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P.Yu. Tsyba[*1,2], O.V. Razina[1,2], N.T. Suikimbayeva[1,3]

[1]*L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan*
[2]*LLP Ratbay Myrzakulov Eurasian International Centre for Theoretical Physics, Nur-Sultan, Kazakhstan*
[3]*M.Kh. Dulaty Taraz Regional University, Taraz, Kazakhstan*
([*]E-mail: pyotrtsyba@gmail.com)

Reconstruction of cosmological models are inspired by generalization of the Chaplygin gas

This paper considers models arising from the composition of the modified Gauss–Bonnet gravity (the Gauss–Bonnet invariant) and the general relativity (the Ricci scalar) against the background of a flat, homogeneous, and isotropic space-time described by the Friedmann–Robertson–Walker metric. Advantages arising from applying a theory containing higher-order invariants (Gauss–Bonnet invariant) consist in the presence of additional degrees of freedom, which makes it possible to study the influence of small-order effects on the dynamics of the system under study, which are in search and confirmed by cosmological observational data. We reconstructed two models with a power-law and exponential dependence on the Gauss–Bonnet invariant, where the model ansatz is a combination of the inverse Weierstrass elliptic function and the power-law function describing the Hubble parameter. This facilitates obtaining a quasi-Dieter law of the change of the scale factor in the initial and late epochs of the Universe. The application of the special function is inspired by generalization equation of state of the Chaplygin gas type, the Weierstrass gas. The application of the equation of state with such dependence makes allows obtaining a quasi-periodic universe. The equations of state are based on the Chaplygin gas are model equations of state and describe well the evolution of both the early and the modern universe. The obtained two particular models are investigated for the fulfillment of the energy conditions, which makes it possible to carry out analysis at a late stage of evolution of the universe and using perturbation theory covering the period of the early universe. For the power-law and exponential models, the perturbations of the Hubble parameter decrease in a finite time are shown, providing a way out of the inflationary stage of evolution of the universe.

Keywords: Friedmann equations, $f(G)$ gravity, Chaplygin gas, Weierstrass gas, energy conditions, perturbation theory, inflation, accelerated expansion.

Introduction

Cosmological observation data [1–4] testify to the discovery of the phenomenon of accelerated expansion and inform on cosmological parameters in the early epoch of the universe. To correctly understand the genesis and process of evolution of the universe, it is required to introduce changes in the classical general relativity by modifying it according to observational cosmology.

There are many possible theoretical descriptions of models responsible for this process. In particular, dark energy models [5] inspired by the modified gravity $f(R)$ have been proposed. Applying the modified theory of gravity, such as $f(R)$, gravity created the prerequisites for understanding the evolution of the universe to explain the accelerated expansion of the universe in recent times. An interesting alternative theory is the modified Gauss–Bonnet gravity [6], or $f(G)$ gravity. Concrete realistic models of $f(G)$ gravity were built to explain cosmic acceleration. By taking into account the corrections for the curvature of a higher order, the features of the future finite time are provided. To study the quantum and general theory of gravity, it is interesting to study the generalization of gravity, such as gravity $f(G)$, where G is the Gauss–Bonnet invariant. In [7], inflationary phenomenology coming from a scalar field, with quadratic curvature terms in the view of GW170817 was investigated. The dynamics of inflationary phenomenology were described and proved that theories with the Gauss–Bonnet term can be compatible with recent observations. In the work [8], de Sitter's solution in the framework of the non-minimal coupling of Gauss–Bonnet gravity with a scalar field was considered and search for the stability of de Sitter solutions, which corresponds to the minimum of the effective potential was made. In [9], inflationary phenomenology of the Einstein–Gauss–Bonnet theory corrected for k-inflation was studied and the problem of non-gaussianity under slow and constant roll conditions was considered.

In this paper, $R + f(G)$ gravity model in the framework of flat and homogeneous space-time described by the Friedmann–Robertson–Walker (FRW) metric is considered. Reconstruction of the resulting model with a special form of the Hubble parameter inspired by the generalization of the Chaplygin gas equation of state – the Weierstrass gas, first presented in [10], is carried out with $f(G) = G^n$ и $f(G) = Ge^{\beta\frac{G}{G_0}}$. The obtained two particular models are investigated for the fulfillment of the energy dominance conditions and through perturbation theory, thus, embracing evolution in the early and late epochs of the universe.

