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Non-standard analysis in electrical engineering. Transient analysis in second-order electrical circuits with violation of switching laws

For the first time, the authors proposed the use of the mathematical apparatus of non-standard analysis to solve certain non-trivial problems of theoretical electrical engineering. It was established that the axiomatics of non-standard analysis allows to simplify the analysis of transient processes in high-order electric circuits when the switching laws are violated. It is shown that the application of non-standard analysis methods in theoretical electrical engineering provides an opportunity to use the traditional classical method of transient processes analysis of circuits with violation of switching laws. Only by using methods of non-standard analysis, it is possible to strictly prove the fulfillment of the law of energy conservation in such schemes. Also, examples of solving such tasks are given. It is recommended to expand the scope of application of non-standard methods of analysis in problems of various branches of science and technology, which use differential calculus and boundary transitions, and the solution of which is limited or impossible by standard approaches.

Keywords: infinitesimal number, infinitely large number, hyperreal number, non-standard number, standard number, second-order transition process.

Introduction

In solving various scientific and technical problems, such as calculating derivatives and determining the coefficients of infinite series in mathematical analysis, DC circuit analysis with ideal reactive elements and determining the coefficients of ideal reactive four-pole circuits in theoretical electrical engineering, the researcher occasionally meets uncertainty like $\frac{0}{0}$. At the same time, there is a number of problems when using classical methods that quite often lead to certain complications. In the case of classical mathematical analysis, derivative calculations sometimes is time-consuming, but generally accepted and does not cause any problems, then in problems of theoretical electrical engineering, the situation is more complicated. So, for example, to analyze the steady modes of DC circuits with ideal inductances and capacitances by classical method, it is necessary to calculate the transient process and obtain the established value based on these results. If the electrical circuit is relatively simple, then this task does not cause complications. But if a circle has a significant number of reactive elements, the characteristic equation of the transition process has a large order, then the solution can be obtained only by numerical methods. Therefore, for such tasks it is proposed to use ideas and methods of non-standard analysis. It is interesting that the ideas of non-standard analysis (viz. the direct use of infinitesimal numbers) were the foundation on which Leibniz and Newton intuitively built the principles of differential and integral calculations. But later, in the works of Cauchy and other mathematicians, infinitesimal numbers were "withdrawn from circulation" and the mathematical apparatus of differential and integral calculus was based on numerical and functional sequences and limit ratios [1-5]. This increased the axiomatic rigor of the mathematical apparatus, but unfortunately made it difficult to solve a certain range of problems.

The revival of the ideas of non-standard analysis took place in the 60s of the last century, when A. Robinson proposed a new axiomatics of mathematical analysis, which is based on a set of hyperreal numbers, containing in addition to the so-called standard (ordinary real), non-standard numbers (infinitely large and their combinations with ordinary real) numbers [6-8].

In previous works, the problems of DC circuits analysis with ideal reactive elements in steady state [9-10], as well as the transient analysis with violation of the switching laws in first-order circuits [11]. Methods of nonstandard analysis are being developed at the present time and are used in different fields of science

[12-15]. Of interest is the use of nonstandard analysis methods in the problems of identifying the internal parameters of electrical motors, which in many cases cannot be solved by traditional methods [16-20].

Typically, the energy characteristics of inductors and capacitors are used in conjunction with the laws of electrical engineering to solve such problems, which greatly complicates the analysis of such circuits, especially for complex circuits. Therefore, it is important to use the mathematical apparatus of non-standard analysis, which will use known unified methods to calculate such circuits.

The purpose of this work is to analyze the transients with the violation of the switching laws in the second order circuits. The following section summarizes the basic principles of non-standard analysis that are needed to solve the above electrical problems.

Basic principles of non-standard analysis

Let R be an ordered set of real numbers. The number α will be called infinitesimal number if and only if

$$\forall r \in R(\alpha < r), \tag{1}$$

where r is an arbitrary real number.

We will call the number $\beta = \frac{1}{\alpha}$ an infinitely large number. In this case, it is possible to write

$$\forall r \in R(\beta > r). \tag{2}$$

All algebraic operations (addition, subtraction, multiplication, division, squaring, etc.) and theorems (commutativity, associativity, etc.) can be applied to infinitesimal and large numbers.

We will distinguish between infinitesimal and large numbers of different orders, such as:

- $\alpha > \alpha^2 > \alpha^3 > \alpha^k$- infinitesimal numbers of the first, second, third, k-th order;
- $\beta < \beta^2 < \beta^3 < \beta^k$- infinitely large numbers of the first, second, third, k-th order.

