_{ij}^r)_y + (\vec{F}_i^h)_y, \\ \frac{dv_i}{dt} = \sum_{\substack{j=1 \\ j \neq i}}^N (\vec{F}_{ij}^e)_z + \sum_{\substack{j=1 \\ j \neq i}}^N (\vec{F}_{ij}^r)_z + (\vec{F}_i^h)_z. \end{cases} \quad (6)$$

The initial conditions are the coordinates of the initial position of the drops in space and their initial velocities (in all the results below, it is assumed that the drops at the initial moment of time are at rest).

For numerical simulation, consider water droplets in oil. Basic parameters for numerical simulation are given in the Table 1.

Table 1

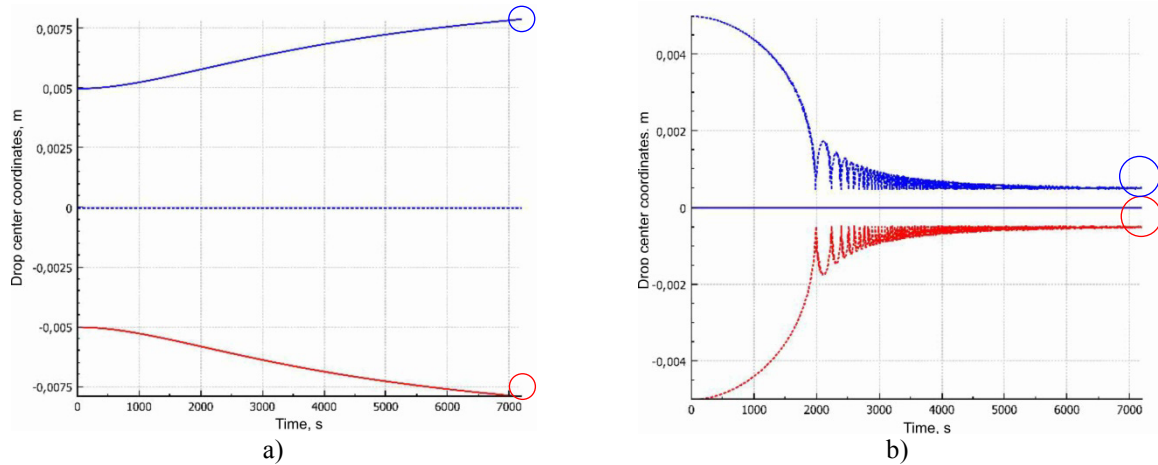
Basic parameters for numerical simulation

Parameter	Symbol	Unit of Measurement	Value
Drop diameter	d	m	10^{-3}
Electric field strength	E_0	V/m	10^6
Parameter characterizing the repelling force of drops	κ	–	10^2
Environment			
Relative dielectric constant	ϵ_c	–	7.3
Dynamic viscosity	μ_c	Pa·s	10^{-1}
Drops			
Relative dielectric constant	ϵ_d	–	23.6
Dynamic viscosity	μ_d	Pa·s	10^{-3}

We begin numerical modeling by considering a system consisting of two drops in view of the possibility of carrying out a relatively simple qualitative analysis.

Results

Figure 3 shows the time dependences of the coordinates of the centers of drops of the same diameter for the cases when the centers of the drops at the initial moment of time are located on one of the coordinate axes symmetrically about the origin. The droplet diameters are the same, and the initial distance between them is 1 cm, i.e., ten times the diameter of the droplets. Curves of different colors correspond to different drops. If at the initial moment of time the centers of the drops are located on the y -axis (i.e., the vector connecting the centers of the drops is perpendicular to the z -axis and accordingly to the vector of the electric field strength), then the drops begin to move along the y -axis in opposite directions (Fig. 3 a).