Experimental

2. Basic of $R + f(G)$ gravity

We consider the following action, which describes General Relativity plus a function of the Gauss-Bonnet term:

$$S = \int d^4 x\sqrt{-g}\left[\frac{1}{2\kappa^2}R + f(G)\right], \tag{1}$$

where $\kappa^2 = 8\pi G_N$, G_N being the Newton constant, and the Gauss-Bonnet invariant is defined as usual

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}. \tag{2}$$

By varying the action (1) over $g_{\mu\nu}$, the following field equations are obtained

$$0 = \frac{1}{2k^2}\left(-R^{\mu\nu} + \frac{1}{2}g^{\mu\nu}R\right) + T^{\mu\nu} + \frac{1}{2}g^{\mu\nu}f(G) - 2f_G RR^{\mu\nu} + 4f_G R^{\mu}_{\rho}R^{\nu\rho} - 2f_G R^{\mu\rho\sigma\tau}R^{\nu}_{\rho\sigma\tau}$$
$$- 4f_G R^{\mu\rho\sigma\nu}R_{\rho\sigma} + 2(\nabla^{\mu}\nabla^{\nu}f_G)R - 2g^{\mu\nu}(\nabla^2 f_G)R - 4\left(\nabla_{\rho}\nabla^{\mu}f_G\right)R^{\nu\rho} - 4\left(\nabla_{\rho}\nabla^{\nu}f_G\right)R^{\mu\rho}$$
$$+4(\nabla^2 f_G)R^{\mu\nu} + 4g^{\mu\nu}\left(\nabla_{\rho}\nabla_{\sigma}f_G\right)R^{\rho\sigma} - 4\left(\nabla_{\rho}\nabla_{\sigma}f_G\right)R^{\mu\rho\nu\sigma}, \tag{3}$$

where we made the notations $f_G = f'(G)$ and $f_{GG} = f''(G)$. We shall assume throughout the paper a spatially-flat FRW universe, whose metric is given by

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \tag{4}$$

In case (4), Einstein–Hilbert action (1) contains the modified Gauss–Bonnet gravity term and GR, we can rewrite as point-like action is defined by the expression:

$$S = \int dt\sqrt{-g}\left(\frac{R}{2} + \frac{f}{2} - \frac{f'}{2}\left(G - 24\frac{\dot{a}^2\ddot{a}}{a^3}\right)\right), \tag{5}$$

where $\sqrt{-g} = a^3$, $R = 6\left(2H^2 + \dot{H}\right)$ – the Ricci scalar, $G = 24\frac{\dot{a}^2\ddot{a}}{a^3} = 24H^2\dot{H} + 24H^4$ – the Gauss–Bonnet invariant, $a = a(t)$ – the scale factor, and $H = \frac{\dot{a}}{a}$ – the Hubble parameter.

From the action (5), Lagrange point takes the form

$$L = -3\dot{a}^2 a + \frac{f}{2}a^3 - \frac{f'}{2}Ga^3 - 4f''\dot{G}\dot{a}^3. \tag{6}$$

By applying Euler–Lagrange equation to the (5), we get equation of motion

$$2\dot{H} + 3H^2 = -\frac{f}{2} + \frac{f'}{2}G - 4f'''\dot{G}^2H^2 - 4f''\ddot{G}H^2 - 8f''\dot{G}H(H^2 + \dot{H}). \tag{7}$$

Accordingly, the first Friedmann $2\dot{H} + 3H^2 = -p$ equation, pressure p takes the form

$$p = \frac{f}{2} - \frac{f'}{2}G + 4f'''\dot{G}^2H^2 + 4f''\ddot{G}H^2 + 8f''\dot{G}H(H^2 + \dot{H}). \tag{8}$$

On the other hand, the total energy (Hamiltonian) corresponds to Lagrangian

$$3H^2 = -\frac{f}{2} + \frac{f'}{2}G - 4f''\dot{G}H^3. \tag{9}$$

From the second Friedmann $3H^2 = \rho$ equation, energy density ρ takes the form

$$\rho = -\frac{f}{2} + \frac{f'}{2} G - 4f''\dot{G}H^3. \tag{10}$$

3. Research methods

We can introduce energy condition in pressure p and energy density terms ρ as [11]

$$\text{NEC} \Rightarrow p + \rho \geq 0 \tag{11}$$

$$\text{WEC} \Rightarrow \rho \geq 0, p + \rho \geq 0,$$
$$\text{SEC} \Rightarrow 3p + \rho \geq 0, p + \rho \geq 0,$$
$$\text{DEC} \Rightarrow \rho \geq 0, -\rho \leq p \leq \rho.$$

To study the stability, we consider the conservation equation energy-matter perfect fluid

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{12}$$

To do that, first, we have presumed a linear perturbation of the Hubble parameter as

$$H(t) = H_0(t)\big(1 + \delta(t)\big), \tag{13}$$

where $H(t)$ is the perturbed Hubble parameter and $\delta(t)$ is the perturbation term.