Together with real numbers $r \in R$, infinitesimal and large numbers form an ordered set of hyperreal numbers $*R$. It is customary to call real numbers $r \in R$ standard or Archimedean in contrast to non-standard (non-Archimedean) numbers $*r \in *R$.

Each non-standard number contains a standard part

$$*r = r \pm \alpha, \tag{3}$$

i.e.

$$r = st(*r), \tag{4}$$

in other words, an ordinary real number is a standard part of some non-standard number (obviously, such numbers quantity can be infinite).

Two standard numbers a and b are called equal if and only if

$$a - b = 0. \tag{5}$$

Two non-standard numbers $*a$ and $*b$ are called equivalent (or infinitely close to each other) if and only if

$$*a - *b \approx \alpha. \tag{6}$$

The notation $\approx$ will mean the equivalence of two non-standard numbers.

For standard numbers m and n write some relations that follow from (1 — 6):

$$\frac{1}{\alpha^k} = \beta^k, \frac{m}{\alpha} = m\beta, \frac{m}{\alpha^k} = m\beta^k \tag{7}$$

$$\frac{m\alpha}{n\alpha} = \frac{m}{n}, \frac{m\alpha}{n} = \frac{m}{n}\alpha, \frac{m}{n\alpha} = \frac{m}{n}\beta \tag{8}$$

$$m\alpha + n \approx n, m\beta + n \approx m\beta, m\alpha^k + n \approx n, m\beta^k + n \approx m\beta^k \tag{9}$$

$$\sin\alpha \approx \alpha, \cos\alpha \approx 1, tg\alpha \approx \alpha, ctg\alpha \approx \beta \tag{10}$$

Here are some examples of usage. These methods in mathematical analysis.

Find, for example, the first derivative of the function $y = x^5$, for which we introduce a substitution $dx = \alpha$.

$$\frac{dy}{dx} = \frac{(x+\alpha)^5 - x^5}{\alpha} = \frac{x^5 + 5x^4\alpha + 10x^3\alpha^2 + 10x^2\alpha^3 + 5x\alpha^4 + \alpha^5 - x^5}{\alpha} =$$
$$= 5x^4 + 10x^3\alpha + 10x^2\alpha^2 + 5x\alpha^3 + \alpha^4 \approx 5x^4.$$

The derivative of the function $y = \frac{1}{x^2}$ can be found as

$$\frac{dy}{dx}=\frac{\frac{1}{(x+\alpha)^2}-\frac{1}{x^2}}{\alpha}=\frac{\frac{1}{x^2+2x\alpha+\alpha^2}-\frac{1}{x^2}}{\alpha}=\frac{\frac{x^2-x^2-2x\alpha-\alpha^2}{x^4+2x^3\alpha+x^2\alpha^2}}{\alpha}$$
$$=\frac{-2x\alpha-\alpha^2}{x^4\alpha+2x^3\alpha^2+x^2\alpha^3}=\frac{-2x-\alpha}{x^4+2x^3\alpha+x^2\alpha^2}\approx\frac{-2x}{x^4}=\frac{-2}{x^3}.$$

For $y=sin\,x$ get

$$\frac{dy}{dx}=\frac{sin(x+\alpha)-sin\,x}{\alpha}=\frac{sin\,x\cdot cos\,\alpha+sin\,\alpha\cdot cos\,x-sin\,x}{\alpha}\approx\frac{sin\,x\cdot 1+\alpha\cdot cos\,x-sin\,x}{\alpha}\approx cos\,x.$$

Let $y=cos\,x$. Then

$$\frac{dy}{dx}=\frac{cos(x+\alpha)-cos\,x}{\alpha}=\frac{cos\,x\cdot cos\,\alpha-sin\,x\cdot sin\,\alpha-cos\,x}{\alpha}\approx\frac{cos\,x\cdot 1-sin\,x\cdot\alpha-cos\,x}{\alpha}\approx -sin\,x.$$

