DOI 10.31489/2022PH3/25-35

UDC 517.958:539.3(2)

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On the issue of a new solution of the materials resistance contact problem on compression of elastic cylinders in contact with parallel generators

As a result of logarithmic singularity in the plane classical problem of elastically deformable material mechanics, it is proved that the reference formula for determining the convergence of two statically compressed parallel cylinders made of a homogeneous, isotropic, and physically linear material is not applicable. In the special case of elastic interaction of a cylinder with a half-plane, it is established that the convergence becomes equal to infinity. This paradoxical result confirms the inadequacy of Flaman's model of a simple radial stress state in determining displacements. Based on this model, it is possible to determine only the stresses in parallel contacting cylinders, while the calculation of displacements, in this case, is not possible. Based on a previously developed and mathematically approximated by the authors flat design scheme of Flaman the algorithm exception of conflicts has been proposed. The algorithm is based on the integral Fredholm equation solution and can be seen as a new fundamental and applied elasticity theory problem, which is of great importance when assessing the contact of refined strength and stiffness of the cylindrical parts of the supporting structures subject to the general and local deformations (cylindrical rollers, gears, pavements with their seal steel rollers, etc.).

Keywords: displacement, convergence, cylinder, stress, force, load, compression, contact pressure, half-plane, uniformity, isotropy, elasticity.

In various branches of modern mechanical engineering and construction, supporting structural elements in the form of cylindrical caps that interact in contact over a surface of finite dimensions are widely used. Typical examples of such parts are plain and rolling bearings; supporting parts of bridges, overpasses and sluice gates; wheels of railway rolling stock, etc. [1–5].

The well-known structurally nonlinear [6–9] theory of small elastic contact deformations of two physically linear, isotropic, and homogeneous circular cylinders is based on the following assumptions (Figure 1) [1–3, 10, 11]:

1) the radii R_1, R_2 of cylindrical bodies are large in comparison with the size $2a$ of the contact area, i.e.,

$$R_1 \gg 2a\,, \quad R_2 \gg 2a, \tag{1}$$

where a is half the width of the pressure band;

2) the cylinders have strictly parallel longitudinal axes O_1, O_2 and length $l \gg 2a$ and their initial contact occurs along a straight line which is called the generatrix when the distance between arbitrary points A_1, A_2 before deformation is expressed by the formulas given in Figure 2 [1, 3, 10–12];
3) within the limits of assumption (1), the contact area can be considered as part of the plane tangent to the guide (circle) of non-deformable cylinders;

4) there is no friction between touching surfaces that are supposed to be absolutely smooth.
5) the contacting elements are pressed against each other by two equal in magnitude and oppositely directed external forces-resultant Q, distributed over a given length l of the cylinders in the form of a constant static load