a) the initial position of the drops on the y -axis

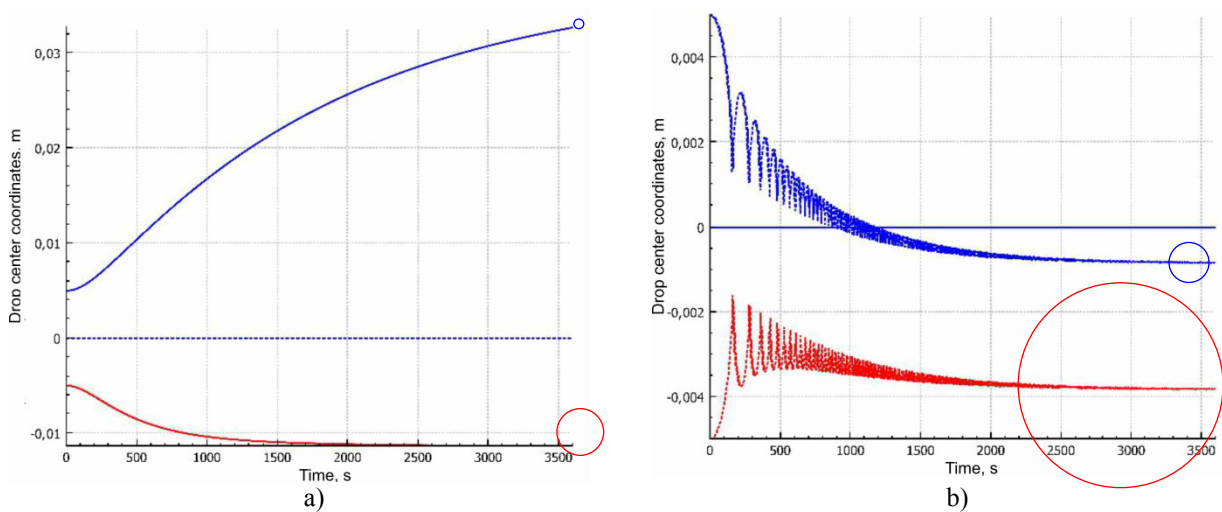
b) the initial position of the drops on the z axis

Figure 3. Dependence of the coordinates of the centers of drops of the same diameter on time

A similar result was described in [5]: the existence of a critical value of the electric field strength was experimentally established, above which oppositely charged droplets began to repel. It can be seen from the Figure 3a that in two hours the distance between the drops increased by 1.5 times. At large times, the distance between drops will continue to increase slightly, and then the drops will stop at a certain distance determined by the force of hydrodynamic resistance.

A qualitatively different behavior of drops is observed in the case when the centers of the drops are located along the vector of the electric field strength (Fig. 3 b). In this case, the motion of the drops has an oscillatory character: first, the drops come closer to each other until the repulsive force becomes greater than the force of electrical interaction, after which the drops change the direction of movement to the opposite. Then the strength of the electrical interaction increases, the drops approach again, etc. After some time, the position of the drops stabilizes, and they are located in the direction of the electric field strength vector in close proximity to each other (the final positions of the drops are schematically shown in Figure 3 and on all the following red and blue circles).

Figure 4 shows the time dependences of the coordinates of the centers of drops of different diameters for the cases when the centers of drops at the initial moment of time are located on one of the coordinate axes symmetrically about the origin. The diameter of the red drop is five times the diameter of the blue drop. The initial distance between the centers of the droplets is still 1 cm.



a) the initial position of the drops on the y -axis; b) the initial position of drops on the z axis

Figure 4. Dependence of the coordinates of the centers of drops of different diameters on time

In principle, the nature of the droplet motion remains the same as for droplets of the same radius: droplets located perpendicular to the electric field strength vector are repelled, and droplets located along the strength vector are attracted, making oscillations. A drop with a smaller diameter (blue in Figure 4) is more affected. Figure 4 b shows that the small drop moves 6 mm, while the large drop moves a little more than 1 mm. An important parameter that determines the nature of the interaction between drops is the dimensionless parameter κ , characterizing the repulsive force between the drops (see formula (4)). This parameter depends on the physical properties of the drop material and the environment, as well as on the properties of the envelope surrounding the drop. Specific parameter value κ should be determined from appropriate experiments. Figure 5 shows the dependences of the coordinates of the centers of drops of the same diameter for different values of the parameter κ .