If $y=tgx$

$$\frac{dy}{dx}=\frac{\frac{sin(x+\alpha)}{cos(x+\alpha)}-\frac{sin\,x}{cos\,x}}{\alpha}=\frac{\frac{sin\,x\cdot cos\,\alpha+sin\,\alpha\cdot cos\,x}{cos\,x\cdot cos\,\alpha-sin\,x\cdot sin\,\alpha}-\frac{sin\,x}{cos\,x}}{\alpha}=$$
$$\frac{cos\,x\cdot sin\,x\cdot cos\,\alpha+sin\,\alpha\cdot cos^2 x-sin\,x\,cos\,x\cdot cos\,\alpha+sin^2 x\cdot sin\,\alpha}{\alpha\,cos^2 x\cdot cos\,\alpha-\alpha\,cos\,x\cdot sin\,x\cdot sin\,\alpha}\approx$$
$$\approx\frac{\alpha(cos^2 x+sin^2 x)}{\alpha\,cos^2 x-\alpha^2\,cos\,x\cdot sin\,x}\approx\frac{cos^2 x+sin^2 x}{cos^2 x-\alpha\,cos\,x\cdot sin\,x}\approx\frac{1}{cos^2 x}.$$

In the case of $y=ctgx$

$$\frac{dy}{dx}=\frac{\frac{cos(x+\alpha)\,sin(x+\alpha)}{sin(x+\alpha)}-\frac{cos\,x}{sin\,x}}{\alpha}=\frac{\frac{cos\,x\cdot cos\,\alpha-sin\,x\cdot sin\,\alpha}{sin\,x\cdot cos\,\alpha+sin\,\alpha\cdot cos\,x}-\frac{cos\,x}{sin\,x}}{\alpha}=$$
$$\frac{sin\,x\,cos\,x\cdot cos\,\alpha-sin^2 x\cdot sin\,\alpha-cos\,x\,sin\,x\cdot cos\,\alpha-sin\,\alpha\cdot cos^2 x}{\alpha\,sin^2 x\cdot cos\,\alpha+\alpha\,sin\,\alpha\cdot cos\,x\,sin\,x}\approx$$
$$\approx\frac{-\alpha(sin^2 x+cos^2 x)}{\alpha\,sin^2 x+\alpha^2\,cos\,x\cdot sin\,x}\approx\frac{-(sin^2 x+cos^2 x)}{sin^2 x+\alpha\,cos\,x\cdot sin\,x}\approx\frac{-1}{sin^2 x}.$$

In addition, the problems of classical transient analysis in electrical circuits require the direct use of standard numbers 0 and infinite quantities ∞, so we formulate their non-standard interpretation.

The standard number 0 in non-standard analysis can be considered as an infinitesimal number of infinitely large order, that is $0\approx\alpha^{\beta}$, therefore

$$\frac{0}{\alpha}\approx 0,\ 0\cdot\beta\approx 0,\ e^{-\beta\cdot 0}\approx 1,\ e^{-\alpha}\approx 1 \tag{11}$$

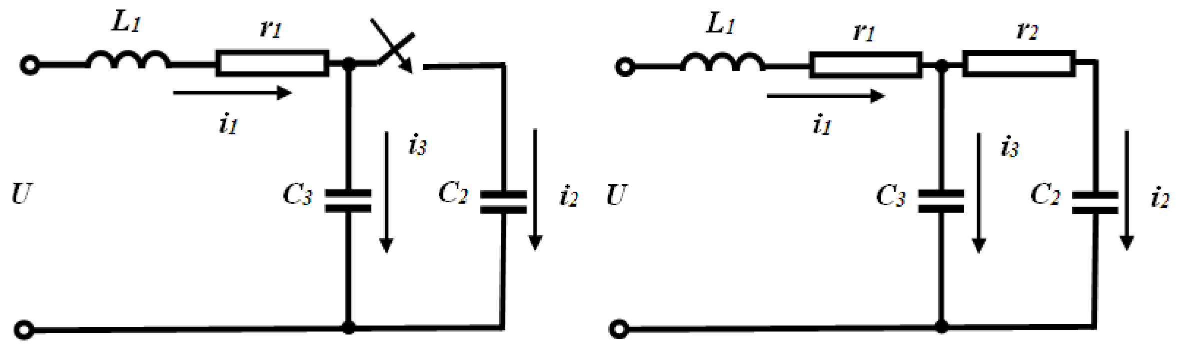
An infinite quantity ∞ in a non-standard analysis can be represented as an infinitely large number of infinitely large order, that is $\infty\approx\beta^{\beta}$, therefore

$$\frac{\infty}{\beta}\approx\infty,\ \infty\cdot\alpha\approx\infty,\ e^{-\infty\cdot\alpha}\approx\alpha,\ e^{-\beta}\approx\alpha \tag{12}$$

Before proceeding to the use of the above expressions to solve various applied problems, we will note that there are no general rules for choosing a parameter that should be equated to an infinitesimal (or infinitely large) number. This choice is made by the researcher depending on the context of a particular task. It should be borne in mind that in the case of the need to replace infinitesimal numbers of several dissimilar parameters of one problem, determining the relationship between these numbers is a difficult problem and sometimes requires additional research.