$$P = \frac{Q}{l} = const \tag{2}$$

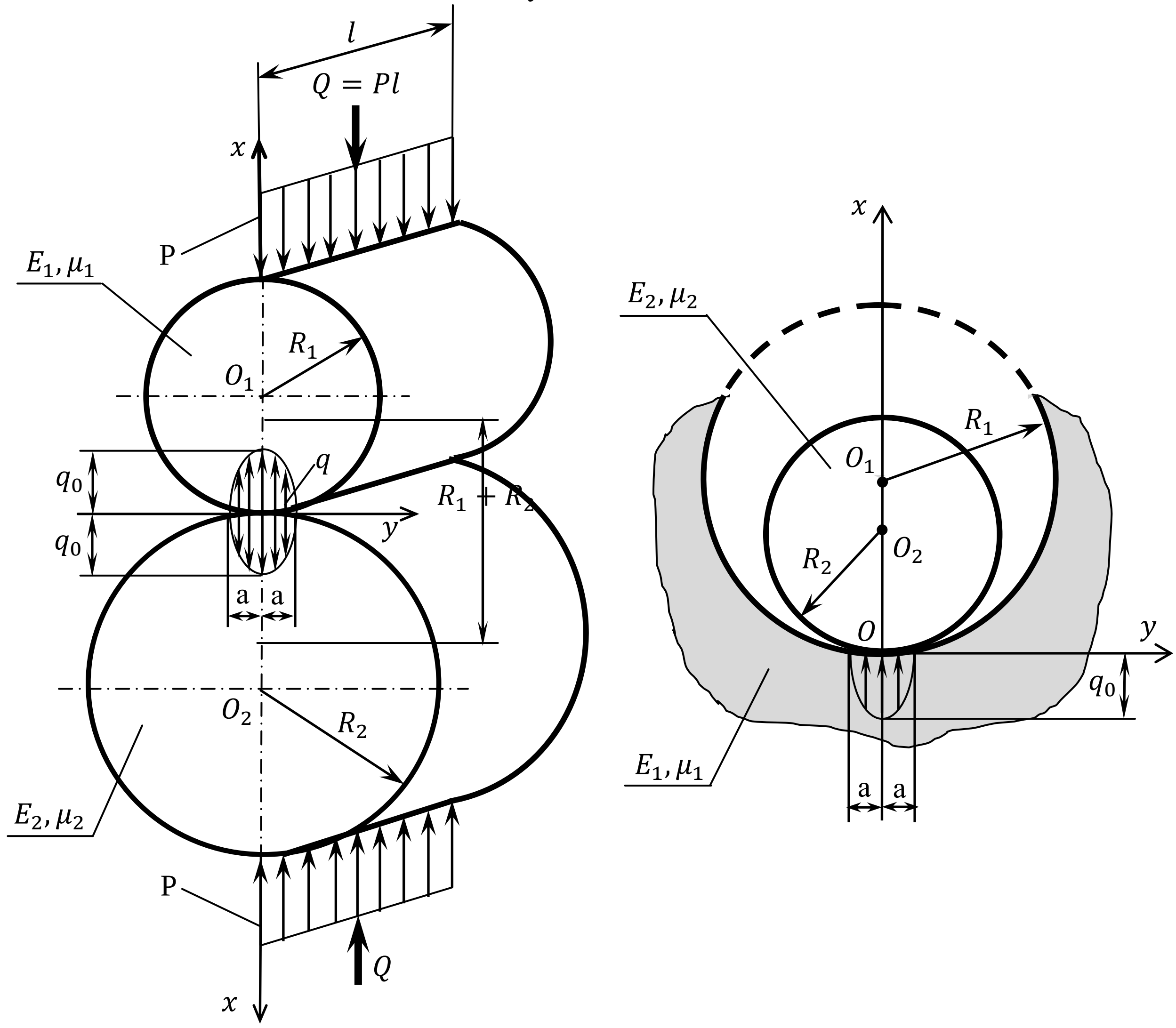


Figure 1. Design schemes of contacting elstic cylinders with radii R_1 and R_2:
a) the basic model of length l, where: $R_1 \lessgtr R_2$;
b) For roller cylindrical rolling bearings at $R_1 \gg R_2$ [1, 18]

if the equilibrium condition is met [11, 13–21]

$$l \cdot \int_{-a}^{a} q(y)dy = P \cdot l = Q, \tag{3}$$

where $q = q(y)$ is the reactive boundary voltage approximated by the Hertz elliptic function [11, 12, 14–17, 20–24]

$$q(y) = q_o \sqrt{1 - \frac{y^2}{a^2}}, \qquad -a \ll y \ll a, \tag{4}$$

having an extreme value [1, 11–16, 20–22, 23, 25]