For small values of the parameter κ the value of the repulsive force is too small and non-physical interpenetration of drops into each other is observed (Fig. 5 a). The interpenetration of drops stops when the parameter is κ close to 100 (Fig. 5 b). A further increase in the parameter leads to a slight increase in the amplitude of oscillations, which does not affect the steady state of the drops (Fig. 5 c). Therefore, in further calculations the value of the parameter will be used $\kappa = 100$.

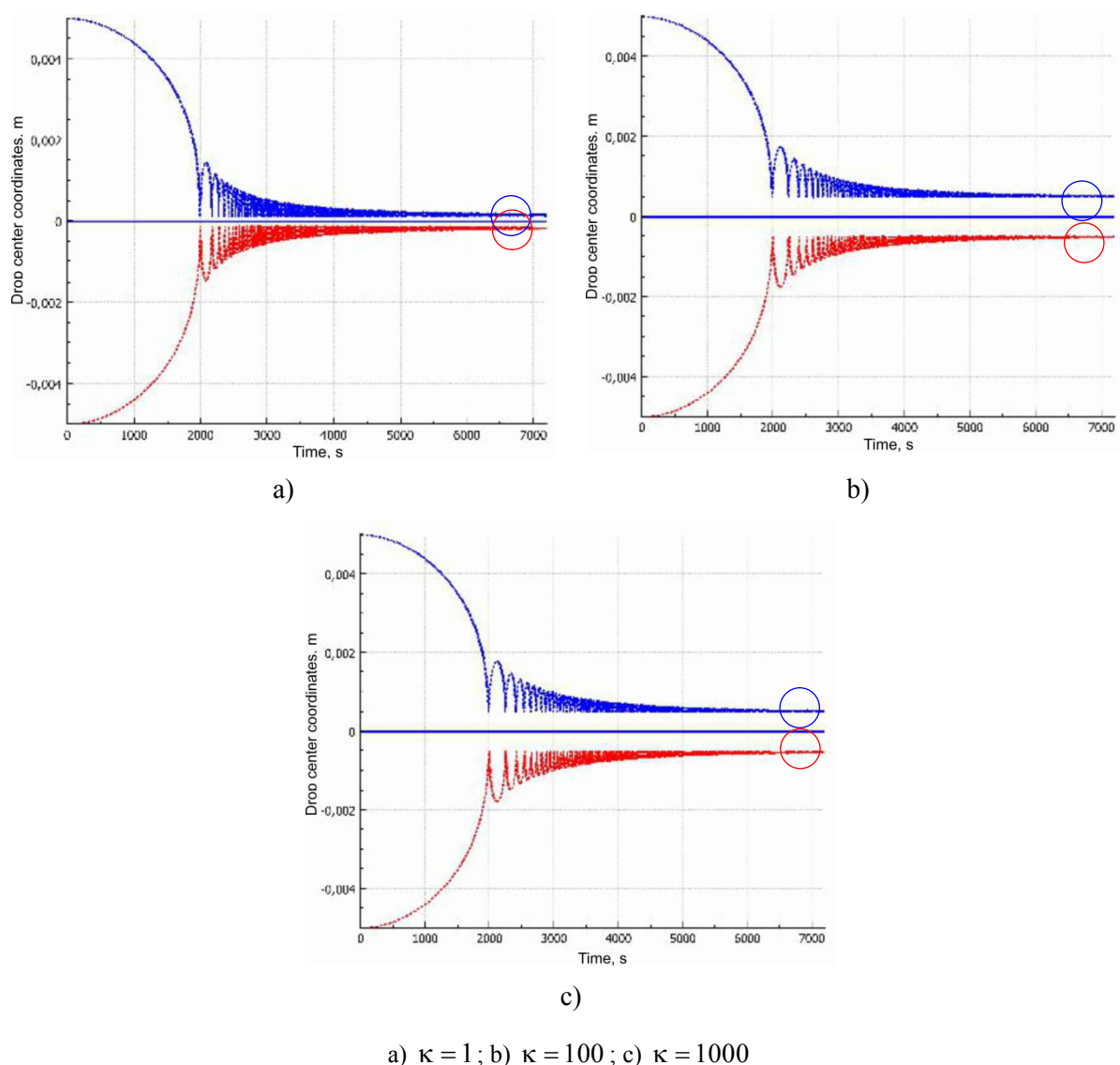
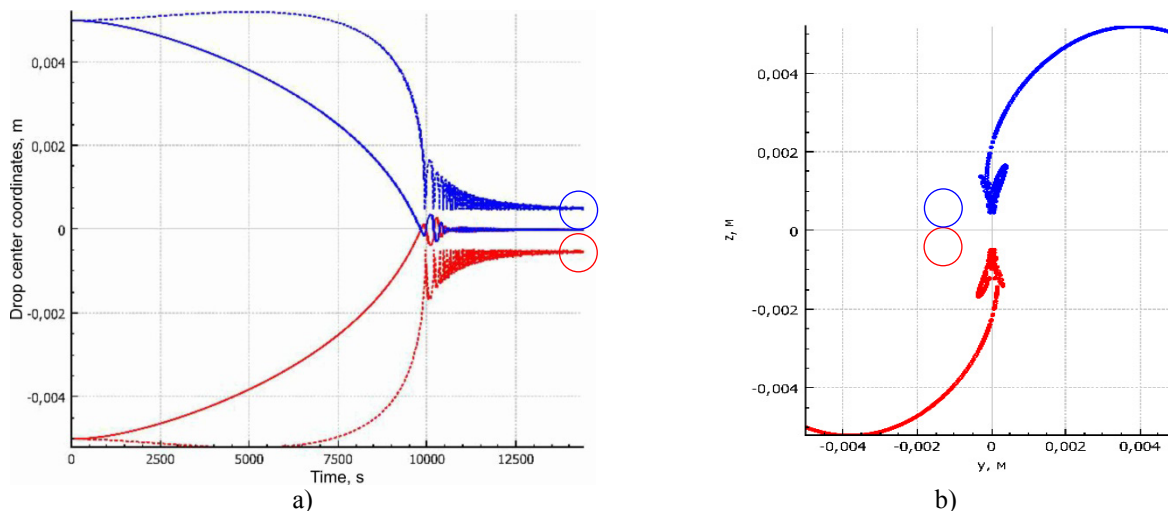