Energy-matter perfect fluid [12]:

$$\dot{H} = \partial_t \left(A_1 \wp^{-1}(t^2 + A_2\, exp(t)\,; g2, g3) + A_3 t^{-\frac{1}{7}} \right), \tag{14}$$

where $\dot{H} = \frac{dH}{dt}$, A_1, A_2, A_3 – arbitrary constants, $\wp^{-1}$ – inverse Weierstrass function, and $g2, g3$ – invariants. Figure 1 represents $\dot{H}$ and H time dependence.

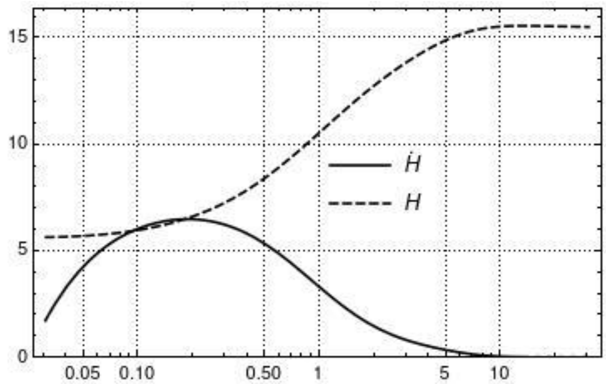


Figure 1. $\dot{H}$ and H dependence on cosmic time t at $A_1 = 10, A_2 = 0.95, A_3 = 0.8$.

Results and Discussion

4 Energy condition analysis

4.1 Power-law model

Let us consider particular case of $R + f(G)$ gravity in form $f(G) = G^n$. Take into account equations (8), (10), (14) and

$$f(G) = G^n, f'(G) = nG^{n-1}, f''(G) = n(n-1)G^{n-2}, f'''(G) = n(n-2)(n-1)G^{n-3}, \tag{15}$$

pressure p and energy density ρ in Hubble terms take the form

$$p_1 = \frac{G^n(1-n)}{2} + 4n(n-1)(n-2)G^{n-3}\dot{G}H^2 + 4n(n-1)G^{n-2}\ddot{G}H^2 + 8n(n-1)G^{n-2}\dot{G}H\big(H^2 + \dot{H}\big), \tag{16}$$

$$\rho_1 = -\frac{G^n(1-n)}{2} - 4n(n-1)G^{n-2}\dot{G}H^3, \tag{17}$$

the graphical dependence of which on the cosmic time t is shown in Figure 2 at $n = 4$. This choice of n provides a contribution to the pressure p and the energy density ρ of all terms in (16) and (17).

