DOI 10.31489/2022PH2/49-57

UDC 537.533.34

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Calculation of the electron-optical scheme of a new type mirror energy analyzer of charged particles

Further studies of the electron-optical properties of electrostatic multipole-cylindrical fields, synthesized from the fields of a cylindrical mirror and circular multipoles, are continued in the work. The implementation of electron spectroscopy methods is based on the use of complex equipment, one of the main elements of which is an electron energy analyzer of low and medium energies. Application of the multipole approach to the synthesis of deflecting fields makes it possible to develop effective methods for energy analysis of charged particle beams. The electron-optical scheme of new type mirror energy analyzer of charged particle beams based on an electrostatic axially symmetric octupole-cylindrical field is proposed in the work. An axially symmetric octupole-cylindrical field is constructed as a superposition of a basic cylindrical field and a circular octupole. When the fields were added, the central circle of the octupole was combined with the zero equipotential of the logarithmic field. The motion of charged particles in the electrostatic octupole-cylindrical field. An integrodifferential equation for the motion of charged particles in an electrostatic octupole-cylindrical field is derived. Calculation of trajectories in an energy analyzer with an octupole-cylindrical field was performed on the basis of the method of expansion into a fractional-power series of the particle motion equation presented in the integrodifferential form. Coefficients of the series, representing the trajectory of motion in an analytical form, accessible for further studies of the electron-optical characteristics of the octupole-cylindrical field, are obtained. Based on an octupole-cylindrical field, high luminosity energy analyzers can be built to determine the composition of charged particle beams with energies from units of eV to tens of keV in space plasma.

Keywords: energy analyzer of charged particles, electron mirrors, electrostatic axially symmetric octupole-cylindrical field, approximate-analytical calculation, motion of charged particles.

Introduction

The operation of all analyzing devices is based on the use of the features of the movement of charged particles in the fields created by the corresponding electrode systems. The suitability of a particular field for the purposes of energy analysis is characterized by the dispersion of the field in terms of energies.

In reflector-type analyzers, charged particles enter between the electrode plates and, if their energy matches the tuning energy, move in the region of the equipotential surface, then fall on the detector. The advantage of these analyzers is the ability to analyze high-energy charged particles beams with a relatively small potential difference between the electrodes. The disadvantage is a small specific dispersion and a strong influence of edge fields. Deflectors, in addition to surface analysis, have found wide application in mass spectrometry, as well as in the analysis of high-energy particles. This has found application in space plasma research. For these purposes, an electrostatic hemispherical analyzer with a circular field of view, called “top-hat” was widely used [1]. This analyzer was designed in the laboratory of the University of Texas (USA). Then it was successfully used in various projects in the USA and Europe.

Until now, top-hat energy analyzers have been successfully used in experimental space plasma physics [2–5].

The measurement of charged particles with energies from a few eV to tens of keV is a significant part of space experiments. A plasma of such energy inhabits the solar wind, planetary ionospheres, interplanetary space, the earth's ionosphere, and the magnetosphere. Near the earth, the study of the boundaries of particle precipitation is necessary for fundamental studies of the magnetosphere.

The cylindrical mirror type energy analyzers have found wide application in the study of resonant phenomena in gases, in spectroscopy for chemical analysis, to obtain spectra of secondary electrons, photoelectrons, auto-electrons, Auger electrons, as well as in space research, in the study of the interaction of atomic particles with the solid body surface and plasma diagnostics. The cylindrical mirror analyzer has become the basic element of electron spectrometers for various purposes, produced in the countries near and far abroad by leading instrument-making companies [6, 7].

The class of potential fields, called multipole-cylindrical, was first substantiated and classified by Zashkvara V.V. and Tyndyk N.N. in [8, 9]. Application of the multipole approach to the synthesis of deflecting fields makes it possible to develop effective methods for energy analysis of charged particle flows. This method is based on the principle of superposition of the simplest fields of cylindrical type and various order circular multipoles.