Figure 5. Dependence of the coordinates of the centers of drops of the same diameter on time

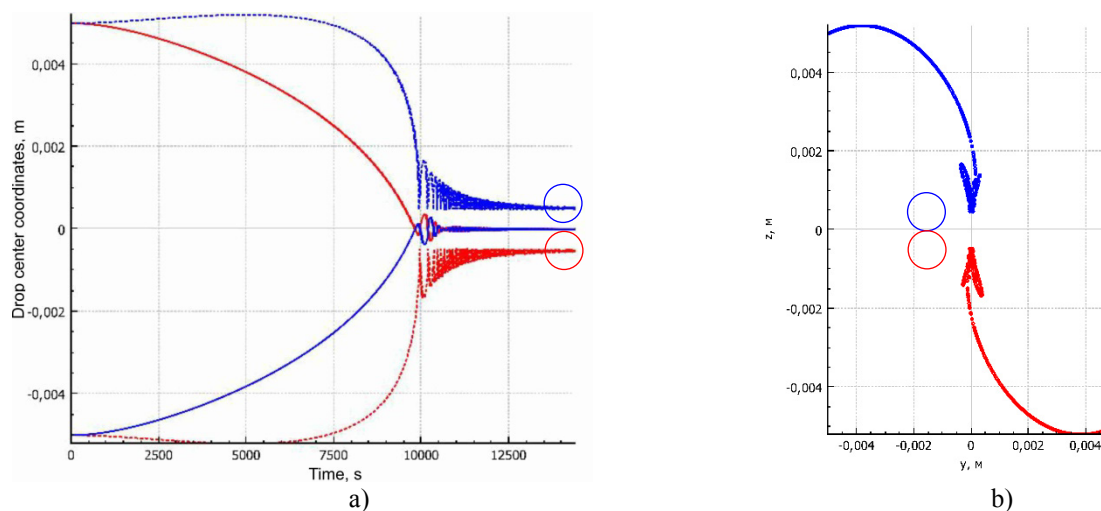
Consider the situation where the vector is connecting the centers of drops at an angle α to the vector of the electric field strength. Figures 6 and 7 show the dependences of the coordinates of the centers of droplets of the same diameter on time and the trajectory of the droplets at angles $\alpha = +45^\circ$ and $\alpha = -45^\circ$ accordingly.

