DOI 10.31489/2022Ph1/75-85

UDC 53.082

O. Lyashuk[1*], L. Serilko[2], I. Hevko[1], D. Serilko[2], I. Lutsiv[1], Y. Vovk[1], M. Levkovich[1], O. Tson[1]

[1]*Ternopil Ivan Puluj National Technical University, Ukraine;*
[2]*National University of Water and Environmental Engineering, Ukraine*
(E-mail: Oleglashuk@ukr.net)

The investigation of a physical pendulum motion, which move along a horizontal axis

The article presents a study of the physical pendulum, taking into account the force of friction in the kinematic pair, as a result of which oscillations are damped. Graphs of the dependence of the pendulum deflection angle α and the angular velocity on time for different values of the velocity *v* have been given. It has been established that the speed of the sleeve significantly reduces the amplitude and angular velocity of the pendulum, and the frequency of its oscillations does not depend on the presence of dry friction in the system. The dependences of the change in the amplitude of pendulum oscillations have been given and the results of numerical integration of the differential equation of pendulum motion have been obtained. The graphical dependences of the pendulum deflection angle and the movement of the sleeve *x* along the horizontal axis from time to time have been obtained at different values of the coefficient of friction. It has been found that during the first five seconds of the system movement, the axial speed of the sleeve is practically independent of the coefficient of friction (at $f = 0.3 \ldots 0.5$). To verify the obtained results, an experimental laboratory installation has been designed and manufactured. Theoretical studies are satisfactorily consistent with experimental data, with an error not exceeding 16%. The obtained dependencies can be used in the design and study of various mechanisms, the motion of which is described by similar differential equations. Such mechanisms include inertial conveyors, the gutter of which performs in addition to longitudinal and transverse oscillations. In addition, the proposed technique can be used in the study of the motion of bulk materials in an inclined cylinder, which performs torsional oscillations around the axis of symmetry.

Keywords: physical pendulum, oscillations, speed, amplitude, sleeve, experimental laboratory installation.

Introduction

The study of a mathematical pendulum motion is a classic problem of nonlinear oscillations, the solution of which has an exact analytical solution, especially at small values of the amplitude of oscillations. In contrast to the mathematical pendulum, when studying the operation of a physical pendulum, it is necessary to take into account the force of friction in the kinematic pair, as a result of which the oscillations have been damped.

However, in technology, there are oscillating processes in which dry friction does not reduce the amplitude, but, conversely, sometimes leads to self-oscillations. The motion of a spring-loaded cargo on an infinite moving belt [1, 2], or the motion of a Freud pendulum is an example of such oscillations, that rotates uniformly with some angular velocity [3].

The motion of a mechanical system consisting of a rectilinear rod with a ring passing through an inclined rectilinear guide and a rod oscillating in a vertical plane passing through a guide has been considered in [4]. Therefore, there is a problem of studying the motion of the pendulum when the rod will oscillate in a vertical plane that passes perpendicular to the guide. However, the ring will move along the guide.

This study aims to determine the motion law of a physical pendulum, the suspension point of which moves along the axis relative to which the oscillation occurs.

The theory of linear oscillations in the presence of viscous friction forces has been the most thoroughly developed [5, 6]. Therefore the study of oscillations with dry friction, and especially nonlinear, has been connected with considerable mathematical difficulties, and in some cases, only numerical solutions of the obtained differential equations of the motion or the approximate solution have been possible [7–9].

The averaging method is one of the effective approximate methods of system analysis with nonlinear friction. It allows to study not only the stationary mode of system motion but also the process of establishing the stationary mode [10–15].

Experimental

The motion of a physical pendulum consists of a sleeve 1, which is installed with the ability to slide along a fixed horizontal rod 2, as well as rotate around the axis of the rod. The rod 3 is fixed to the sleeve 1, at the end of which the load 4 is placed (Fig. 1).

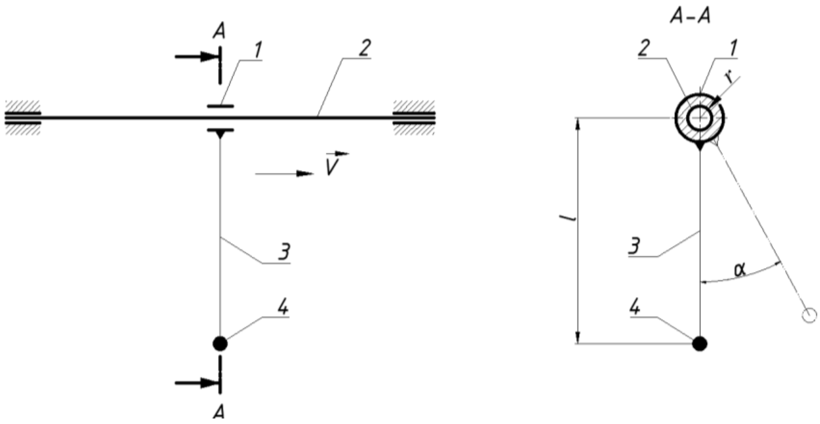


Figure 1. Scheme motion of a physical pendulum

Since the sleeve can move along the horizontal axis, there are two options for this motion:

1. To set the variation speed law $\vec{v}$ of the sleeve relative to the rod $\vec{v}$=$\vec{v}$(t);
2. To set the variation law of the horizontal force $\vec{F}$, which is attached to the sleeve $\vec{F}$=$\vec{F}$(t).

The first option applies to systems with kinematic transformation of dry friction, and the second — with dynamic transformation.

In the presence of dry friction and the absence of axial movement of the sleeve, the equation of motion will have the form:

$$I \cdot \ddot{\alpha} + mgl \sin\alpha + M_T \operatorname{sgn}\dot{\alpha} = 0, \tag{1}$$

where, M_T – the moment of friction forces

$$M_T = m \cdot g \cdot r \cdot f \cdot \cos\alpha,$$

r – shaft radius; f – consolidated coefficient of friction$f = \frac{4}{\pi} \cdot f_0$ [16]; f_0 – the coefficient of sliding friction between the sleeve and the shaft.

At small values of an angle α: $\sin\alpha = \alpha$; $\cos\alpha = 1$; $M_T = m \cdot g \cdot r \cdot f$.