The electron-optical schemes of mirror energy analyzers based on electrostatic quadrupole-cylindrical [10], hexapole-cylindrical [11], and decapole-cylindrical fields [12] were previously studied in sufficient detail. In particular, the monograph [13] is devoted to the study of their electron-optical characteristics and potential capabilities, to the search for optimal electron-optical schemes with high focusing properties and energy resolution.

In previous studies, a family of equipotentials of cylindrical octupoles with a plane of symmetry and antisymmetry was calculated [14]. Calculation and analysis of equipotential portraits of electrostatic axially symmetric octupole-cylindrical fields for different weight contributions of the cylindrical field and the circular octupole were carried out.

In the present work, the electron-optical scheme of a new type of energy analyzer based on an electrostatic octupole-cylindrical field is investigated, and a trajectory calculation in an electrostatic octupole-cylindrical field is carried out.

1. Focusing field of the energy analyzer

The field is built based on the superposition of the fields of a cylindrical mirror and a circular octupole. The potential distribution for the proposed electrostatic system can be expressed as follows:

$$U(r,z)=\mu\ln r+\omega U_{oct}(r,z) \tag{1}$$

here

$$U_{oct}(r,z)=\frac{1}{4!}z^4+\frac{1}{2}z^2\left\{\frac{1}{4}(1-r^2)+\frac{1}{2}\ln r\right\}+ \\ +\frac{1}{64}r^4+\frac{1}{16}r^2-\frac{1}{8}\ln r\left[\frac{1}{2}+r^2\right]-\frac{5}{64} \tag{2}$$

is circular octupole, μ is coefficient specifying the weight contribution of the cylindrical field $ln\,r$, ω is weight component of the circular octupole.

The octupole-cylindrical field was chosen as the object of research, in which the component of the cylindrical field is equal to the multipole part.

Figure 1 shows a schematic view in the longitudinal section (upper part) of the electron-optical scheme of an energy analyzer based on an octupole-cylindrical field, where the value of the weight contributions of the cylindrical field μ = 1 and octupole ω= 1. Potential distribution Eq. (1) differs significantly from the classical field cylindrical mirror.

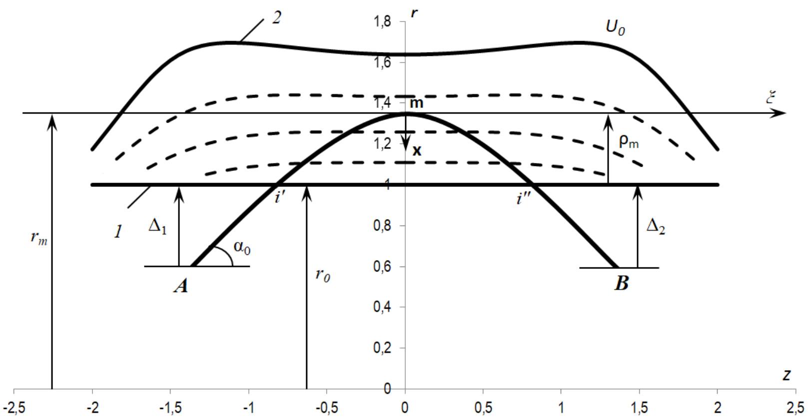

Figure 1. Schematic view in longitudinal section (upper part) of the electron-optical scheme of the energy analyzer based on an octupole-cylindrical field: 1 – inner cylindrical electrode, 2 – outer deflecting electrode, *A* – ring source of charged particles, *i'* – entrance ring window, *i''* – exit ring window, *B* – ring image

The analyzer contains two coaxial electrodes, the inner electrode *1* has a cylindrical shape of radius r_0 and is under zero potential the outer electrode *2* has a curvilinear profile and is under the deflecting potential U_0. The field is created between the electrodes that decelerate and deflect charged particles and has the properties of an electrostatic mirror. The profile of the outer deflecting electrode *2* repeats the equipotential surface of the electrostatic octupole-cylindrical field. The cylindrical electrode *1* cuts through the entrance *i'* and exits *i''* windows for the movement of the charged particles beam.

According to the scheme, the charged particles beam from the ring source *A* through the entrance window *i'* in the inner cylinder *1* enters the energy analyzer field, further reflected by the field, then through the exit window *i''* on the inner cylinder *1* returns to the zero potential region and focused into the ring image *B*.