$$q_o = \frac{2}{\pi} \cdot \frac{Q}{a \cdot l} = max. \qquad (5)$$

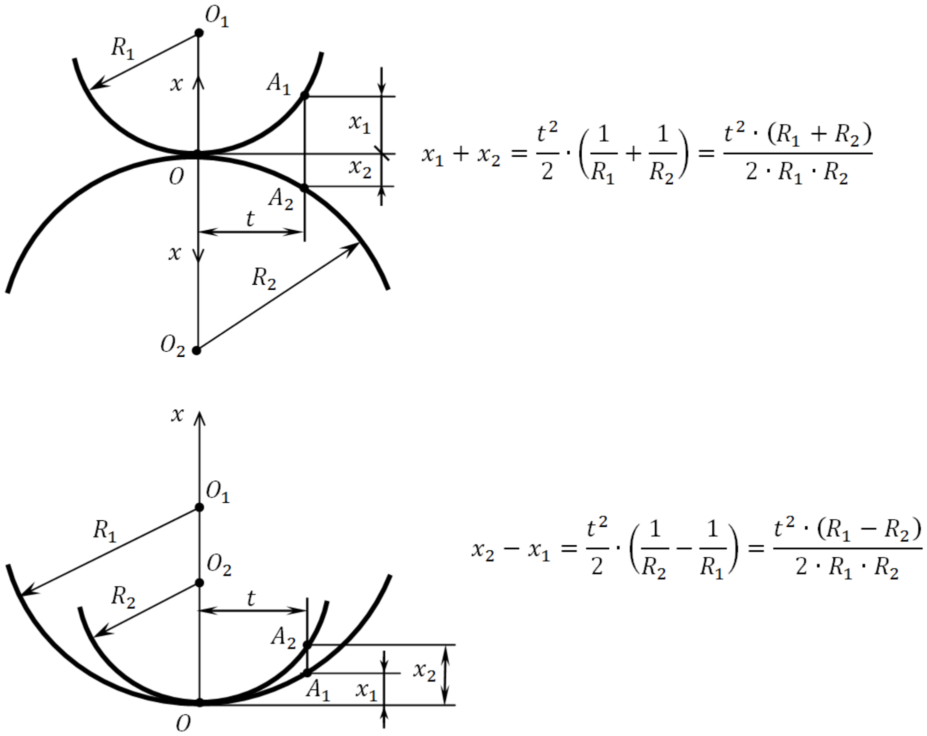


Figure 2. Geometry of the contact of two elasticall desormable bodies-cylinders:
a) to modeling the external interaction according to Fig. 1a ($R_1 \lessgtr R_2$);
b) for contacting a cylinder of radius R_2 with a cylindrical cavity having a radial size $R_1 \gg R_2$
Figure 1b

If the cylinders are made of materials that have elastic modulus E_1, E_2, Poisson's ratios μ_1, μ_2, then the formulas for a, q_o and the total kinematic displacement δ (convergence of the axes O_1, O_2) have the following form [1, 2, 11, 25]:

$$a = 2\sqrt{\frac{Q}{\pi \cdot l} \cdot \frac{R_1 \cdot R_2}{(R_1 \pm R_2)} \cdot \left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)}, \qquad (6)$$

$$q_o = \sqrt{\frac{Q}{\pi \cdot l} \cdot \frac{(R_1 \pm R_2)}{R_1 \cdot R_2} \cdot \left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)^{-1}}, \qquad (7)$$

$$\delta = \frac{2 \cdot Q}{\pi \cdot l} \cdot \left[\frac{1-\mu_1^2}{E_1} \cdot \left(ln\frac{2 \cdot R_1}{a} + 0{,}407\right) + \frac{1-\mu_2^2}{E_2} \cdot \left(ln\frac{2 \cdot R_2}{a} + 0{,}407\right)\right]; \qquad (8)$$

where the "+" sign refers to the main calculation scheme shown in Figures 1, 2, and "-" corresponds to the model of Figure 1b.

Relations (6)–(8) are used to quantify the load-bearing capacity of cylindrical systems in Figure 1. From a practical point of view, these are design calculations for the contact strength and stiffness of friction and gear gears, roller parts of bridge supports, and other critical elements of engineering structures.