When the pendulum is deflected at an angle φ_0 the equation of motion (1) will be:

$$I \cdot \ddot{\alpha} + mgl\alpha = mgrf. \tag{2}$$

If the mass of the load is much greater than the mass of the rod and sleeve, it can be assumed that the moment of inertia of the pendulum will be equal to: $I = m \cdot l^2$

Enter the notation $\omega^2 = \frac{g}{l}$; $\frac{rf}{l} = b$.

After replacing the equation (2) takes the form:

$$\alpha + \omega^2 \cdot \ddot{\alpha} = \omega^2 \cdot b. \tag{3}$$

The coefficient b is the pendulum deflection under the action of the maximum moment of friction.

If the pendulum is deflected by a value less than or equal to b, the motion will not occur, because the moment of gravity will be less than the moment of resistance.

The general solution (3) has the form [5]:

$$\alpha = b + c_1 \cos\omega t + c_2 \sin\omega t. \tag{4}$$

Taking into account the sustainable integration ($t = t_0 = 0$; $\alpha = \alpha_0$; $\dot{\alpha} = \dot{\alpha_0} = 0$, has been get:

$$\alpha = b + (\alpha_0 - b)\cos\omega t. \tag{5}$$

The law of motion will be fair till $\dot{\alpha} < 0$. Because $\dot{\alpha} = -\omega \cdot (\alpha_0 - b) \cdot \sin\omega t$, then the speed will be negative by the time point $t_1 = \frac{\pi}{\omega}$.

At this point, the pendulum will stop:

$$\alpha_1 = b + (\alpha_0 - b) \cdot \cos\pi = -(\alpha_0 - 2b). \tag{6}$$

Consider the first variant of motion, when the sleeve moves relative to the shaft at a constant speed $\vec{v}$.

With simultaneous oscillation of the pendulum and the sleeve motion, the direction of friction between the sleeve and the shaft will depend on the speed $\vec{v}$ of the sleeve motion and speed $\vec{u}$ of the sleeve rotation motion, which occurs due to oscillations of the rod:

$$u = \dot{\alpha} \cdot r, \tag{7}$$

where, $\dot{\alpha}=\frac{d\alpha}{dt}$ – angular velocity of the pendulum; α – the pendulum deflection angle from the vertical.

The normal reaction N of the sleeve surface will be equal:

$$N = m \cdot g \cdot \cos\alpha + m \cdot \dot{\alpha}^2 \cdot l. \tag{8}$$

$$\cos\gamma = \frac{u}{\sqrt{u^2+v^2}} = \frac{\dot{\alpha}\cdot r}{\sqrt{(\dot{\alpha}\cdot r)^2+v^2}}. \tag{9}$$

The differential equation of rotational motion of the pendulum relative to the axis should be written:

$$I \cdot \ddot{\alpha} = m \cdot g \cdot l \cdot \sin\alpha - f \cdot (m \cdot g \cdot l \cdot \cos\alpha + m\dot{\alpha}^2 r) \cdot r \cdot \frac{r\cdot\dot{\alpha}}{\sqrt{v^2+(r\cdot\dot{\alpha})^2}} \tag{10}$$

Consider small oscillations $\sin\alpha = \alpha$; $\cos\alpha = 1$; $r \cdot \dot{\alpha}^2 \ll g$

Taking into account the assumptions, equation (10) will have the form:

$$\ddot{\alpha} = \frac{g}{l}\alpha - fg\frac{r}{l^2} \cdot \frac{r\dot{\alpha}}{\sqrt{v^2+(r\dot{\alpha})^2}} \tag{11}$$

A replacement should be done: $\frac{g}{l} = \omega^2$; $fg\frac{r}{l^2} = \lambda$.

Then,

$$\ddot{\alpha} = -\omega^2\alpha - \lambda \cdot \frac{r\dot{\alpha}}{\sqrt{v^2+(r\dot{\alpha})^2}} \tag{12}$$

The equation (12) is not reduced to quadratures and its solution can be obtained by numerical or approximate method.

Figures 2, 3 are graphs of the dependence of the pendulum deflection angle α and the angular velocity $\dot{\alpha}$ of time to different values of the velocity v, at $r=0{,}01$ m, $f=0{,}4$, $l=0{,}5$ m, $\alpha_0=15^0$, $\dot{\alpha}_0=0$ rad/s .

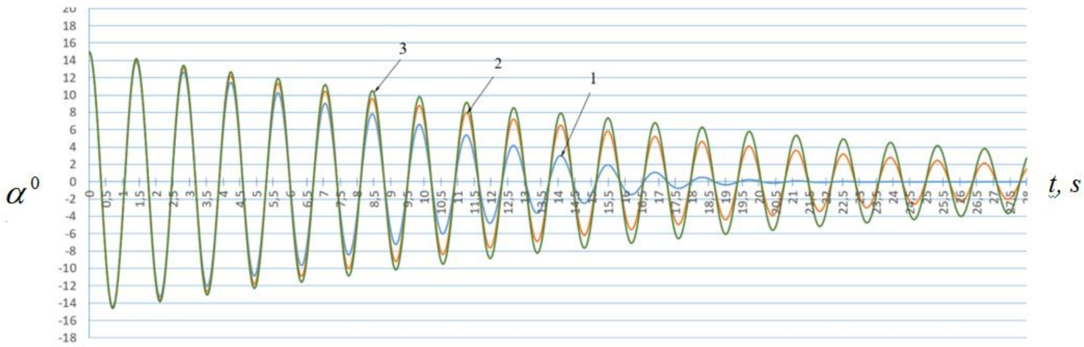


Figure 2. Graph of the pendulum deflection angle α of time for different values of velocity u
1-at u=0.001m/s; 2- at u=0.006m/s; 3- at u=0.008m/s