2. Calculations

Consider the motion of a charged particle in the octupole-cylindrical field. To calculate the trajectories of charged particles motion in the octupole-cylindrical field, let us move the reference point of the trajectory to its vertex **m** and place the origin x,ξ at the same point (see Fig. 1). All linear dimensions are calculated in the radii of the inner cylindrical electrode r_0 to maintain the following dimensionless parameters:

$$\frac{r}{r_0}=\frac{r_0+r_0\,\rho}{r_0}=1+\rho\,,\qquad x=\frac{r_m-r}{r_0}=\rho_m-\rho\,,\qquad \xi=\frac{z}{r_0}\,. \tag{3}$$

The distribution of the octupole-cylindrical field Eq. (1) in the new coordinates x,ξ ((for any μ and ω) has the following form:

$$U\left(x,\xi\right)=U_0 g\left(x,\xi\right)=U_0 g_x\,, \tag{4}$$

where

$$\begin{gathered}g\left(x,\xi\right)=g_x=\mu\ln\left(R-x\right)+\omega\left[\frac{1}{4!}\xi^4+\frac{1}{2}\xi^2\left\{\frac{1}{4}\left[1-\left(R-x\right)^2\right]+\frac{1}{2}\ln\left(R-x\right)\right\}+\right.\\ \left.+\frac{1}{64}\left(R-x\right)^4+\frac{1}{16}\left(R-x\right)^2-\frac{1}{8}\ln\left(R-x\right)\left[\frac{1}{2}+\left(R-x\right)^2\right]-\frac{5}{64}\right]\end{gathered}\,,\quad R=1+\rho_m\,. \tag{5}$$

The motion of a charged particle in the octupole-cylindrical field for the case under study: the contributions of the cylindrical field and the circular octupole, respectively, μ = 1, ω= 1, in this case, the potential distribution in this system is described in the x,ξ coordinate system as follows:

$$U\left(x,\xi\right)=U_0 g\left(x,\xi\right)=U_0 g_x\,, \tag{6}$$

where

$$\begin{gathered}g\left(x,\xi\right)=g_x=\ln\left(R-x\right)-\frac{1}{4!}\xi^4-\frac{1}{2}\xi^2\left\{\frac{1}{4}\left[1-\left(R-x\right)^2\right]+\frac{1}{2}\ln\left(R-x\right)\right\}+\\ -\frac{1}{64}\left(R-x\right)^4-\frac{1}{16}\left(R-x\right)^2+\frac{1}{8}\ln\left(R-x\right)\left[\frac{1}{2}+\left(R-x\right)^2\right]+\frac{5}{64}\end{gathered}\,. \tag{7}$$

The motion of a charged particle in the axially symmetric octupole-cylindrical field (6) has the following form:

$$m\ddot{x}=qU_0\varepsilon_1,\qquad \varepsilon_1=-\frac{\partial g\left(x,\xi\right)}{\partial x}\,, \tag{8a}$$

$$m\ddot{\xi}=qU_0\varepsilon_2\,,\qquad \varepsilon_2=-\frac{\partial g\left(x,\xi\right)}{\partial \xi}\,. \tag{8b}$$

According to the law of conservation of energy when moving in a static potential field, the change in the kinetic energy of a charged particle is determined by the passed potential difference. Further, integrating the sum of Eqs. (8a) and (8b) along the particle trajectory from the vertex m to an arbitrary point, we obtain the law of conservation of energy for a particle moving in the electrostatic octupole-cylindrical field, which relates the change in the kinetic energy of the particle to the potential difference:

$$\frac{m\upsilon_m^2}{2}-\frac{m}{2}\left(\dot{x}^2+\dot{\xi}^2\right)=-q\left(U_m-U\left(x,\xi\right)\right)=-qU_0\left(g_0-g_x\right), \tag{9}$$

here $U_m = U_0 g(x_m, \xi_m) = U_0 g_0$ is field potential at point m, while $x_m = \xi_m = 0$, $g_x = g(x, \xi(x))$; $g_0 = g(x,\xi)\big|_{\substack{x=0\\ \xi=0}}$.