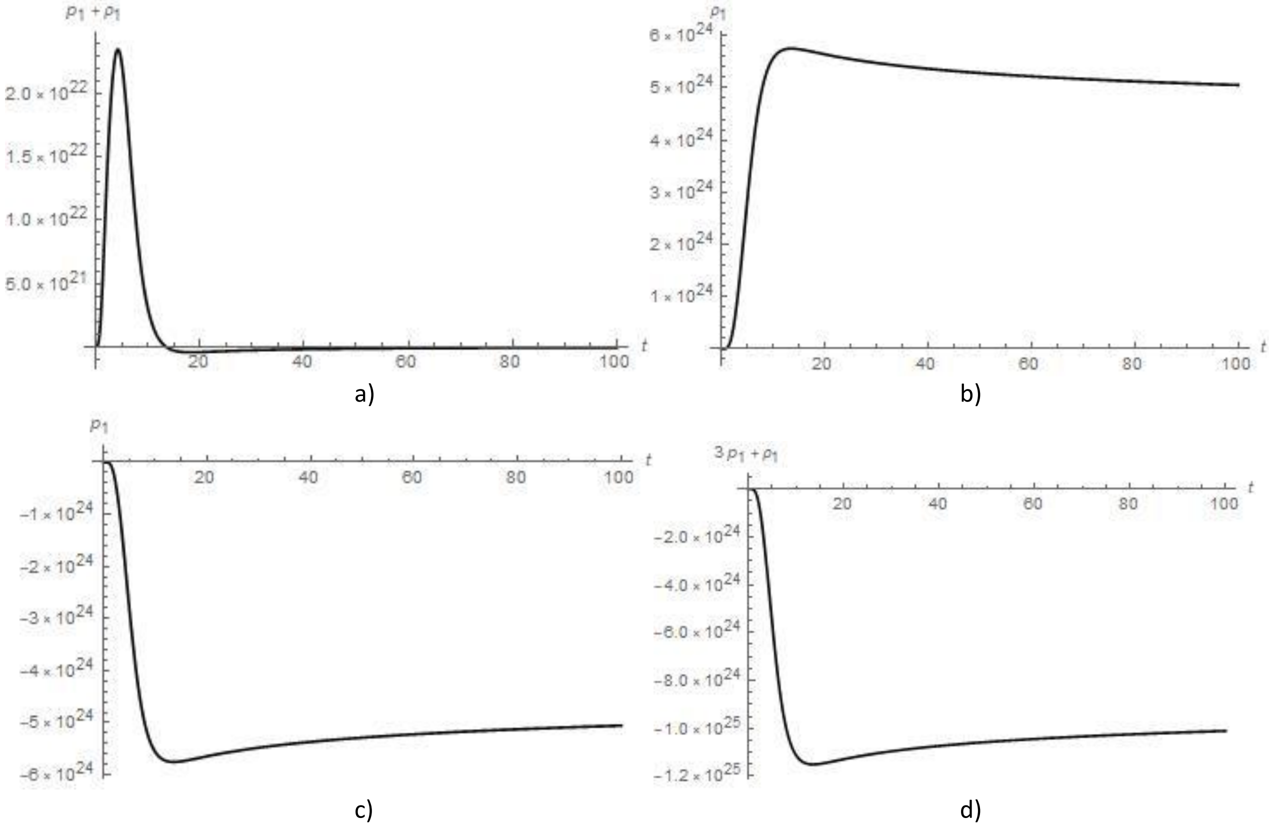


Figure 2. Dependence on cosmic time t: a) pressure p_1; b) energy density ρ_1; c) $p_1 + \rho_1$ и d) $3p_1 + \rho_1$.

Note that, with decreasing n, the values of pressure and energy density decrease, so for $n = 3$ the form of the time dependence remains, but the amplitude decreases by a factor of 10^8.

4.2 Exponential model

Let us consider another particular case $R + f(G)$ gravity in form $f(G) = f_0\left(1 - Ge^{\frac{G_0}{G}}\right)$. Take into account equation (8), (10), (14) and

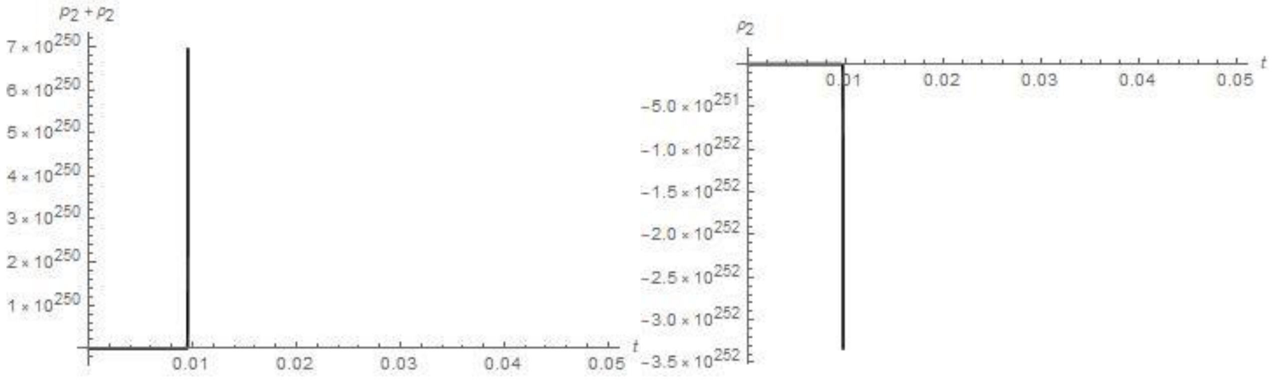
$$f(G) = -f_0\left(1 - e^{\frac{G}{G_0}}\right), f'(G) = \frac{f_0}{G_0}e^{\frac{G}{G_0}}, f''(G) = \frac{f_0}{G_0^2}e^{\frac{G}{G_0}}, f'''(G) = \frac{f_0}{G_0^3}e^{\frac{G}{G_0}}. \tag{18}$$