Transient analysis in the 2nd order circuits with violation of switching laws

Example. Determine the transient voltages at the capacitors and the current in the inductor in the circuit shown in Fig. 1, *a*.
Circuit parameters: $U=100$ V, $r_1=50$ Ohm, $L_1=100$ mH, $C_2=100$ uF, $C_3=150$ uF.



a) b)

Figure 1

To ensure the possibility of using the second law of switching, we assume that the branch with capacitance C_2 contains a resistor $r_2 = \alpha \approx 0$ (Fig. 1, *b*).
The initial conditions can be found as

$$u_{C_3}(0+) = U = 100 \text{ V}, u_{C_2}(0+) = 0 \text{ V}, i_1(0+) = 0 \text{ A}. \quad (13)$$

Forced components are defined as

$$u_{C_2np} = U = 100 \text{ V}, u_{C_3np} = U = 100 \text{ V}, i_{1np} = 0 \text{ A}. \quad (14)$$

By the method of input resistance

$$Z_{ex}(p) = r_1 + pL_1 + \frac{\left(r_2 + \frac{1}{pC_2}\right)\frac{1}{pC_3}}{r_2 + \frac{1}{pC_2} + \frac{1}{pC_3}} = r_1 + pL_1 + \frac{\left(\alpha + \frac{1}{pC_2}\right)\frac{1}{pC_3}}{\alpha + \frac{1}{pC_2} + \frac{1}{pC_3}} = , \quad (15)$$
$$= \frac{r_1\left(\alpha C_2C_3p^2 + (C_2+C_3)p\right) + pL_1\left(\alpha C_2C_3p^2 + (C_2+C_3)p\right) + \alpha C_2 p + 1}{\alpha C_2C_3p^2 + (C_2+C_3)p}$$

form a characteristic equation:

$$\alpha L_1C_2C_3p^3 + [\alpha r_1C_2C_3 + L_1(C_2 + C_3)]p^2 + [r_1(C_2 + C_3) + \alpha C_2]p + 1 = 0, \quad (16)$$

or in numerical form

$$15 \cdot 10^{-10}\alpha p^3 + [75 \cdot 10^{-8}\alpha + 25 \cdot 10^{-6}]p^2 + [0.0125 + 15 \cdot 10^{-5}\alpha]p + 1 = 0, \quad (17)$$

This cubic equation has three roots, the first two of which we determine by performing the transformation

$$15 \cdot 10^{-10}\alpha p^3 + [75 \cdot 10^{-8}\alpha + 25 \cdot 10^{-6}]p^2 + [0.0125 + 15 \cdot 10^{-5}\alpha]p + 1 \approx$$
$$\approx 25 \cdot 10^{-6}p^2 + 0.0125p + 1 = 0, \quad (18)$$

where $p_1 = -400$ s^{-1}, $p_2 = -100$ s^{-1}.
The third root is found using the theorem of factorization, according to which expression (17) can be represented as $15 \cdot 10^{-10}\alpha(p - p_1)(p - p_2)(p - p_3) = 0$.
It follows from this theorem that $(-15 \cdot 10^{-10}\alpha p_1p_2p_3) = 1$,
where $p_3 = -\frac{1}{15\cdot 10^{-10}\alpha p_1p_2} = -\frac{16667}{\alpha}$ s^{-1}.
Then

$$u_{C_2}(t) = U + A_1e^{p_1t} + A_2e^{p_2t} + A_3e^{p_3t} = 100 + A_1e^{-400t} + A_2e^{-100t} + A_3e^{-\frac{16667}{\alpha}t}, \quad (19)$$

and

$$u_{C_3}(t) = u_{C_2}(t) + \alpha C_2\frac{du_{C_2}(t)}{dt} = 100 + A_1e^{-400t} + A_2e^{-100t} + A_3e^{-\frac{16667}{\alpha}t} +$$

$$+10^{-4}\left(-400A_1e^{-400t}-100A_2e^{-100t}-\frac{16667}{\alpha}A_3e^{-\frac{16667}{\alpha}t}\right)\alpha\approx \tag{20}$$
$$\approx 100+A_1e^{-400t}+A_2e^{-100t}-0.6667A_3e^{-\frac{16667}{\alpha}t}.$$