By integrating Eq. (8 b) within the range from point m to an arbitrary point of the trajectory, we can determine the value of $\frac{m\dot{\xi}^2}{2}$. At the same time, we take into account that $\upsilon_m^2 = \dot{\xi}_m^2 + \dot{x}_m^2 = \dot{\xi}_m^2$, since at the vertex of the trajectory $\dot{x}_m = 0$. Further, using the relation $\dot{\xi} = \frac{d\xi}{dt} = \frac{d\xi}{dx}\frac{dx}{dt} = \xi'\dot{x}$, we obtain

$$\frac{m\upsilon_m^2}{2} - \frac{m\dot{\xi}^2}{2} = qU_0 \int_x^{x_m} \frac{\partial g(x,\xi)}{\partial \xi}\frac{d\xi}{dx}dx = qU_0 \int_0^x \frac{\partial g(x,\xi)}{\partial \xi}\xi' dx \ . \tag{10}$$

According to the scheme (Fig. 1), $\frac{m\dot{\xi}^2}{2} = \frac{m\upsilon_0^2}{2} = W\cos^2\alpha_0$ at $x = \rho_m$, therefore, Eq. (10) can be rewritten relatively $\frac{m\upsilon_m^2}{2}$ as follows:

$$\frac{m\upsilon_m^2}{2} = W\cos^2\alpha_0 + qU_0 f_m \ , \tag{11}$$

where

$$f_m = \int_0^{\rho_m} \frac{\partial g(x,\xi)}{\partial \xi}\xi' d\xi \ . \tag{12}$$

Substituting Eqs. (11) and (12) into Eq. (10), we obtain the integro-differential equation of motion of a charged particle in the octupole-cylindrical field (6):

$$(\xi')^2\left[g_0 - g_x + f_x\right] = P^2\mathrm{ctg}^2\alpha_0 + f_m - f_x \ , \tag{13}$$

or

$$(\xi')^2 = \frac{P^2\mathrm{ctg}^2\alpha_0 + f_m - f_x}{g_0 - g_x + f_x} \ , \tag{14}$$

where

$$f_x = \int_0^x \frac{\partial g(x,\xi)}{\partial \xi}\xi' dx \ , \tag{15}$$

and $P_0^2 = \frac{W}{qU_0}\sin^2\alpha_0$ is the reflection parameter relating the geometric and energy parameters of the octupole-cylindrical field.

The solution of the integro-differential equation (13) can be found as an expansion in a power series with indeterminate coefficients. These coefficients can be calculated from Eq. (13) by substituting a power series into it.

The integro-differential equation (13) has a singular point at the point x=0, since the factor $(\xi')^2$, in this case, vanishes, therefore, to integrate the equation, the method of expanding the solution of the equation ξ into a fractional power series is used [15]:

$$\xi = \sqrt{x}\sum_{n=0}^{\infty} C_n x^n \ , \tag{16}$$

or

$$\xi = \sqrt{x}\left(C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + \ldots\right) \ . \tag{17}$$

The radial component $R = 1 + \rho_m$ of the turning point of the trajectory, necessary for calculating the value of ξ, can be determined by using the integro-differential equation of the trajectory (13) for the point $x = \rho_m$. In this case:

$$(\xi')^2 = \mathrm{ctg}^2\alpha_0, \quad g_{x=\rho_m} = 0 ,$$

and

$$g_0 + f_m = P^2 .$$

Substituting g_o from Eq. (3) into Eq.(16) we arrive at the expression:

$$\ln R = \frac{8\left(P^2 - f_m\right) + \frac{1}{8}R^4 + \frac{1}{2}R^2 - \frac{5}{8}}{\left(\frac{17}{2} + R^2\right)} .$$

The value R can be determined by the method of successive approximations. As a zero approximation, the parameters of a cylindrical mirror analyzer are used: $R_0 = \exp\left(P^2\right) = 1 + P^2 + \frac{1}{2}P^4 + \frac{1}{6}P^6 + \frac{1}{24}P^8 + ...$ and $f_{m_0} = 0$.