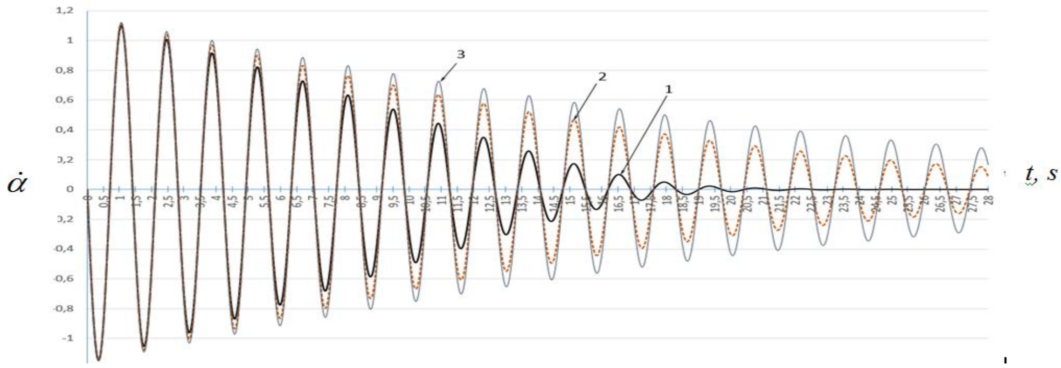


Figure 3. Graph of the dependence of the angular velocity $\dot{\alpha}$ of time for different values of the velocity u
1- at u=0.001m/s; 2- at u=0.006m/s; 3- at u=0.008m/s

The graphs show that the speed of the sleeve significantly affects the reduction of the amplitude and angular velocity of the pendulum, but the frequency of its oscillations does not depend on the presence of dry friction in the system.

Dimensionless quantities should be used to obtain an approximate solution of the equation of pendulum motion:

$$\tau=\omega\cdot t;\ \xi=\frac{\omega\cdot r\cdot\alpha}{v};\ \frac{fgr^2}{l^2v\omega}=\mu;\ \dot{\alpha}=\frac{d\alpha}{dt};\ \dot{\xi}=\frac{d\xi}{dr};\ d\tau=\omega\cdot dt.$$

Then, $\frac{d\xi}{d\tau}=\frac{d}{d\tau}\cdot\left(\frac{\omega r\alpha}{v}\right)=\frac{d}{\omega dt}\cdot\left(\frac{\omega r\alpha}{v}\right)=\frac{\dot{\alpha}r}{v};\ \ddot{\xi}=\frac{d\dot{\xi}}{d\tau}=\frac{d}{d\tau}\left(\frac{\dot{\alpha}r}{v}\right)=\frac{1}{\omega dt}\left(\frac{\dot{\alpha}r}{v}\right)=\frac{r}{\omega v}\ddot{\alpha}.$

Then the equation (12) takes the form

$$\ddot{\xi}+\xi+\mu\frac{\dot{\xi}}{\sqrt{1+\dot{\xi}^2}}=0. \quad (13)$$

The μ value has been suggested small and the averaging method has been used [4].

The variables $\xi_1=\xi;\ \xi_2=\dot{\xi}$ has been involved and rewritten (13) in the normal form of Cauchy

$$\dot{\xi_1}=\xi_2;\ \dot{\xi_2}=-\xi_1-\mu\frac{\xi_1}{\sqrt{1+{\xi_1}^2}} \quad (14)$$

Replace variables

$$\xi_1=a\sin\varphi;\ \xi_2=a\cos\varphi.$$

Turn to the equations in the standard form of the averaging method

$$\dot{a}=-\mu\cdot a\frac{\cos^2\varphi}{\sqrt{1+a^2\cos^2\varphi}} \quad (15)$$

$$\dot{\varphi}=1+\mu\frac{\sin\varphi\cdot\cos\varphi}{\sqrt{1+a^2\cdot\cos^2\varphi}} \quad (16)$$

Average the right parts (15) and (16) on the fast variable φ

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{\sin\varphi\cos\varphi}{\sqrt{1+a^2\cos^2\varphi}}d\varphi=I_1 \quad (17)$$

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{a\cos^2\varphi}{\sqrt{1+a^2\cos^2\varphi}}d\varphi=I_2 \quad (18)$$

In equation (17) a replacement should be made

$$z=1+a\cos^2\varphi. \quad (19)$$

$$dz=-a2\cos\varphi\sin\varphi\,d\varphi. \quad (20)$$

Then

$$I_1=\frac{1}{2\pi}\int_0^{2\pi}-\frac{dz}{2a\sqrt{z}}=-\frac{1}{4\pi a}\int_0^{2\pi}z^{-\frac{1}{2}}dz=-\frac{1}{4\pi a}\cdot 2z^{\frac{1}{2}}. \quad (21)$$

Returning to (17) it should be:

$$I_1=-\frac{1}{2\pi a}\sqrt{1+a\cos^2\varphi}\,\Big|_0^{2\pi}=-\frac{1}{2\pi a}\left(\sqrt{1+a\cos^2 2\pi}-\sqrt{1+a\cos^2\varphi}\right)=0.$$

So,

$$\frac{d\varphi}{d\tau}=1. \quad (22)$$

In equation (15) a replacement should be made

$$k=\frac{a}{\sqrt{1+a^2}}. \quad (23)$$

where $a^2=\frac{k^2}{1-k^2}$

Then:

$$\sqrt{1+a^2\cos^2\varphi}=\sqrt{\frac{1-k^2+k^2\cos^2\varphi}{1-k^2}}=\sqrt{\frac{1-k^2(1-\cos^2\varphi)}{1-k^2}}=\frac{1}{\sqrt{1-k^2}}\sqrt{1-k^2\sin^2\varphi}. \quad (24)$$

$$I_2=4\cdot\frac{1}{2\pi}\int_0^{\frac{\pi}{2}}\frac{k\cos^2\varphi d\varphi}{\sqrt{1-k^2\sin^2\varphi}}=\frac{2k}{\pi}\int_0^{\frac{\pi}{2}}\frac{\cos^2\varphi d\varphi}{\sqrt{1-k^2\sin^2\varphi}}=\frac{2k}{\pi}(\frac{1}{k^2}F(k)-\frac{1}{k^2}E(k), \quad (25)$$

where $F(k)$ – the complete elliptic integral has been made with the module k; $E(k)$ – complete elliptic integral of the second kind [17].

$$\frac{da}{d\tau}=-\frac{2\pi}{\pi}k\cdot B(k), \quad (26)$$

where $B(k)=\frac{1}{k^2}\left(F(k)-E(k)\right)$

The system of equations (22), (26) can be integrated in quadratures.