The current in the inductor is defined as

$$i_1(t)=C_2\frac{du_{C_2}(t)}{dt}+C_3\frac{du_{C_3}(t)}{dt}=10^{-4}\left(-400A_1e^{-400t}-100A_2e^{-100t}-\frac{16667}{\alpha}A_3e^{-\frac{16667}{\alpha}t}\right)+$$
$$+1.5\cdot10^{-4}\left(-400A_1e^{-400t}-100A_2e^{-100t}+\frac{11111}{\alpha}A_3e^{-\frac{16667}{\alpha}t}\right)= \tag{21}$$
$$=-0.1A_1e^{-400t}-0.025A_2e^{-100t}.$$

To determine the integration constants it is necessary to substitute the value of the initial moment of time $t=0_+\approx\alpha_1$ in expressions (19) and (20) instead of the variable *t* (the initial moment of time is denoted by a symbol α_1, because it differs from resistance in its physical nature $r_3=\alpha$). Uncertainty arises $e^{-16667\frac{\alpha_1}{\alpha}}$.

The ratio of infinitesimal numbers α and α_1 is impossible to establish purely mathematically, because they belong to heterogeneous parameters. Let's analyze them from a physical point of view. Recall that α_1 this is the starting point of time, and α this is the active conductivity of the circuit breaker, which we specifically introduce to fulfill the standard switching laws. Since these values are independent of each other, it is always possible to choose their value to ensure the condition $\alpha_1\approx\alpha^2$. This way it is possible to write

$$e^{-16667\frac{\alpha_1}{\alpha}}=e^{-16667\frac{\alpha^2}{\alpha}}=e^{-16667\alpha}\approx 1. \tag{22}$$

From expressions (19), (20) and (21), taking into account (22) we find the integration constants, for which we compose a system of equations:

$$\begin{aligned} u_{C_2}(0+)&=100+A_1+A_2+A_3=0,\\ u_{C_3}(0+)&=100+A_1+A_2-0.6667A_3=100,\\ i_1(0+)&=-0.1A_1-0.025A_2=0. \end{aligned} \tag{23}$$

Hence $A_1=13.333$, $A_2=-53.333$, $A_3=-60$

Thus, the transient voltages on the capacitors

$$u_{C_2}(t)=100+13.333e^{-400t}-53.333e^{-100t}-60e^{-\frac{16667}{\alpha}t}\approx$$
$$\approx 100+13.333e^{-400t}-53.333e^{-100t}\text{ V}, \tag{24}$$
$$u_{C_3}(t)=100+13.333e^{-400t}-53.333e^{-100t}+40e^{-\frac{16667}{\alpha}t}\approx$$
$$\approx 100+13.333e^{-400t}-53.333e^{-100t}\text{ V}, \tag{25}$$

and the transient current in inductance

$$i_1(t)=-1.333e^{-400t}+1.333e^{-100t}\text{ A}. \tag{26}$$

Let now consider what values the voltages on the capacitors and the current in the inductance at the moments $t=0$ and $t=0_+$ in the real circuit for which $r_2=0\approx\alpha^\beta$.

As already determined, before switching at $t<0$ (in particular at $t=0_-$) $u_{C_3}=100$ V, $u_{C_2}=0$ V, $i_1=0$ A.

At the time, $t=0_+\approx\alpha_1$ expressions (24) and (25), taking into account (12), will take the form

$$u_{C_2}(0_+)=100+13.333e^{-400\alpha_1}-53.333e^{-100\alpha_1}-60e^{-\frac{16667}{0}\alpha_1}\approx$$
$$\approx 100+13.333e^{-400\alpha_1}-53.333e^{-100\alpha_1}-60\alpha\approx 60\text{ V},$$

$$u_{C_3}(0_+) = 100 + 13.333e^{-400\alpha_1} - 53.333e^{-100\alpha_1} + 40e^{-\frac{16667}{0}\alpha_1} \approx$$
$$\approx 100 + 13.333e^{-400\alpha_1} - 53.333e^{-100\alpha_1} + 40\alpha \approx 60 \text{ V},$$