At the same time, it should be noted that the definition of the displacement δ at Eq.(8) has significant mechanical-mathematical incorrectness [2, 10], as in the classic problem of Flaman [1, 10, 12–17, 22, 27, 28] on the effects of concentrated and distributed force $P = const$ (Figure 1) on an elastic isotropic half-plane lying in the basis of the Eq. (8). In Eq. (8) the displacement is calculated relative to a fairly remote from the contact point. Its position is unknown arbitrary. In Eq. (8) as of such points taken the centers of curvature of O_1 and O_2 (Figures 1, 2) in the hypothetical assumption that the parameter δ is determined only by the total deformations of cylinders [2], excluding contact components, which according to [29], can represent a significant percentage (30 to 90%) in the overall balance of the elastic displacements of the contacting parts.

The indicated uncertainty (multivariance) in choosing the coordinate of a fixed point when determining displacements directed perpendicular to the boundary of the half-plane is a consequence of the general logarithmic feature of Flaman's physical and mathematical model [10, 12, 20, 22]. In this relation, Galin [10] states that, based on the Flaman solution, it is possible to determine only the stresses in parallel-located contacting cylinders, and the calculation of displacements, in this case, is not possible. Thus, it can be stated that the estimation of contact stiffness by formula (8) will not adequately characterize the deformed state of the cylinders. Another negative consequence of the presence of the logarithm ln in Eq. (8) is manifested in its special case when one of the radii, for example

$$R_2 = \infty. \tag{9}$$

In this case, we will have a common engineering problem in design calculations about the contact of a compressible cylinder with an elastically deformable half-plane (Figure 3) [1–5].

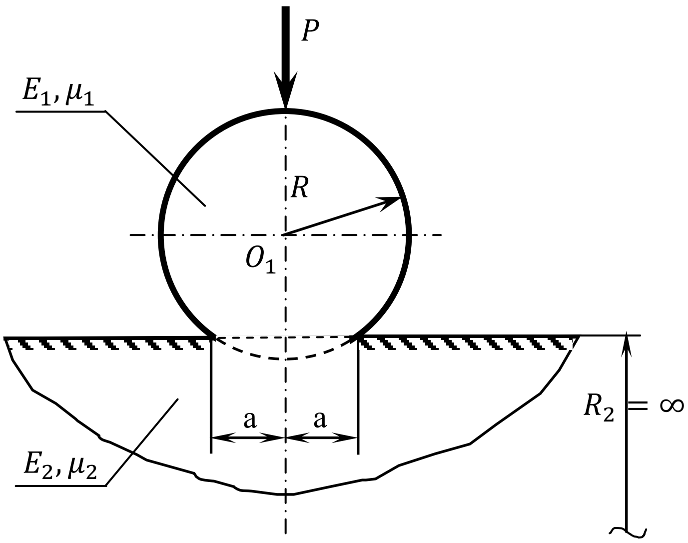


Figure 3. Model of pressure of a cylinder of radius R and length 1 on a linearly elastic half-plane

Considering the condition (9) and the equality $R_1 = R$, we obtain the final value for linear size a under Eq. (6):

$$a = 2\sqrt{\frac{Q \cdot R}{\pi \cdot l} \cdot \left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)}. \tag{10}$$

Substituting Eq. (10) in Eq. (8) for $R_2 = \infty$, according to Eq. (9), leads to the paradoxical answer

$$\delta = \infty, \tag{11}$$

which contradicts the physical meaning of the contact problem under consideration and confirms the unacceptability of the Flaman mathematical model in determining the displacements δ [10, 21].

The incorrect result (11) also does not correspond to the basic fundamental axisymmetric problem of Boussinesca [18, 20–22, 23, 24, 28] on a concentrated force directed perpendicular to an elastic half-space, in which there are no contradictions mentioned above.