Perform differentiation by τ and get:

$$\frac{da}{d\tau} = (1-k^2)^{-\frac{3}{2}}\frac{dk}{d\tau} \tag{27}$$

Then the equation (26) takes the form

$$\frac{dk}{dr} = -\frac{2\mu}{\pi}\cdot k\cdot(1-k^2)^{\frac{3}{2}}\cdot B(k). \tag{28}$$

Integrate (22) and (28) under initial conditions $\varphi(\tau_0) = \varphi_0$; $k(\tau_0) = k_0$;

$$\varphi = \tau + \varphi_0 - \tau_0. \tag{29}$$

$$\int_{k_0}^{k}\frac{dk}{k\cdot(1-k^2)^{\frac{3}{2}}\cdot B(k)} = \int_0^{\tau_0} -\frac{2\pi}{\pi}d\tau. \tag{30}$$

The left part (30) is easily tabulated, because it depends only on the dimensionless value *k*. Table 1 illustrates the value of *G(k)* [4].

$$\int_{k_0}^{k}\frac{dk}{k\cdot(1-k^2)^{\frac{3}{2}}\cdot B(k)} = G(k) - G(k_0). \tag{31}$$

Table 1

The values free oscillations of the pendulum in the presence of dry friction and axial motion of the sleeve of *G(k)*

k	*G(k)*	*k*	*G(k)*	*k*	*G(k)*
0.01	-4.3939	0.3	-0.8268	0.9	2.2917
0.02	-3.9758	0.4	-0.3884	0.92	2.6095
0.03	-3.6380	0.5	0	0.94	3.0267
0.05	-3.1181	0.6	0.3828	0.96	3.6102
0.1	-2.2915	0.7	0.8054	0.98	4.5019
0.2	-1.3903	0.8	1.3510	0.99	5.1568

In Figure 4, the dependences of the change in the amplitude of pendulum oscillations obtained as a result of numerical integration of the differential equation of pendulum motion and the approximate solution are given.

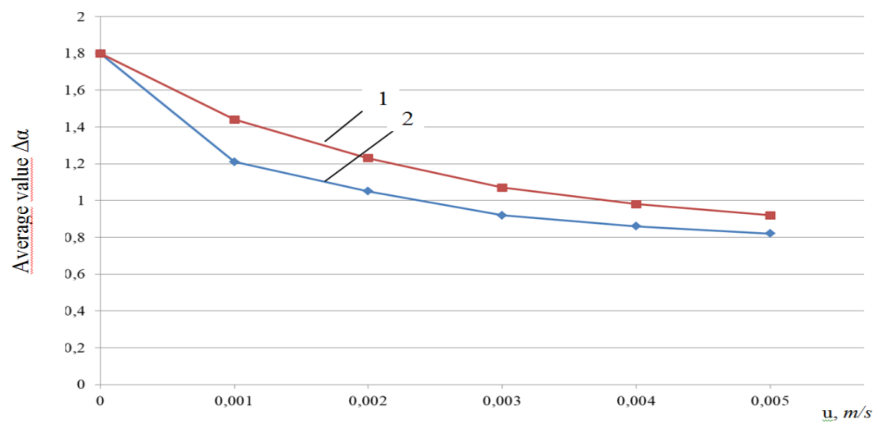


Figure 4. Graph of the dependence of pendulum oscillations amplitude decrease on the speed of the sleeve *u*

The graphs show that the approximate solution can be used in the study of such mechanical systems, especially when the values of the speed of the sleeve *u>0.005 m/s*, when the error of the results does not exceed 12%.

Consider the second variant of system motion, namely assume that the sleeve moves along the guide under the action of the horizontal force *F*.

In this case, the differential equations take the form:

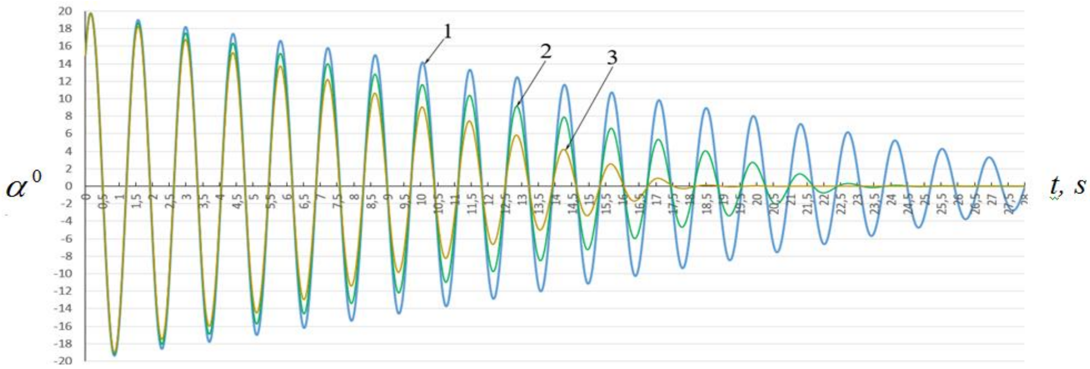
$$\begin{cases} m\ddot{x} = F - fmg\frac{\dot{x}}{\sqrt{\dot{x}^2 + (r\dot{\alpha})^2}} \\ I\ddot{\alpha} = -mgl\sin\alpha - mgfr\frac{r\dot{\alpha}}{\sqrt{\dot{x}^2 + (r\dot{\alpha})^2}} \end{cases} \tag{32}$$

Taking into account the accepted assumptions, the system (32) will be:

$$\begin{cases} \ddot{x} = F_1 - fg\dfrac{\dot{x}}{\sqrt{\dot{x}^2 + (r\dot{\alpha})^2}} \\ \ddot{\alpha} = -\omega^2\alpha - \dfrac{fgr}{l^2}\dfrac{r\dot{\alpha}}{\sqrt{\dot{x}^2 + (r\dot{\alpha})^2}} \end{cases} \tag{33}$$

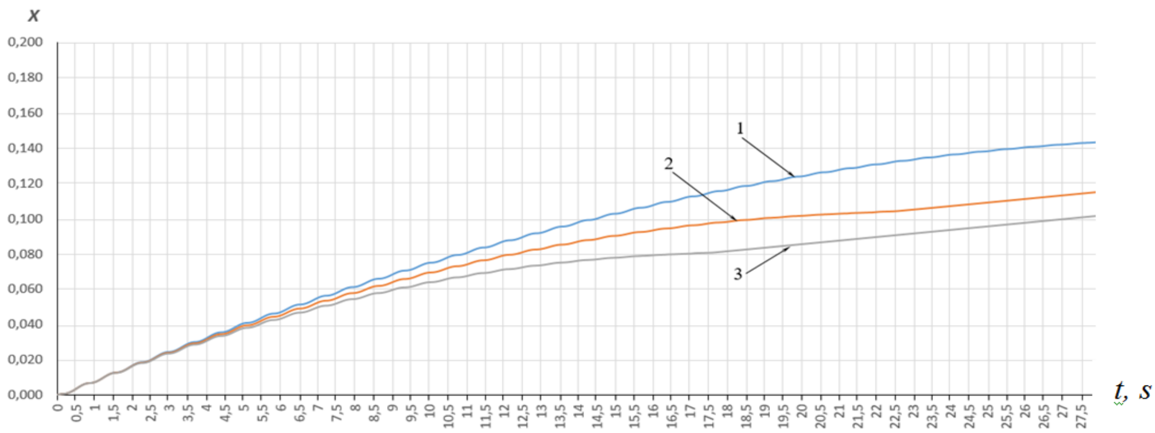
where $F_1 = \frac{F}{m}$.