As expected, the movement patterns in Figures 6 and 7 turned out to be symmetrical, therefore we will only analyze in detail the results in Figure 6. In contrast to the previous results, Figure 6 a shows four curves. Curves of different colors correspond to different droplets, the solid curve shows the change in the y coordinate, and the dashed curve shows the change in the z coordinate. It can be seen that under the action of an external electric field the droplets are subjected to a rotational moment and move to a stationary position along a curved trajectory (Fig. 6 b) simultaneously experiencing damped oscillations. In a stationary position, the drops are located on the z axis (i.e., in the direction of the electric field strength vector) symmetrically relative to the origin.



a) time dependence of the coordinates of the centers of drops; b) droplet trajectories

Figure 6. Dynamics of drops of the same diameter at an angle $\alpha = +45^\circ$



a) time dependence of the coordinates of the centers of drops; b) droplet trajectories

Figure 7. Dynamics of drops of the same diameter at an angle $\alpha = -45^\circ$

As shown earlier (see Figure 3 a), if the angle α between the vector of the electric field strength and the vector connecting the centers of the drops is equal, then at the selected value of the strength the drops move in opposite directions. For its angle values is α close, but smaller, drops also begin to move in opposite directions, however, the radial component of the electric interaction force increases, the trajectory is curved, the direction of movement of the drops changes to and the drops begin to approach (Fig. 7, a, b). If the angle α is less than 45° , then the opposite motion of drops is not observed (Fig. 8).