pressure p and energy density ρ in Hubble terms take the form

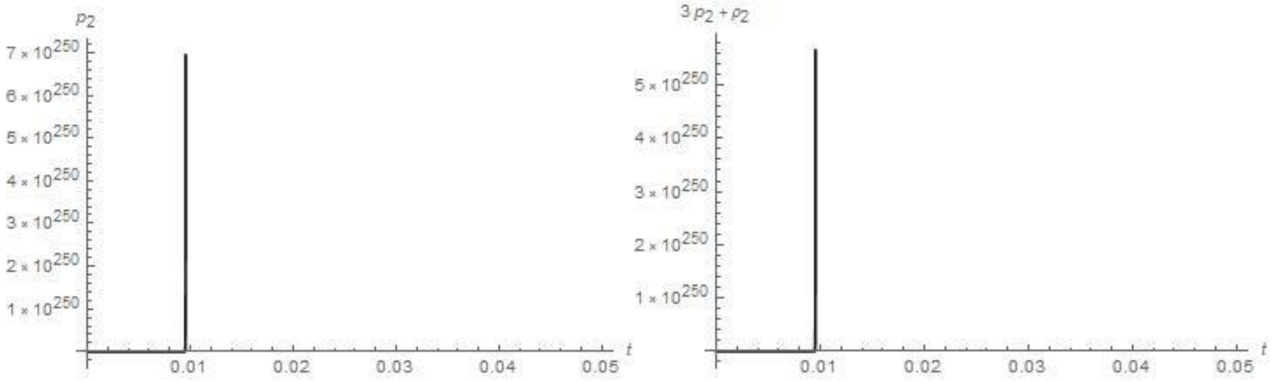
$$p_2 = -\frac{f_0}{2}\left(1 - e^{\frac{G}{G_0}}\right) + 2\frac{f_0}{G_0}e^{\frac{G}{G_0}}G + 4\frac{f_0}{G_0^3}e^{\frac{G}{G_0}}\dot{G}^2H^2 + 4\frac{f_0}{G_0^2}e^{\frac{G}{G_0}}\ddot{G}H^2 + 8\frac{f_0}{G_0^2}e^{\frac{G}{G_0}}\dot{G}H(H^2 + \dot{H}), \tag{19}$$

$$\rho_2 = \frac{f_0}{2}\left(1 - e^{\frac{G}{G_0}}\right) - 2\frac{f_0}{G_0}e^{\frac{G}{G_0}}G - 4\frac{f_0}{G_0^2}e^{\frac{G}{G_0}}\dot{G}H^2, \tag{20}$$

the graphical dependence of which on the cosmological time t is shown in Figure 3.



a) b)



b) d)

Figure 3. Dependence on cosmic time t: a) pressure p_2; b) energy density ρ_2; c) $p_2+\rho_2$ and d) $3p_2+\rho_2$.

5 Perturbation analysis

5.1 Power-law model

In this section, we are interested in investigating the power law's stability through perturbation analysis. Substituting in (12) (8), (10), and (13) taking into account $f(G)=G^n$, at $n=4$, we get equation

$$H\left(\dot{H}+H^2\right)\left(\dot{G}^2+12\ddot{G}\left(\dot{H}+H^2\right)H^2+36\dot{G}H\left(\dot{H}+H^2\right)^2\right)=0,$$

which, in the case of a search for a particular solution in terms of the Hubble parameter, transforms into a differential equation

$$\dot{H}_0(t)b(t)+H_0\dot{b}(t)+H_0^2(t)b^2(t)=0, \qquad (21)$$

where $b(t)=1+\delta_1(t)$. From equation (18), we obtain a particular solution describing the perturbation $\delta_4(t)$ in view

$$\frac{1}{H_0\left(1+\delta_4(t)\right)}=t+c_1, \qquad (22)$$

where H_0 is described by expression (14) and c_1 is integration constant. Solution (22) is illustrated in Figure 4.

It should be noted that at $c_1\geq 1$ perturbation $\delta_4(t)$ takes the positive value. Perturbation $\delta_4(t)$ for the most rapidity tend to be zero at $c_1=1$. Perturbation $\delta_4(t)$ takes the negative value at $c_1<1$.