Based on the classical interpretation of plane linear-elastic deformation [2, 11–15, 19, 21–23] and the refined innovative solution of the Flaman problem [30], which includes three stresses (compared to one radial component [12, 13]) and parameter a, in the work [30] a fundamentally new formula for calculation of displacement

$$v_g = v_g(y) = \frac{2 \cdot P \cdot (1-\mu^2)}{3 \cdot \pi \cdot E} \cdot \left(\frac{a}{y}\right)^2 \quad (12)$$

of a boundary of the half-plane in the unlimited range

$$-\infty \leq y \leq \infty. \quad (13)$$

of variable change has been obtained.

In contrast to the incorrect logarithmic dependence [11–14, 21, 22, 24]

$$v_{gf} = v_{gf}(y) = \frac{2 \cdot P \cdot (1-\mu^2)}{\pi \cdot E} \cdot ln\frac{l_k}{|y|}\ , \quad (14)$$

containing the distance to an arbitrary point K (Figure 3) and approximating only the relative value of displacements v_{gf} over a closed interval

$$-l_k \leq y \leq l_k, \quad (15)$$

in the formula (12) derived in [30] allows us to determine the absolute precipitation of the boundary $x = 0$ of the half-plane without reference to the parameter l_k over a theoretically infinite interval (13).

The behavior of functions (12) and (14) is illustrated in dimensionless forms

$$v_g^*(y) = v_g(y) \cdot \frac{\pi \cdot E}{P \cdot (1-\mu^2)} = \frac{2}{3} \cdot \left(\frac{a}{y}\right)^2, \quad (16)$$

$$v_{gf}^*(y) = v_{gf}(y) \cdot \frac{\pi \cdot E}{P \cdot (1-\mu^2)} = 2 \cdot ln\frac{l_k}{|y|} \quad (17)$$

in Figure 4, using data of Table.

Table

Values of functional relations (16), (17), when $l_k = 6a$.

y	0	$\pm a$	$\pm 2a$	$\pm 4a$	$\pm 6a$	$\pm 8a$	$\pm 10a$	$\pm\infty$
v_g^*	∞	0,6667	0,1667	0,0416	0,0186	0,0104	0,0066	0
v_{gf}^*	∞	3,5836	2,1972	0,8110	0	−0,5751	−1,0216	$-\infty$