So the equation for determining ρ_m:

$$\rho_m = \exp\left[\frac{8\left(P^2 - f_m\right) + \frac{1}{8}R^4 + \frac{1}{2}R^2 - \frac{5}{8}}{\left(\frac{17}{2} + R^2\right)} - 1\right].$$

The left side of Eq.(13) was found from the calculations:

$$\begin{aligned}
&\left(\frac{d\xi}{dx}\right)^2 \left[g_0 - g_x + \int_0^x \frac{\partial g\left(x,\xi\right)}{\partial \xi}\xi' dx\right] = \\
&= h_0\left[b_1 - e_1\right]x + \left[h_0\left(\frac{b_2}{2} - e_2\right) + h_1\left(b_1 - e_1\right)\right]x^2 + \\
&+\left[h_0\left(\frac{b_3}{3} - e_3\right) + h_1\left(\frac{b_2}{2} - e_2\right) + h_2\left(b_1 - e_1\right)\right]x^3 \\
&+\left[h_0\left(\frac{b_4}{4} - e_4\right) + h_1\left(\frac{b_3}{3} - e_3\right) + h_2\left(\frac{b_2}{2} - e_2\right) + h_3\left(b_1 - e_1\right)\right]x^4 + \\
&+\left[h_0\left(\frac{b_5}{5} - e_5\right) + h_1\left(\frac{b_4}{4} - e_4\right) + h_2\left(\frac{b_3}{3} - e_3\right) + h_3\left(\frac{b_2}{2} - e_2\right) + h_4\left(b_1 - e_1\right)\right]x^5 + \\
&+\left(h_0\left(\frac{b_6}{6} - e_0\right) + h_1\left(\frac{b_5}{5} - e_5\right) + h_2\left(\frac{b_4}{4} - e_4\right) + h_3\left(\frac{b_3}{3} - e_3\right) + h_4\left(\frac{b_2}{2} - e_2\right) + h_5\left(b_1 - e_1\right)\right)x^6.
\end{aligned} \tag{18}$$

Further, the integral on the right side of Eq.(13) is found:

$$\begin{aligned}
&P^2\text{ctg}^2\alpha_0 + f_m - f_x = P^2\text{ctg}^2\alpha_0 + F\left(\rho_m\right) - \\
&-\left\{b_1 x + \frac{b_2 x^2}{2} + \frac{b_3 x^3}{3} + \frac{b_4 x^4}{4} + \frac{b_5 x^5}{5} + \frac{b_6 x^6}{6} + ...\right\} - \phi\left(x\right).
\end{aligned} \tag{19}$$

Thus, both parts of the equation of motion of a charged particle in the octupole-cylindrical field (13) are presented in the form of power series. Further, by equating the terms at the same powers of x in expressions (18) and (19), the coefficients of series Eq. (16) are determined. The coefficients of series allow for further analyzing the corpuscular-optical parameters of the considered system.

The results of calculating the coefficients of the C_n series (16), which determine the trajectories of motion of charged particles in the investigated octupole-cylindrical field, are given below. We equate the terms at the same powers in Eqs. (18) and (19), and the following system of equations is obtained:

$$h_0\left(b_1 - e_1\right) = -b_1 , \tag{20a}$$

$$h_0\left(\frac{b_2}{2}-e_2\right)+h_1\left(b_1-e_1\right)=-\frac{b_2}{2}, \tag{20b}$$

$$h_0\left(\frac{b_3}{3}-e_3\right)+h_1\left(\frac{b_2}{2}-e_2\right)+h_2\left(b_1-e_1\right)=\frac{b_3}{3}, \tag{20c}$$

$$h_0\left(\frac{b_4}{4}-e_4\right)+h_1\left(\frac{b_3}{3}-e_3\right)+h_2\left(\frac{b_2}{2}-e_2\right)+h_3\left(b_1-e_1\right)=\frac{b_4}{4}, \tag{20d}$$