As a result of the numerical solution of the system (33), obtain the dependences of the pendulum deflection angle α and the movement of the sleeve x along the horizontal axis of time, at different values of the coefficient of friction (Fig. 5- 6)



1- at f=0.3; 2- at f=0.4; 3- at f=0.5

Figure 5. Graph of the pendulum deflection angle α° of time t, at different values of the coefficient of friction f (r=0.01 m; l=0.5 m; F_1=2; $\alpha_0=0^0$; $\dot{\alpha}_0$=1 rad/s)



1- at f=0.3; 2- at f=0.4; 3- at f=0.5

Figure 6. Graph of the dependence of the sleeve motion x along the horizontal axis of time, at different values of the coefficient of friction (r=0.01 m; f=0.4; l=0,5 m; $\alpha_0=15^0$; $\dot{\alpha}_0$=1 rad/s)

The graphs demonstrate that during the first five seconds of the system motion, the axial speed of the sleeve is practically independent of the coefficient of friction (at *f=0,3…0,5*). Further, the speed will decrease, and for different values of the coefficient of friction, this change will be different. This is due to the fact that the amplitude of oscillations and the angular velocity of the pendulum will decrease, and, consequently, the axial component of the friction force between the sleeve and the rod will increase.

Since the oscillation frequency of the physical pendulum does not depend on the force of friction, to obtain an approximate solution of the equation of the sleeve motion, assume that at the initial moment of time the motion of the pendulum occurs by law:

$$\dot{\alpha} = \dot{\alpha}_0 \cos \omega t \tag{34}$$

Consider that the pendulum moves from the equilibrium position due to the initial velocity $\dot{\alpha}_0$.

Then the first equation of system (33) takes the form:

$$\ddot{x} = F_1 - fg\frac{\dot{x}}{\sqrt{\dot{x}^2 + (r\dot{\alpha}_0\cos\omega t)^2}} \tag{35}$$

Proceed to dimensionless quantities:

$$\tau = \omega t,\ d\tau = \omega dt,\ \xi = \frac{x\omega}{r\dot{\alpha}_0},\ x = \frac{\xi r\alpha_0}{\omega},\ \dot{x} = \frac{dx}{dt},\ \dot{\xi} = \frac{d\xi}{d\tau}$$

Then,

$$\dot{x} = \frac{d}{dt}\left(\frac{\xi r\alpha_0}{\omega}\right) = \frac{d}{d\tau}\left[\left(\frac{\xi r\alpha_0}{\omega}\right)\cdot\omega\right] = \dot{\xi}\dot{\alpha}_0 r;$$

$$\ddot{x} = \frac{d}{dt}\left(\dot{\xi}\dot{\alpha}_0 r\right) = \ddot{\xi}\dot{\alpha}_0 r\omega$$

Then equation (35) takes the form:

$$\ddot{\xi}\dot{\alpha}_0 r = F_1 - fg\cdot\frac{\dot{\xi}}{\sqrt{\dot{\xi}^2 + \sin^2\tau}} \tag{36}$$

or

$$\ddot{\xi} = \mu\left[\gamma - \frac{\dot{\xi}}{\sqrt{\dot{\xi}^2 + \sin^2\tau}}\right] \tag{37}$$

where, $\gamma = \frac{F}{mfg}$; $\mu = \frac{fg}{\dot{\alpha}_0 r\omega}$.

In our case at $f = 0{,}4$, $r = 0{,}01$, $\dot{\alpha}_0 = 1$ *rad/s*, $\alpha_0 = 1{,}2$, $\omega = 4{,}92$ c^{-1}.

$$\mu = \frac{0{,}4\cdot 10}{1{,}26\cdot 0{,}01\cdot 4{,}92} = 100 \gg 1.$$

Find the approximate periodic solution of equation (37). To do this, write it as follows:

$$\mu^{-1}\frac{d\dot{\xi}}{d\tau} = \gamma - \frac{\dot{\xi}}{\sqrt{\dot{\xi}^2 + \cos^2\tau}} \tag{38}$$

The resulting equation is an equation with a small parameter for the derivative. According to Tikhonov's theorem [18], limiting the degenerate approximation and assuming the left-hand side of (38) is zero, obtain:

$$\dot{\xi} = \frac{\gamma}{\sqrt{1-\gamma^2}}[\cos\tau] \tag{39}$$

The obtained dependence determines with accuracy the order of μ^{-1} the main in this problem slow component of the speed of the sleeve motion. Average each part over the period of the pendulum oscillation. Denote the average speed of the sleeve ϑ.

$$\vartheta = \langle\dot{\xi}\rangle = \frac{1}{2\pi}\int_0^{2\pi}\dot{\xi}(r)d\tau =$$

$$\frac{1}{2\pi}\int_0^{2\pi}\frac{\gamma}{\sqrt{1-\gamma^2}}\cos\tau d\tau = \frac{4\gamma}{2\pi\sqrt{1-\gamma^2}}\int_0^{\frac{\pi}{2}}\cos\tau d\tau = \frac{2\gamma}{\pi\sqrt{1-\gamma^2}} \tag{40}$$

So,

$$v = \frac{2}{\pi}\cdot\frac{\gamma}{\sqrt{1-\gamma^2}} \tag{41}$$

From equation (40) it is seen that the stationary mode is possible only under the condition $\gamma < 1$, and at $\gamma \geq 1$ the stationary mode will be absent and the sleeve will move with some non-zero acceleration.