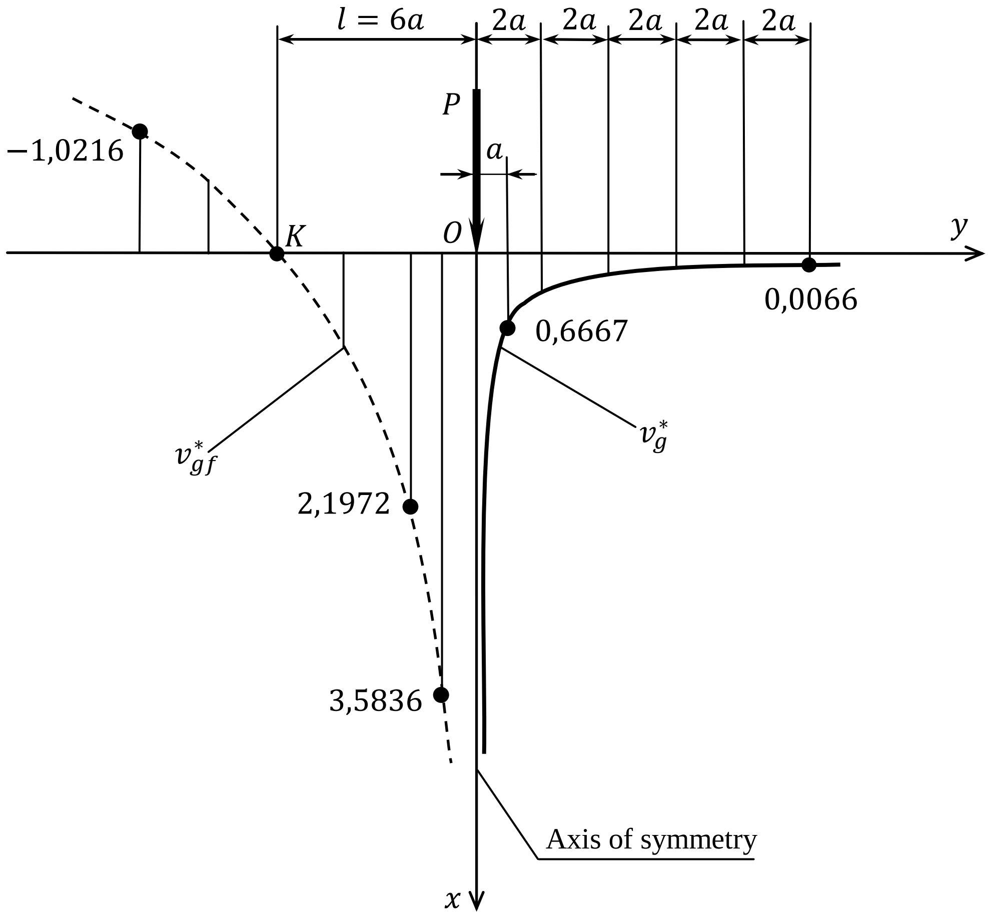


Figure 4. General view of changes in functional dependencies v^*_g, v^*_{gf}:
$v^*_g(y)$ is a solid curve according to the new formula (16) [30];
$v^*_{gf}(y)$ is a dashed line according to Flaman's solution (17) [12–14, 21, 22, 24]

A mechanical system in which a local uniformly distributed stationary force P acts on an elastic isotropic medium (Figures 1, 3) should be considered as an abstract system that does not reflect the actual possible conditions. However, using the formally idealized mathematical solution (12), we proceed to a real simulation of the reactive load $q = q(y)$ that occurs between contacting elastically deformable bodies Figure 1). Therefore, by analogy with the developed theory of calculating the sediment of a belt foundation [30] and guided by [11, 15–17, 21–24, 31–33], to answer this question, we present the following Fredholm equation of the first kind [13, 16, 20-22, 31, 32] with an unknown function $q(y)$ under the sign of a certain integral:

$$-\frac{2 \cdot a^2}{3 \cdot \pi} \cdot \left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right) \cdot \int_{-a}^{a} \frac{q(y) \cdot dy}{(t-y)^2} = \delta - \frac{1}{2} \cdot \left(\frac{1}{R_2} \pm \frac{1}{R_1}\right) \cdot t^2 \,, \tag{18}$$

where $q = q(y), q_o, a, \delta$ are the desired physical and geometric characteristics of the interaction pro cess of two round cylindrical elements, while meeting the obvious requirements