$$h_0\left(\frac{b_5}{5}-e_5\right)+h_1\left(\frac{b_4}{4}-e_4\right)+h_2\left(\frac{b_3}{3}-e_3\right)+h_3\left(\frac{b_2}{2}-e_2\right)+h_4\left(b_1-e_1\right)=\frac{b_5}{5}, \tag{20e}$$

$$h_0\left(\frac{b_6}{6}-e_0\right)+h_1\left(\frac{b_5}{5}-e_5\right)+h_2\left(\frac{b_4}{4}-e_4\right)+h_3\left(\frac{b_3}{3}-e_3\right)+h_4\left(\frac{b_2}{2}-e_2\right)+h_5\left(b_1-e_1\right)=\frac{b_6}{6}. \tag{20f}$$

The problem of determining the trajectories of charged particles in the field under study is reduced to calculating the coefficients in expression (16). Expressions for the coefficients h_i, e_i and b_i are obtained:

$$h_0=\frac{C_0^2}{4}, \tag{21a}$$

$$h_1=\frac{3C_0C_1}{2}, \tag{21b}$$

$$h_2=\frac{9C_1^2}{4}+\frac{5C_0C_2}{2}, \tag{21c}$$

$$h_3=\frac{7C_0C_3}{2}+\frac{15C_1C_2}{2}, \tag{21d}$$

$$h_4=\frac{25C_2^2}{4}+\frac{9C_0C_4}{2}+\frac{21C_1C_3}{2}, \tag{21e}$$

$$h_5=\frac{11C_0C_5}{2}+\frac{27C_1C_4}{2}+\frac{35C_2C_3}{2}\quad, \tag{21f}$$

$$h_6=\frac{49C_3^2}{4}+\frac{13C_0C_6}{2}+\frac{33C_1C_5}{2}+\frac{45C_2C_4}{2}\quad. \tag{21g}$$

$$e_0=-\frac{3}{16}\rho_m-\frac{5}{32}\rho_m^2-\frac{1}{16}\rho_m^3-\frac{1}{64}\rho_m^4+\ln\left(1+\rho_m\right)\left(\frac{19}{16}+\frac{1}{4}\rho_m+\frac{1}{8}\rho_m^2\right), \tag{22a}$$

$$e_1=\frac{3}{16}+\frac{5}{16}\rho_m+\frac{3}{16}\rho_m^2+\frac{1}{16}\rho_m^3+\frac{C_0^2}{2}\rho_m\left(1+\rho_m\right)-\ldots\quad, \tag{22b}$$

$$e_2=\frac{3}{32}-\frac{C_0^2}{2}-\frac{\rho_mC_0^2}{2}-\frac{C_0^4}{24}-\frac{1}{8}\rho_m-\frac{19}{16\left(1+\rho_m\right)^2}-\frac{1}{8}\ln\left(1+\rho_m\right)-\ldots, \tag{22c}$$

$$e_3=\frac{1}{16}+\frac{C_0^2}{4}+\frac{1}{4\left(1+\rho_m\right)}\left(1+\frac{1}{2}\right)-\frac{19}{16\left(1+\rho_m\right)^3}+\frac{\rho_mC_1^2}{2}+\frac{\rho_m^2C_1^2}{4}-\ldots\quad, \tag{22d}$$

$$e_4=-\frac{1}{64}-\frac{C_1^2}{2}-\rho_mC_0C_2+\rho_mC_0C_3+\rho_mC_1C_2+\frac{1}{2\left(1+\rho_m\right)^2}-\frac{3\rho_m}{8\left(1+\rho_m\right)^4}+\ldots\;, \tag{22d}$$

$$e_5=\frac{C_1^2}{4}+\frac{1}{8\left(1+\rho_m\right)^3}-\frac{19}{16\left(1+\rho_m\right)^5}-\frac{\rho_m}{4\left(1+\rho_m\right)^5}-\frac{\rho_m^2}{8\left(1+\rho_m\right)^5}+\frac{\rho_mC_2^2}{2}-\ldots, \tag{22e}$$