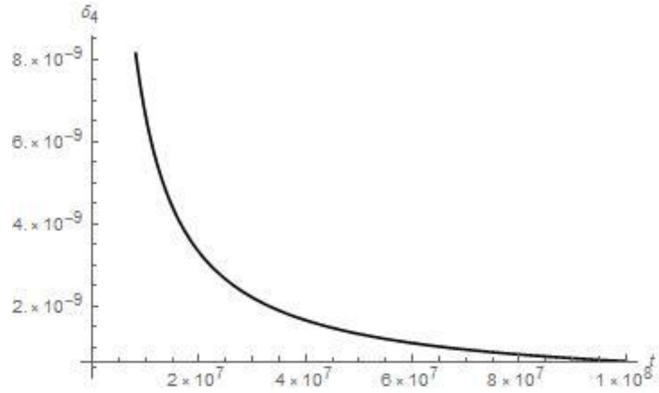


Figure 4. $\delta_4(t)$ dependence on cosmic time t at $n = 4$ and $c_1 = 1$.

5.2 Exponential model

Let us research perturbation in exponential model. Substituting in (12), (8), (10), (13) and $f(G) = f_0\left(1 - Ge^{\frac{G}{G_0}}\right)$ neglecting terms higher than the first order, we obtain the equation

$$\dot{H} = -H^2. \quad (23)$$

From equation (23), we obtain a particular solution describing the perturbation $\delta(t)$ the same as (22).

Conclusions

Here $R + f(G)$ cosmological model was considered in flat, homogeneous, and isotropic Friedmann–Robertson–Walker space-time. Power-law model and exponential model were chosen as particular cases of modified Gauss–Bonnet gravity in the $f(G)$ term. We provided energy conditions and made a perturbation analysis.

In the power-law model, the null energy condition performs, which is depicted in Figure 2a and described by $p_1 + \rho_1 \geq 0$. Weak energy condition performs and is shown in Figures 2a and 2b. It is described inequality $\rho_1 \geq 0$ and $p_1 + \rho_1 \geq 0$. Strong energy condition $p_1 + \rho_1 \geq 0$ and $3p_1 + \rho_1 \geq 0$ is represented in Figures 2a and 2d. This condition partially performs because the first inequality performs only. Violation of this condition provides accelerated expansion. Dominant energy condition $\rho_1 \geq 0, -\rho_1 \leq p_1 \leq \rho_1$ is shown in Figures 2b and 2c, and performs. The simultaneous fulfillment of weak and dominant energy conditions ensures the acceleration mode.

For the exponential model, the null energy condition $p_2 + \rho_2 \geq 0$ the same as power-law model. Weak energy condition $\rho_2 \geq 0$ and $p_2 + \rho_2 \geq 0$ partially performs because the second inequality performs only. These conditions are shown in Figures 3a and 3b. Strong energy condition $p_1 + \rho_1 \geq 0$ and $3p_1 + \rho_1 \geq 0$ is represented in Figures 3a and 3d. Figures 3a and 3c illustrate dominant energy condition $\rho_2 \geq 0, -\rho_2 \leq p_2 \leq \rho_2$. It not performs for exponential model. In this model, the energy density has a negative value, which corresponds to cosmology with a phantom field. This behavior provides a superacceleration mode.

By comparing results of energy condition models, we have $|p_1 + \rho_1| \ll |p_2 + \rho_2|$ and $|3p_1 + \rho_1| \ll |3p_2 + \rho_2|$. The power-law model $f(G)$ describes the accelerated expansion of the universe at a late stage in the evolution of the universe and this expansion will be eternal, and the results of the exponential model predict a transition to the super acceleration regime, that is, the disintegration of the universe in a finite period of time.

The analysis of the perturbation of the studied models shows that both in the power-law and exponential models, the perturbations tend to be zero. This ensures a way out of the inflationary stage at an early stage in the universe's evolution.