$$-a \le y \le a, \qquad q(\pm a) = 0, \quad q(0) = q_o = max \tag{19}$$

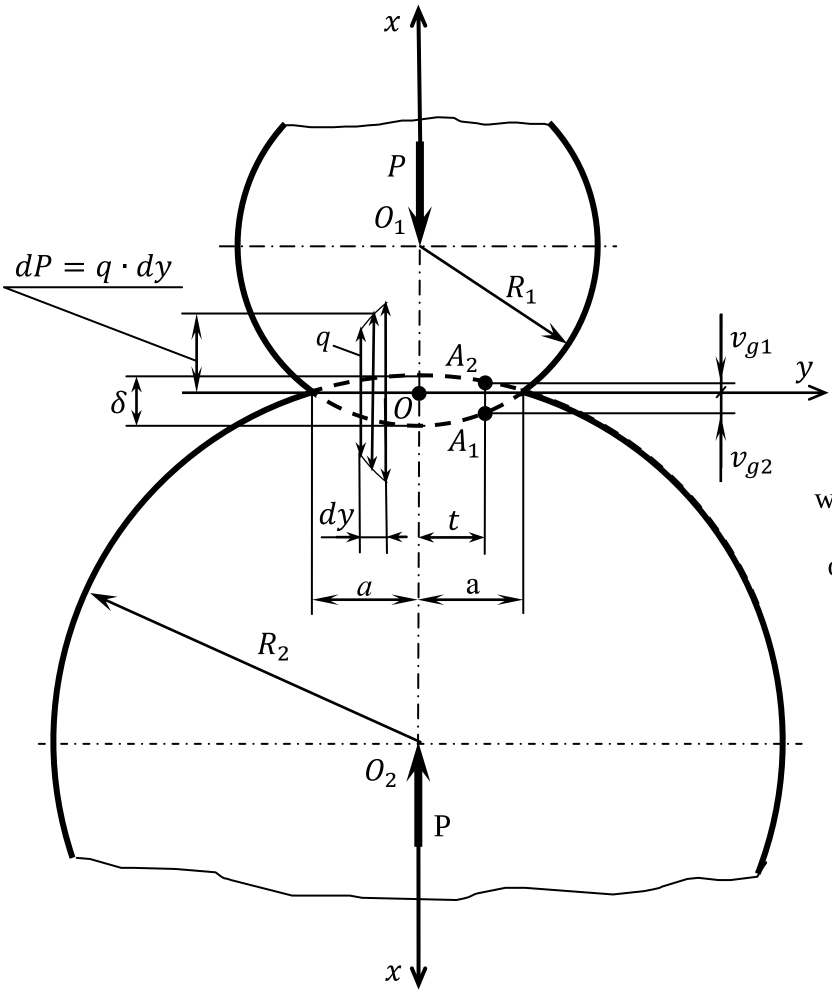
and condition (3) is met;

t is an auxiliary variable that varies within $-a \le t \le a$ and represents the horizontal coordinate of arbitrary points A_1, A_2, whose mutual vertical displacement depends on the elementary load (Figures 2, 5).

$$dP = q(y) \cdot dy \tag{20}$$

equal to (see (12) and (18))

$$dv_g = dv_{g1} + dv_{g2} = -\frac{2\cdot a^2}{3\cdot\pi}\cdot\left(\frac{1-\mu_1^2}{E_1}+\frac{1-\mu_2^2}{E_2}\right)\cdot\frac{q(y)\cdot dy}{(t-y)^2}. \quad (21)$$



$$v_g = v_{g1} + v_{g2} = \delta - \frac{1}{2}\left(\frac{1}{R_2} \pm \frac{1}{R_1}\right)t^2,$$

where is the sign "-" for the model of cylinder pressure on a concave cylindrical surface (Figures 1b, 2b)

Fig*ure*.5. Basic scheme of contact interaction of cylinders (Fig*ure* 1a) to the interpretation of the integral equation (18)

As noted by the authors of [30], in comparison with the absolutely idealized original (12), in which the directions of force P and displacement $v_g > 0$ coincide (Figure 4), in formulas (18), (21) the "minus" sign indicates the opposite effect of the contact pressure $q(y)$ and the kinematic components v_{g1}, v_{g2} within each cylinder (Figures 1, 5).

Analysis of the conducted theoretical studies allows us to draw the following CONCLUSIONS:

1. The impossibility [10, 11, 21] of using the reference dependency (8) [1, 2, 25] for calculation of the convergence δ of cylinders (Figures 1, 3) due to the logarithmic feature noted in well-known fundamental mechanical and mathematical works [10, 14, 21] is confirmed and proved.

2. For the same reason (see point 1), the use of the basic Flaman model [11–16, 21–25, 28] for solving any plane problems of the elasticity theory is proved incorrect (Figure 4), related to the evaluation of contact stiffness [29] of cylindrical parts of load-bearing structures that are in parallel contact.

3. To completely eliminate the considered contradictions, a linear integral equation (18), which makes it possible to determine the reaction pressure function $q(y)$ and its extremum $q_0 = max$; the half-width a of the contact site and the total mutual displacement δ of points O_1, O_2 (Figures 1, 5) is formulated.