$$e_6=-\frac{1}{8\left(1+\rho_m\right)^4}+\frac{1}{4\left(1+\rho_m\right)^5}-\frac{C_2^2}{2}-\frac{C_1^4}{24}-\frac{19}{16\left(1+\rho_m\right)^6}+\frac{\rho_m}{4\left(1+\rho_m\right)^5}-\ldots\;. \tag{22f}$$

$$b_1=\frac{\rho_mC_0^2}{4}\left(1+\frac{\rho_m}{2}\right)-\frac{C_0^2}{16}\ln(1+\rho_m), \tag{23a}$$

$$b_2 = -\frac{C_0^2}{4} - \frac{C_0^4}{12} + \rho_m C_0\left(C_1 - \frac{C_0}{4}\right) + \frac{C_0^2}{16(1+\rho_m)} + \dots, \tag{23b}$$

$$b_3 = \frac{C_0^2}{8} - \frac{C_0^3 C_1}{2} - C_0 C_1(1_1 + \rho_m) + \frac{3\rho_m}{2}\left(\frac{C_1^2}{2} + C_0 C_2\right)\left(1 + \frac{\rho_m}{2}\right) + \dots, \tag{23c}$$

$$b_4 = \frac{C_0 C_1}{2} - \frac{3C_0 C_2}{2} - \frac{3C_1^2}{4} - C_0^2 C_1^2 - \frac{2C_0^3 C_2}{3} + 3\left(\frac{C_1^2}{2} + C_0 C_2\right)\left(\frac{1}{8(1+\rho_m)} - \frac{\rho_m}{2}\right) + \dots, \tag{23d}$$

$$b_5 = -2C_1 C_2 - 2C_0 C_3 - C_0^3 C_4 - \frac{3C_0 C_2}{4} - \frac{5C_0^2 C_1 C_2}{2} + \frac{3C_1^2}{8} - \dots, \tag{23e}$$

$$b_6 = C_0 C_3 + C_1 C_2 - 3\left(C_0 C_1^2 C_2 + C_0^2 C_1 C_3\right) - \frac{1}{2}\left(5C_1 C_3 + 5C_0 C_4 - 3C_0^2 C_2^2\right) - \dots, \tag{23f}$$

$$b_7 = -3\left(C_2 C_3 + C_1 C_4 + C_0 C_5\right) + \frac{5}{4}\left(\frac{C_2^2}{2} - C_0 C_4 + C_1 C_3\right) - \frac{7}{6}\left(C_1^3 C_2 + C_0^3 C_5\right) - \dots. \tag{23g}$$

Considering the system of Eqs. (21)–(23), the coefficients C_i are found. The approximate-analytical method makes it possible to describe with high accuracy the trajectories of motion of charged particles in the field under consideration.

Conclusions

The electron-optical scheme of a new type of mirror energy analyzer based on an electrostatic octupole-cylindrical field has been studied. The calculation of particle trajectories in an electrostatic octupole-cylindrical field is performed. The problem of integrating the differential equations of motion of charged particles and the analytical description of the trajectory equation in the electrostatic octupole-cylindrical mirror is solved. The coefficients of the fractional-power series are obtained, which are necessary for describing the trajectories of motion in an analytical form, accessible for further studies of the electron-optical characteristics of the octupole-cylindrical field.

Acknowledgements

The research was supported by the Ministry of Education and Science of the Republic of Kazakhstan, Grant No. AP09058188.

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Жаңа типті зарядталған бөлшектердің айналы энергия талдағыштың электронды-оптикалық сұлбасын есептеу