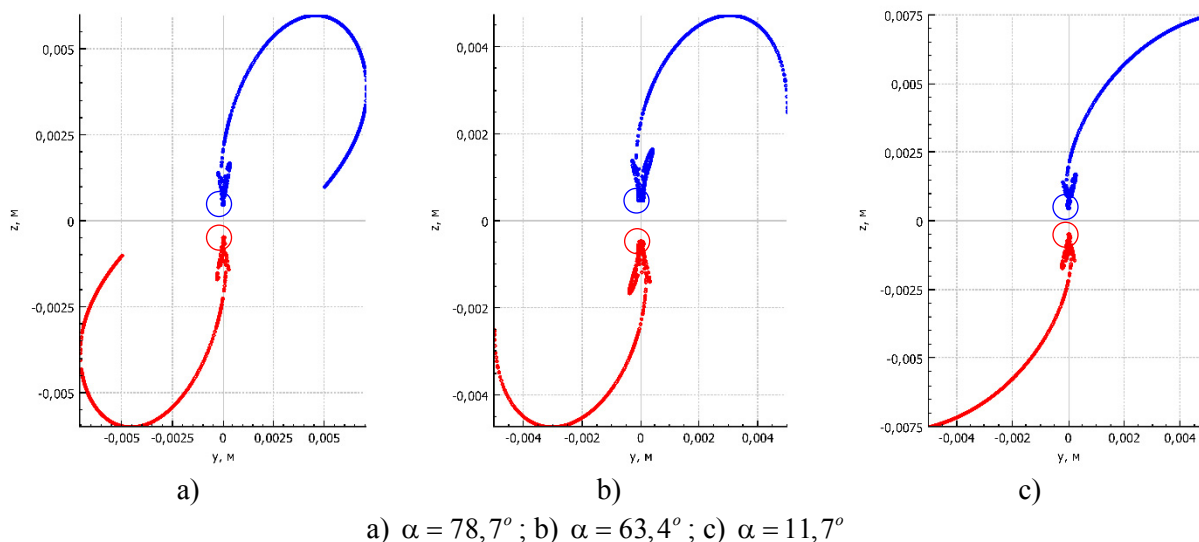


Figure 8. Trajectories of droplets at different angles α

The magnitude of the electric field strength has a significant effect on the time to reach a steady state (Fig. 9). If with strength $2 \cdot 10^6$ V/m drops come to a stationary position in about two hours (Fig. 9, c), then with a four times lower intensity B/m, this takes more than ten hours (Fig. 9, a).

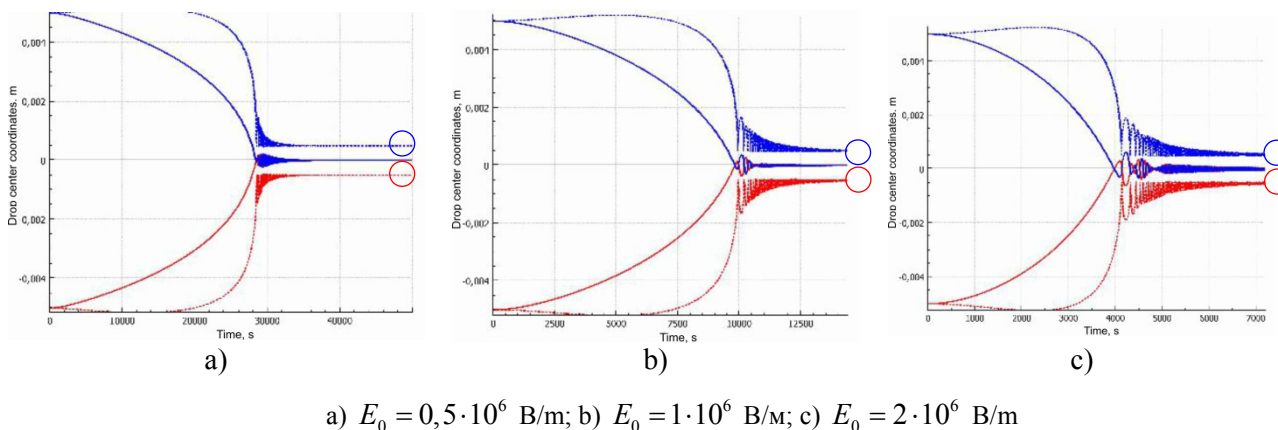


Figure 9. Dependence of the coordinates of the centers of drops of the same diameter on time at different strengths of the electric field

When simulating a system of three or more drops, all of the above regularities are observed, however, the dynamics of interaction is much more complex.

Conclusion

The mathematical model has been constructed and an algorithm has been developed for the numerical solution of the problem of the motion of two or more drops in an electric field, and a computer program has been created that implements this algorithm. The behavior of several drops in an electric field is studied for different physical parameters of the material of the drops and the environment, as well as for different initial distributions of drops and the strength of the electric field. It is shown that emulsion droplets distributed in space, which are initially at rest, begin to move under the action of an electric field, and after a certain time (relaxation time) a new stationary droplet structure is formed. It was found that the relaxation time depends on the electric field strength, the size of the drops and on their initial distribution. The resulting structures, depending on the parameters of the medium and the field, are either threadlike formations of drops oriented in the direction of the electric field, or separate chains of drops. In addition, the type of the structures formed also depends on the type of the selected dependence, which expresses the repulsive force between the drops. The obtained numerical results are in qualitative agreement with the known experimental data.

The work was carried out within the framework of the grant financing project of the Ministry of Education and Science of the Republic of Kazakhstan AP09562291 "Study of gas thermodynamics after an electric discharge in a supersonic flow".