Then the average dimensional speed of the sleeve will be equal:

$$v = \vartheta\cdot r\cdot\dot{\alpha}_0 \tag{42}$$

Figure 7 shows graphs of the dependence of the average dimensionless speed of the sleeve along the rod on the coefficient of friction *f*, and the value of the relative force F_1.

The experimental laboratory setup has been designed and manufactured to verify the results. Its scheme is shown in Figure 8a, and the general view in Figure 8b.

The horizontal rod 1 is rigidly attached to a fixed base 2. On the rod is a sleeve 3, which is installed with the ability to move along the rod and rotate around its axis. The rod 4 is rigidly attached to the sleeve 3, at the lower end of which is the load 5. At the second end of the rod 1 is a block 6, through which is passed a weightless thread, one end of which is attached to the sleeve 3, and the other to the load 8, the weight of which can be changed. During the experiment, the load 5 is given an initial speed V_0, while releasing the sleeve 3, which, under the action of gravity of the load 8 begins to slide along the rod.

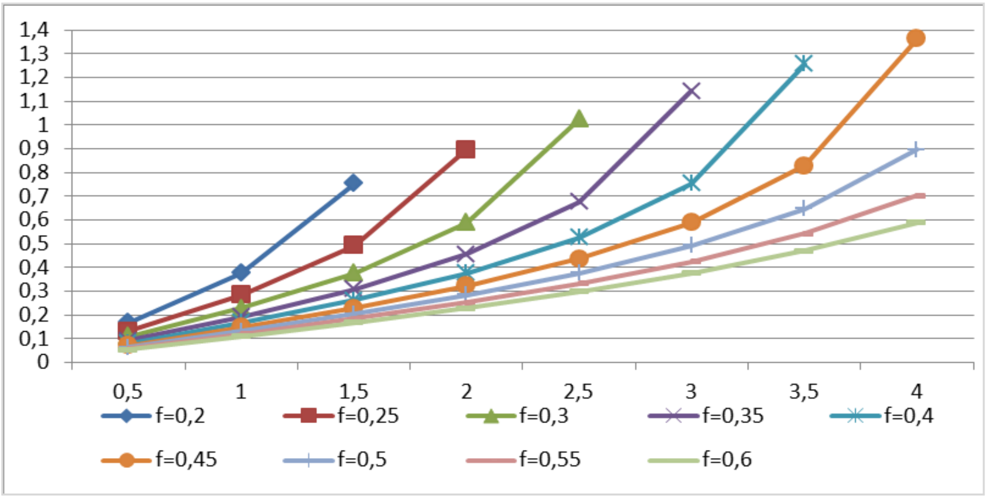


Figure 7. Graphs of the dependence of the average dimensionless speed of the sleeve along the rod on the coefficient of friction *f*, and the value of the relative force F_1

By measuring the time during which the sleeve will go a certain path, we determined the average speed of the sleeve. Table 2 represents the results of the experiment.

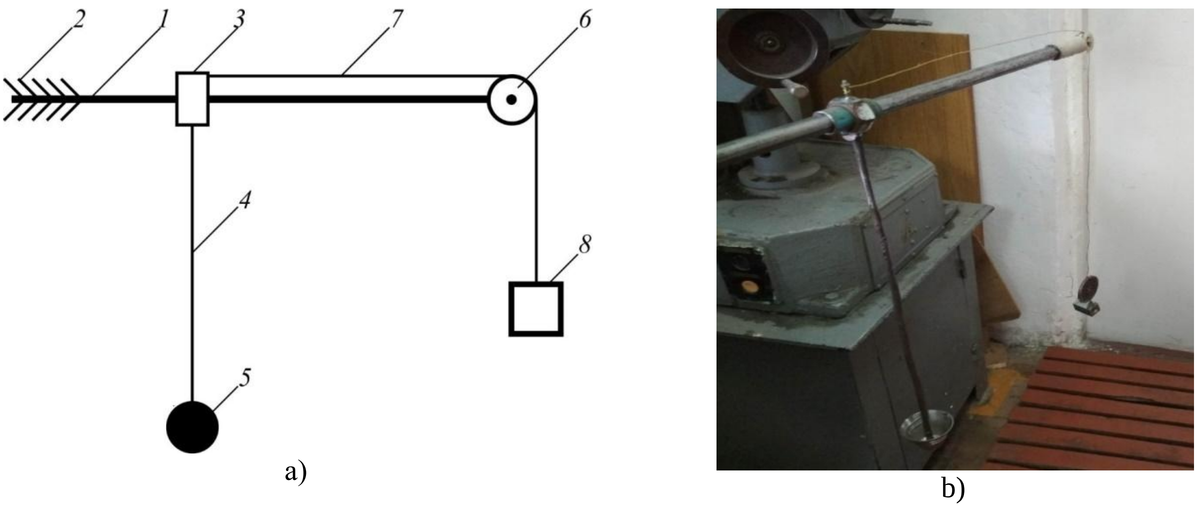


Figure 8. General view: a – constructive scheme; b – the experimental setup

Table 2

The results of the experiment are the average speed of the sleeve

F_1, m/s^2	V_T, m/s	V_E, m/s	δ, %
1	0.0066	0.0057	15.7
1.5	0.0073	0.0065	12.3
2	0.0078	0.007	11.4
2.5	0.0085	0.0079	8.2
3	0.0092	0.0087	5.7

Conclusions

These graphs show that at the value of $F_1=4$, the maximum value of the dimensionless speed will be at $f=0.45$. At higher values of the coefficient of friction, the dependence of velocity on relative force is less pronounced. Given a constant value of the amplitude of pendulum oscillations, the average speed of the sleeve will be a constant value. As the oscillations of the pendulum fade, the speed of the sleeve will decrease. However, at the beginning of the motion, the error does not exceed 12% at $f<0.3$ and 18% at $f<0.45$.As can be seen from Table 2, the results of theoretical studies agree satisfactorily with the experimental data, with an error not exceeding 16%. Similarly, it is possible to determine the speed of the sleeve under the action of gravity, when the rod is inclined at an angle to the horizon that does not exceed the value of the angle of friction. The obtained dependences can be used in the design and study of various mechanisms, the motion of which is described by similar differential equations. The proposed technique can be used in the study of the motion of bulk materials in an inclined cylinder, which performs torsional oscillations around the axis of symmetry.

References

1 Makarenkov Oleg A new test for stick-slip limit cycles in dry-friction oscillators with small nonlinear friction characteristics / Oleg Makarenkov. — Meccanica, 2016. 52. 10.1007/s11012-017-0648-7.