Мақалада цилиндрлік айна мен дөңгелектік мультипольдер өрістерінен синтезделген электрстатикалық мультипольді-цилиндрлік өрістердің электронды–оптикалық қасиеттеріне одан арғы зерттеулер жүргізілген. Электрондық спектроскопия әдістерін іске асырудың негізгі элементтерінің бірі төмен және орта энергиялы электрондарының энергия талдағышы болып табылатын күрделі құрал-жабдықты қолдануға негізделген. Ауытқушы өрістерді синтездеуге мультипольдік әдіс-тәсілді қолдану зарядталған бөлшектер ағындарын энергия талдауының тиімді әдістерін дамытуға мүмкіндік береді. Бұл жұмыста электрстатикалық осьтік симметриялық октупольді – цилиндрлік өріс негізінде зарядталған бөлшектер ағындарының жаңа типті айналы энергия талдағышының электронды-оптикалық сұлбасы ұсынылған. Аксиалды–симметриялық октупольді–цилиндрлік өріс базалық цилиндрлік өріс және дөңгелектік октупольдің суперпозициясы түрінде құрастырылған. Берілген өрістерді қосу кезінде дөңгелектік октупольдің орталық шеңбері логарифмдік өрістің нөлдік эквипотенциясымен біріктіріледі. Электрстатикалық аксиалды-симметриялық октупольді-цилиндрлік өрісте зарядталған бөлшектердің қозғалысы зерттелген. Электрстатикалық октупольді–цилиндрлік өрістегі зарядталған бөлшектердің қозғалысының интегралды–дифференциалдық теңдеуі алынған. Энергия талдағыштың октупольді–цилиндрлік өрісінде бөлшектердің траекториялық есептеуі интегралды–дифференциялық түрде берілген зарядталған бөлшектердің қозғалыс теңдеуінің бөлшекті–дәрежелі қатарға жіктеу әдісі негізінде жүргізілген. Октупольді–цилиндрлік өрістің электронды–оптикалық сипаттамаларын одан әрі зерттеуге мүмкіндік беретін аналитикалық түрдегі қозғалыс траекториясын беретін қатар коэффициенттері есептелінген. Октупольді–цилиндрлік өріс негізінде ғарыштық плазмадағы эВ бірліктерінен ондаған кэВ дейінгі энергияға ие зарядталған бөлшектер ағындарының құрамын анықтауға арналған жарық күшті электрстатикалық энергия талдағыштарын құрастыруға болады.

Кілт сөздер: зарядталған бөлшектердің энергия талдағышы, электронды айналар, электрстатикалық остік симметриялық октупольді–цилиндрлік өріс, жуық–аналитикалық есептеу, зарядталған бөлшектердің қозғалысы.

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Расчет электронно-оптической схемы зеркального энергоанализатора заряженных частиц нового типа

В статье продолжены дальнейшие исследования электронно-оптических свойств электростатических мультипольно-цилиндрических полей, синтезированных из полей цилиндрического зеркала и круговых мультиполей. Реализация методов электронной спектроскопии основана на использовании сложного оборудования, одним из главных элементом которого является энергоанализатор электронов низких и средних энергий. Применение мультипольного подхода к синтезу отклоняющих полей дает возможность для развития эффективных методов энергоанализа потоков заряженных частиц. Авторами предложена электронно-оптическая схема зеркального энергоанализатора потоков заряженных частиц нового типа на основе электростатического осесимметричного октупольно-цилиндрического поля. Осесимметричное октупольно-цилиндрическое поле сконструировано в виде суперпозиции базового цилиндрического поля и кругового октуполя. При сложении полей центральная окружность октуполя совмещалась с нулевой эквипотенциалью логарифмического поля. Исследовано движение заряженных частиц в электростатическом октупольно-цилиндрическом поле. Выведено интегро-дифференциальное уравнение движения заряженных частиц в электростатическом октупольно-цилиндрическом поле. Расчет траекторий частиц в зеркальном энергоанализаторе с октупольно-цилиндрическим полем выполнен на основе метода разложения в дробно-степенной ряд уравнения движения заряженных частиц, представленного в интегро-дифференциальной форме. Получены коэффициенты ряда, представляющие траекторию движения в аналитическом виде, доступном для дальнейших исследований электронно-оптических характеристик октупольно-цилиндрического поля. На основе октупольно-цилиндрического поля могут быть построены светосильные энергоанализаторы, предназначенные для определения состава потоков заряженных частиц с энергиями от единиц эВ до десятков кэВ в космической плазме.

Ключевые слова: энергоанализатор заряженных частиц, электронные зеркала, электростатическое осесимметричное октупольно-цилиндрическое поле, приближенно-аналитический расчет, движение заряженных частиц.

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