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Параллель жасаушылармен жанасатын серпімді цилиндрлердің сығылуы туралы матералдардың кедергісі байланыс есебінің жаңа шешімі сұрағына

Серпімді деформацияланатын қатты дененің механиканың классикалық есебінің жазықтағы белгілі логарифмдік ерекшелігіне байланысты біртекті, изотропты және физикалық сызықты материалдан жасалған статикалық сығылған екі параллель цилиндрдің жақындауын анықтауға арналған анықтамалық формуланың қолданылмайтындығы осы жұмыста дәлелденді. Цилиндрдің жартылай жазықтық пен серпімді әсерлесуінің ерекше жағдайында, радиустардың біреуінің ұзындығы шексіз болған жағдайында, жақындаудың шексіздікке тең болатыныгы анықталған. Жақындаудың шексіздікке тең болған парадоксалды нәтижесі мақалада зерттеліп отырған процестің физикалық және механикалық мағынасына қарсы келеді, сонымен қатар ығысуларды анықтауда қарапайым радиалды кернеу күйінің Фламан моделінің сәйкессіздігін растайды. Бұл модельге негізделіп, тек параллель жанасатын цилиндрлердегі кернеулерді анықтауға болады, бұл жағдайда орын ауыстыруларды есептеу мүмкін емес. Осы мақаланың авторлары бұрын әзірлеген және математикалық жуықтайтын, үш кернеу құрамдас бөлігі мен цилиндрдің жанасу аймағының енін ескеретін жалпақ Фламан есептеу схемасы негізінде, шешуге негізделген қайшылықтарды жою алгоритмін ұсынған, яғни бірінші текті Фредгольм интегралдық теңдеуі, әрі қарай серпімділік теориясының жаңа іргелі–қолданбалы мәселесі ретінде қарастырылуы мүмкін, жалпы және жергілікті деформацияларды ескере отырып, ол жүк көтергіш құрылымдардың цилиндрлік бөліктерінің жанасу беріктігін және қаттылығын нақты бағалауда үлкен маңызға ие (цилиндрлік роликтер, тісті доңғалақтар, болат біліктермен тығыздалған кездегі жол төсемдері және т.б.).

Кілт сөздер: жылжыту, жақындау, цилиндр, кернеу, күш, жүктеме, сығылу, жанасу қысымы, жартылай жазықтық, біртектілік, изотропия, серпімділік.

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К вопросу нового решения контактной задачи сопротивления материалов о сжатии упругих цилиндров, соприкасающихся с параллельными образующими

Вследствие известной логарифмической особенности в плоской классической задаче механики упругодеформируемого твердого тела доказана неприменимость справочной формулы для определения сближения двух статически сжатых параллельных цилиндров из однородного, изотропного и физически линейного материала. В частном случае упругого взаимодействия цилиндра с полуплоскостью, когда один из радиусов имеет бесконечную длину, установлено, что и сближение становится равным бесконечности. Этот парадоксальный результат противоречит не только физико-механическому смыслу исследуемого процесса, но и подтверждает неадекватность модели Фламана о простом радиальном напряженном состоянии при определении перемещений. Основываясь на этой модели, можно определять только напряжения в параллельно расположенных контактирующих цилиндрах, в то время как расчет перемещений в этом случае не представляется возможным. На основе ранее разработанной и математически аппроксимированной авторами данной статьи плоской расчетной схемы Фламана, учитывающей три компоненты напряжений и размер ширины площадки контакта цилиндров, предложен алгоритм исключения противоречий, базирующийся на решении интегрального уравнения Фредгольма первого рода, что может быть рассмотрено в дальнейшем как новая фундаментально-прикладная задача теории упругости, имеющая большое значение при уточненной оценке контактной прочности и жесткости цилиндрических деталей несущих конструкций с учетом общих и местных деформаций (цилиндрических катков, зубчатых передач, дорожных покрытий при их уплотнении стальными вальцами и т.д.).

Ключевые слова: перемещение, сближение, цилиндр, напряжение, сила, нагрузка, сжатие, контактное давление, полуплоскость, однородность, изотропность, упругость.

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