2 Журавлев Ф.В. Прикладные методы в теории колебаний / Ф.В. Журавлев, Д.М. Климов. — М.: Наука, 1988.

3 Магнус К. Колебания / К. Магнус. — М.: Мир, 1982.

4 Андронов В.В. Сухое трение в задачах механіки / В.В. Андронов, В.Ф. Журавлев. — Ижевск: НИЦ «Регулярная и хаотическая динамика»; Ин-т компьютерных исследований, 2010.

5 Бидерман В.Л. Теория механических колебаний / В.Л. Бидерман. — М.: Высш. шк., 1980.

6 Пановко Я.Г. Основы прикладной теории колебаний и удара. — 3-е изд., доп. и перераб. / Я.Г. Пановко. — Л.: Машиностроение, 1976.

7 Martynyuk A.A. Nonlinear Oscillations in a System with Dry Friction / A.A. Martynyuk, N.V. Nikitina// Int Appl Mech. — 2005. — No. 41. — P. 435–440. https://doi.org/10.1007/s10778–005–0107–3

8 Nicolae-Doru Nonlinear Oscillations of a Mass with Cubic Stiffness, Harmonic Excitation and Dr y Friction / Nicolae-Doru, Popa Dinel // Applied Mechanics and Materials, 2014. — No. 659. — P. 571–576. www.scientific.net/AMM.659.571.

9 Popp K., Stelter P. Nonlinear Oscillations of Structures Induced by Dry Friction. In: Schiehlen W. (eds) Nonlinear Dynamics in Engineering Systems/ K. Popp, P. Stelter// International Union of Theoretical and Applied Mechanics. Springer, Berlin, Heidelberg. — 1990. DOI: https://doi.org/10.1007/978–3-642–83578–0_29

10 Hevko, I., Hud, V., Lyashuk, O., Hevko, O., & Shust, I. (2020). Research of resonance vibrations of the system "Telescopic screw is a bulk media" condited by tortial vibrations. *Bulletin of the university of Karaganda-Physics,* 2(98), 119–126.

11 Hevko, I., Lyashuk, O., Sokil, M., Slobodian, L., Hud, V., & Vovk Y. (2019). Resonant oscillation of vertical working part of conveyer-loader. *Bulletin of the university of Karaganda-Physics,* 2(94), 73–82.

12 A. Mitropolsky A.Veraging method in non-linear mechanics / A. Mitropolsky. International Journal of Non-Linear Mechanics. — 1967. — № 2(1). — P. 69–96.

13 Jan A. Sanders Ferdinand Verhulst James Murdock / A. Jan. Averaging Methods in Nonlinear Dynamical Systems: Edition 2. — 2007.

14 E.A. Grebenikov Methods of averaging equations in celestial mechanics / E.A. Grebenikov. American Institute of Phisics. 1–965. — № 9(1). — P.190–194.

15 Mzaki Dakel Jean-Jacques Sinou Stability and nonlinear self-excited friction-induced vibrations for a minimal model subjected to multiple coalescence patterns Jve international ltd. / Dakel Mzaki. Journal of vibroengineering. — 2017. — No. 19. — P. 604–628.

16 Бондаренко Л.Н. Момент трения в подшипниках скольжения / Л. Н.Бондаренко, Г. К. Демин, А. П. Буратинский // Вісник Придніпровської державної академії будівництва та архітектури. — 2016. — № 9 (222). — С. 36–41.

17 Янке Е. Специальные функции/ Е. Янке, Ф. Эмде, Ф. Леш. — М.: Наука, 1968.

18 Тихонов А.Н. Система дифференциальных уравнений, содержащих малые параметры при производных / А.Н. Тихонов // Мат. сб. — 1952. — Т 31(73). — С. 575–586.

О. Ляшук, Л. Серилко, И. Гевко, Д. Серилко, И. Луцив,
Ю. Вовк, М. Левковыч, О. Цонь

Көлденең ось бойымен қозғалатын маятниктің физикалық қозғалысын зерттеу

Мақалада нәтижесінде тербелістердің сөнуіне әкелетін кинематикалық жұптағы үйкеліс күшін ескере отырып, физикалық маятник жұмысының зерттеуі берілген. Маятниктің ауытқу бұрышы мен бұрыштық жылдамдықтың уақытқа тәуелділігінің графиктері *v* жылдамдықтың әртүрлі мәндері үшін келтірілген. Маятник тербелістерінің амплитудасы мен бұрыштық жылдамдығының төмендеуіне төлкенің жылдамдығы айтарлықтай әсер ететіні анықталды, ал оның тербеліс жиілігі жүйеде құрғақ үйкелістің болуына байланысты емес. Маятниктің тербеліс амплитудасының өзгеруіне және маятник қозғалысының дифференциалдық теңдеуінің сандық интеграциясының нәтижелеріне тәуелділіктер келтірілген. Үйкеліс коэффициентінің әртүрлі мәндері үшін маятниктің ауытқу бұрышының және төлкенің *x* көлденең ось бойымен уақытқа байланысты орын ауыстыруының графикалық тәуелділігі алынған. Маятник тербелістерінің амплитудасының өзгеруінің тәуелділіктері және маятник қозғалысының дифференциалдық теңдеуінің сандық интегралдау нәтижелері келтірілген. Маятниктің ауытқу бұрышының графикалық тәуелділігі және үйкеліс коэффициентінің әртүрлі мәндері үшін маятниктің ауытқу бұрышының және төлкенің *x* көлденең ось бойымен уақытқа орын ауыстыруының қозғалысы алынды. Жүйе қозғалысының алғашқы бес секундында төлкенің осьтік жылдамдығы үйкеліс коэффициентіне тәуелді емес екендігі анықталды (F=0,3.0,5 кезінде). Нәтижелерді тексеру үшін эксперименттік зертханалық қондырғы жобаланып, дайындалды. Теориялық зерттеулер эксперименттік деректермен келісілген, бұл ретте қателік 16%-дан аспайды. Алынған тәуелділіктерді қозғалысы ұқсас дифференциалдық теңдеулермен сипатталатын әртүрлі механизмдерді жобалау мен зерттеуде қолдануға болады. Мұндай механизмдерге бойлық және көлденең тербелістерден басқа, науа жасайтын инерциялық конвейерлер жатады. Сонымен қатар, ұсынылған әдісті симметрия осінің айналасында айналмалы тербелістер жасайтын көлбеу цилиндрдегі сусымалы материалдардың қозғалысын зерттеуде қолдануға болады.