Acknowledgment

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П.Ю. Цыба, О.В. Разина, Н.Т. Суйкимбаева

Чаплыгин газын жалпылау арқылы космологиялық модельдерді қайта құру

Мақалада Фридман–Робертсон–Уокер метрикасымен сипатталған жазық, бір текті және изотропты кеңістік–уақыт фонындағы жалпы салыстырмалық теориясының (Риччи скаляры) және Гаусс–Бонне (Гаусс-Бонне инварианты) гравитациясының модификацияланған теориясы композициясының нәтижесінде пайда болған модельдер қарастырылған. Жоғарғы ретті (Гаусс–Бонн инварианты) инварианттан тұратын теорияны есепке алу нәтижесінде пайда болатын артықшылықтар, қосымша еркіндік дәрежесінің пайда болуына негізделеді, олар космологиялық бақылаулармен нақтыланған кіші ретті эффекттердің зерттелетін жүйенің динамикасына әсерін зерттеуге мүмкіншілік береді. Авторлар Гаусс–Бонн инвариантына экспоненциалды және дәрежелі тәуелді екі моделдің қайта құрастырылуын жүзеге асырған, мұнда анзац моделі ретінде Вейерштрассаның кері эллипстік функциясының комбинациясы мен Хаббл параметрін сипаттайтын дәрежелік функциясы алынған. Бұл Әлемнің бастапқы және кейінгі дәуірлеріндегі масштаб факторының өзгеруінің квазидезитерлік заңын алуға мүмкіндік береді. Арнайы функцияны қолдану Чаплыгиндік газ түрі — Вейерштрас газының күй теңдеуін жалпылаудан туындаған. Осындай тәуелділігі бар күй теңдеуін қолдану квазипериодты Әлемді алуға мүмкіндік береді. Чаплыгин газына негізделген күй теңдеулері модельдік күй теңдеулері болып табылады және ерте және қазіргі әлемнің эволюциясын жақсы сипаттайды. Алынған екі жеке модель энергия доминантының шарттарын орындау үшін зерттелді, бұл әлемнің эволюциясының кеш кезеңінде және ерте әлемнің кезеңін қамтитын бұзылулар теориясын қолдана отырып талдау жасауға мүмкіндік береді. Дәрежелік және экспоненциалды модельдер үшін Хаббл параметрі соңғы уақытта ауытқиды, осылайша Ғалам эволюциясының инфляциялық сатысынан шығуды қамтамасыз етеді.

Кілт сөздер: Фридман теңдеуі, $f(G)$-гравитация, Чаплыгин газы, Вейерштрасс газы, энергияның доминанттылық шарттары, ұйытқу теориясы, инфляция, үдемелі кеңею.

П.Ю. Цыба, О.В. Разина, Н.Т. Суйкимбаева

Реконструкция космологических моделей, инспирированная обобщением газа Чаплыгина

В статье рассмотрены модели, возникающие в результате композиции модифицированной теории гравитации Гаусса–Бонне (инвариант Гаусса–Бонне) и общей теории относительности (скаляр Риччи) на фоне плоского, однородного и изотропного пространства–времени, описываемого метрикой Фридмана–Робертсона–Уокера. Преимущества, возникающие в результате применения теории, содержащей инварианты высшего порядка (инвариант Гаусса–Бонне), заключаются в наличии дополнительных степеней свободы, которые позволяют изучать влияние эффектов малого порядка на динамику исследуемой системы, находящихся в поиске и подтвержденных космологическими наблюдательными данными. Авторами осуществлена реконструкция двух моделей со степенной и экспоненциальной зависимостью от инварианта Гаусса–Бонне, где в качестве анзаца модели выступает комбинация обратной эллиптической функции Вейерштрасса и степенной функции, описывающей параметр Хаббла. Это позволяет получить квазидеситеровский закон изменения масштабного фактора в начальную и позднюю эпохи Вселенной. Применение специальной функции инспирировано обобщением уравнения состояния типа газа Чаплыгина – газом Вейерштрасса. Применение уравнения состояния с такой зависимостью позволяет получить квазипериодическую Вселенную. Уравнения состояния, основанные на газе Чаплыгина, являются модельными уравнениями состояния и хорошо описывают эволюцию как ранней, так и современной Вселенной. Полученные две частные модели исследованы на выполнение условий энергодоминантности, что дает возможность провести анализ в поздний этап эволюции Вселенной и с помощью теории возмущений охватывающей период ранней Вселенной. Показано, что для степенной и экспоненциальной модели возмущения параметр Хаббла уменьшается за конечное время, тем самым обеспечивая выход из инфляционной стадии эволюции Вселенной.

Ключевые слова: уравнения Фридмана, $f(G)$-гравитация, газ Чаплыгина, газ Вейерштрасса, условия энергодоминантности, теория возмущений, инфляция, ускоренное расширение.