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Электр өрісінің әсерінен бір сұйықтың зарядталған тамшыларының басқа сұйықтағы динамикасын моделдеу

Мақала әртүрлі физикалық өрістердің (электромагниттік күштердің) әсерінен бір тұтқыр сұйықтың тамшылар тобының басқа сұйықтағы қозғалысының ерекшеліктерін зерттеуге арналған. Бір тұтқыр сығылмайтын сұйықтан тұратын және де онымен араласпайтын, екінші тұтқыр сығылмайтын сұйықтың көлемінде таралған, сфералық тамшылардың эмульсиясы қарастырылған. Екі сұйық диэлектрик болып саналады. Электр өрісінің әсерінен екі тамшының динамикасын қарастырғанда, a радиусы бар шар тәрізді тамшылар кернеулігі \vec{E} болатын электр өрісіне орналастырылады және электр өрісінің әсерінен тамшылардың бір-біріне қалай әсер ететіні зерттелген. Осы есептің сандық шешімін табу үшін математикалық модель құрылып, компьютерлік бағдарлама жасалды. Электр өрісіндегі бірнеше тамшылардың қозғалысы тамшылардың материалы мен қоршаған ортаның әртүрлі физикалық параметрлеріне, сонымен қатар тамшылардың әртүрлі бастапқы үлестірімдері мен электр өрісінің кернеулігіне байланысты зерттелді. Электр өрісінің әсерінен кеңістікте таралған эмульсия тамшылары қозғала бастайтындығы және белгілі бір уақыттан (релаксация уақытынан) кейін тамшылардың жаңа стационарлық құрылымы пайда болатыны бірінші рет көрсетілген. Релаксация уақыты электр өрісінің кернеулігіне, тамшылардың мөлшеріне және олардың алғашқы таралуына

байланысты екендігі анықталды. Түрі тамшылар арасындағы \vec{F}_i^r итергіш күшіне тәуелді болып келетін, яғни пайда болған құрылымдар сол орта мен өрістің параметрлеріне байланысты немесе электр өрісі бағытымен бағытталған жіп тәріздес тамшылардың түзілімдері және тамшылардың бөлек тізбектері тәрізді болып келеді.

Кілт сөздер: тамшылардың динамикасы, электромагниттік өріс, дисперсті жүйе, математикалық модель, компьютерлік модель, есептеу аймағы, эмульсия жүйесінің динамикасы.

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Моделирование динамики заряженных капель одной жидкости в другой под действием электрического поля

Статья посвящена исследованию особенностей поведения группы капель одной вязкой жидкости в другой под действием различных физических полей (электромагнитных сил). Рассмотрен состав эмульсии, состоящей из сферических капель одной вязкой несжимаемой жидкости, распределенных в объеме другой вязкой несжимаемой жидкости, несмешивающейся с первой. Обе жидкости считаются диэлектрическими. При рассмотрении динамики двух капель под действием электрического поля капли в форме шара с радиусом a будут помещены в электрическое поле с напряженностью \vec{E} , и исследована реакция друг на друга под действием электрического поля. Построена математическая модель, и разработана компьютерная программа для численного решения данной задачи. Изучено поведение не-

скольких капель в электрическом поле при различных физических параметрах материала капель и окружающей среды, а также при различных начальных распределениях капель и напряженности электрического поля. Впервые показано, что распределенные в пространстве эмульсионные капли под действием электрического поля начинают движение и через определенное время (время релаксации) образуется новая стационарная структура капель. Установлено, что время релаксации зависит от напряженности электрического поля, размеров капель и от их начального распределения. Образующиеся структуры, вид которых зависит от силы отталкивания между каплями \vec{F}_i^r , в зависимости от параметров среды и поля, представляют собой либо нитевидные образования из капель, ориентированные по направлению электрического поля, либо отдельные цепочки капель.

Ключевые слова: динамика капель, электромагнитное поле, дисперсная система, математическая модель, компьютерная модель, расчетная область, динамика эмульсионной системы.

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