Кілт сөздер: физикалық маятник, тербелістер, жылдамдық, амплитуда, төлке, эксперименттік зертханалық қондырғы.

О. Ляшук, Л. Серилко, И. Гевко, Д. Серилко, И. Луцив,
Ю. Вовк, М. Левковыч, О. Цонь

Исследование физического движения маятника, движущегося вдоль горизонтальной оси

В статье приведены исследования работы физического маятника с учетом силы трения в кинематической паре, в результате которой происходит затухание колебаний. Приведены графики зависимости угла отклонения маятника и угловой скорости от времени для различных значений скорости *v*. Установлено, что скорость движения втулки оказывает существенное влияние на уменьшение амплитуды и угловой скорости колебаний маятника, при этом частота его колебаний не зависит от наличия сухого трения в системе. Приведены зависимости изменения амплитуды колебаний маятника и результаты численного интегрирования дифференциального уравнения движения маятника. Получены графические зависимости угла отклонения маятника и перемещение втулки *x* вдоль горизонтальной оси от времени, при разных значениях коэффициента трения. Установлено, что в первые пять секунд движения системы осевая скорость втулки практически не зависит от коэффициента трения (при f=0,3…0,5). Для проверки результатов была спроектирована и изготовлена экспериментальная лабораторная установка. Теоретические исследования согласуются с экспериментальными данными, при этом погрешность не превышает 16 %. Полученные зависимости могут быть использованы при проектировании и исследовании различных механизмов, движение которых описывается аналогичными дифференциальными уравнениями. К таким механизмам относятся инерционные конвейеры, желоб которых совершает, помимо продольных, и поперечные колебания. Кроме того предложенную методику можно использовать при исследовании движения сыпучих материалов в наклонном цилиндре, совершающем крутильные колебания вокруг оси симметрии.

Ключевые слова: физический маятник, колебания, скорость, амплитуда, втулка, экспериментальная лабораторная установка.

References

1 Makarenkov, Oleg. (2016). A new test for stick-slip limit cycles in dry-friction oscillators with small nonlinear friction characteristics. *Meccanica*. 52. 10.1007/s11012–017–0648–7.

2 Zhuravlev, F.V., & Klimov, D.M. (1988). *Prikladnye metody v teorii kolebanii [Applied methods in the theory of oscillations]*. Moscow: Nauka [in Russian].

3 Magnus, K. (1982). *Kolebaniia [Oscillations]*. Moscow: Mir [in Russian].

4 Andronov, V.V., & Zhuravlev, V.F. (2010). *Sukhoe trenie v zadachakh mekhaniki [Dry friction in problems of mechanics]. Reguliarnaia i khaotichnaia dinamika; Institut kompiuternykh issledovanii – "Regular and chaotic dynamics", Computer Research Institute*. Izhevsk: Nauchno-issledovatelskii tsentr [in Russian].

5 Biderman, V.L. (1980). *Teoriia mekhanicheskikh kolebanii [Theory of mechanical oscillations]*. Moscow: Vysshaia shkola [in Russian].

6 Panovko, Ya.G. (1976). *Osnovy prikladnoi teorii kolebanii i udara [Fundamentals of the applied theory of oscillations and shock 3rd ed., Ext. and recycling]*. Leningrad: Mashinostroenie [in Russian].

7 Martynyuk, A.A., & Nikitina, N.V. (2005). Nonlinear Oscillations in a System with Dry Friction. *Int Appl Mech* 41, 435–440. https://doi.org/10.1007/s10778–005–0107–3

8 Nicolae-Doru, & Popa, Dinel (2014). Nonlinear Oscillations of a Mass with Cubic Stiffness, Harmonic Excitation and Dry Friction. *Applied Mechanics and Materials,* 659, 571–576. 10.4028/ www.scientific.net/AMM.659.571.

9 Popp, K., & Stelter, P. (1990). Nonlinear Oscillations of Structures Induced by Dry Friction. In: Schiehlen W. (eds) Nonlinear Dynamics in Engineering Systems. *International Union of Theoretical and Applied Mechanics. Springer, Berlin, Heidelberg.* DOI: https://doi.org/10.1007/978–3-642–83578–0_29

10 Hevko, I., Hud, V., Lyashuk, O., Hevko, O., & Shust, I. (2020). Research of resonance vibrations of the system "Telescopic screw is a bulk media" condited by tortial vibrations. *Bulletin of the university of Karaganda-Physics,* 2(98), 119–126.

11 Hevko, I., Lyashuk, O., Sokil, M., Slobodian, L., Hud, V., & Vovk Y. (2019). Resonant oscillation of vertical working part of conveyer-loader. *Bulletin of the university of Karaganda-Physics*, 2(94), 73–82.

12 Mitropolsky, Yu.A. (1967). Averaging method in non-linear mechanics. *International Journal of Non-Linear Mechanics*, 2(1), 69–96.

13 Jan, A. (2007). Sanders Ferdinand Verhulst James Murdock. *Averaging Methods in Nonlinear Dynamical Systems*: Edition 2.

14 Grebenikov, E.A. (1965). Methods of averaging equations in celestial mechanics. *American Institute of Phisics*, 9(1), 190–194.

15 Mzaki, Dakel (2017). Jean-Jacques Sinou Stability and nonlinear self-excited friction-induced vibrations for a minimal model subjected to multiple coalescence patterns Jve international ltd. *Journal of vibroengineering*, 19, 604–628.

16 Bondarenko, L.N., Demin, G.K., & Buratinsky, A.P (2016). *Moment treniia v podshipnikakh skolzheniia [The moment of friction in sliding bearings]*. *Journal of the Dnieper State Academy of Architecture and Architecture,* 9(222), 36–41 [in Russian].

17 Janke, E., Emde, F., & Lesh, F. (1968). *Spetsialnye funktsii [Special functions]*. Moscow: Nauka [in Russian].

18 Tikhonov, A.N. (1952). Sistema differentsialnykh uravnenii, soderzhashchikh malye parametry pri proizvodnykh [System of differential equations containing small parameters in the derivatives]. *Matematicheskii sbornik — Mathematical Collection*, 31(73), 575–586 [in Russian].