and current in inductance

$$i_1(t) = -1.333e^{-400\alpha_1} + 1.333e^{-100\alpha_1} \approx 0 \text{ A}.$$

At the moment of time the $t = 0 \approx {\alpha_1}^{\beta}$ expression $e^{-16667\frac{0}{0}}$ becomes uncertain, because time and resistance are heterogeneous parameters, so the exact value of voltages at this time cannot be determined. We can only know the intervals of their possible values, like that

$$0 \leq u_{C_2}(0) \leq 60,$$
$$60 \leq u_{C_3}(0) \leq 100.$$

Let now consider the energy ratios in a circle. Since before switching ($t < 0$) the current in the inductor and the voltage on the capacitor C_2 was zero, the energy was stored only in the capacitor C_3 and was equal to

$$W(0_-) = \frac{C_3 u_{C_3}^2(0_-)}{2} = \frac{150 \cdot 10^{-6} \cdot 100^2}{2} = 0.75 \text{ J}.$$

At the first moment of time after switching ($t = 0_+$) the energy is already stored in both capacitors and is equal to

$$W(0_+) = \frac{(C_3 + C_2) u_{C_3}^2(0_+)}{2} = \frac{250 \cdot 10^{-6} \cdot 60^2}{2} = 0.45 \text{ J}.$$

Thus, the energy deficit is $\Delta W = 0.75 - 0.45 = 0.3$ J.

In traditional electrical engineering textbooks, the presence of this deficiency is explained by the loss of energy when charging the capacitor, which is turned on, but does not provide any mathematical evidence. Let's try to prove this within the framework of non-standard analysis.

To do this, first determine the current in the capacitor C_2

$$i_2(t) = C_2 \frac{du_{C_2}(t)}{dt} = 10^{-4} \frac{d\left(100 + 13.333e^{-400t} - 53.333e^{-100t} - 60e^{-\frac{16667}{\alpha}t}\right)}{dt} =$$
$$= -0.533e^{-400t} + 0.533e^{-100t} + \frac{100}{\alpha} e^{-\frac{16667}{\alpha}t} \text{ A}$$

Then

$$\Delta W = \int_0^\infty i_2^2(t) r_2 dt = \int_0^\infty \left(-0.533e^{-400t} + 0.533e^{-100t} + \frac{100}{\alpha} e^{-\frac{16667}{\alpha}t}\right)^2 \alpha dt =$$
$$= \int_0^\infty \begin{pmatrix} 0.284\alpha e^{-800t} + 0.284\alpha e^{-200t} + \dfrac{10000}{\alpha} e^{-\frac{33333}{\alpha}t} - 0.568\alpha e^{-500t} - \\ -106.6e^{-\left(400 + \frac{16667}{\alpha}\right)t} + 106.6e^{-\left(100 + \frac{16667}{\alpha}\right)t} \end{pmatrix} dt =$$
$$= \left.\begin{pmatrix} -3.55 \cdot 10^{-4}\alpha e^{-800t} - 1.42 \cdot 10^{-3}\alpha e^{-200t} - 0.3e^{-\frac{33333}{\alpha}t} + 1.136 \cdot 10^{-3}\alpha e^{-500t} + \\ + \dfrac{106.6}{400 + \dfrac{16667}{\alpha}} e^{-\left(400 + \frac{16667}{\alpha}\right)t} - \dfrac{106.6}{100 + \dfrac{16667}{\alpha}} e^{-\left(100 + \frac{16667}{\alpha}\right)t} \end{pmatrix}\right|_0^\infty \approx$$
$$\approx -0.3e^{-\frac{33333}{\alpha}\infty} + 0.3e^{-\frac{33333}{\alpha}0} \approx 0.3e^{-33333 \cdot 0} \approx 0.3 \text{ J}.$$

The law of energy conservation is fulfilled.

Conclusions

1. The application of ideas and methods of non-standard analysis in the field of theoretical electrical engineering makes it possible to use the traditional classical method of transient analysis of circuits with violation of the switching laws.

2. Only using the methods of non-standard analysis, it is possible to strictly prove the implementation of the energy conservation law in such circuits.

3. In order to expand the scope of non-standard analysis methods, it is necessary to distinguish similar problems from various fields of science and technology, which use differential calculus and boundary transitions and which solution is limited or impossible by standard approaches.

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Электротехникадағы стандартты емес талдау. Ауысу заңдылықтарын бұза отырып екінші ретті электр тізбектеріндегі өтпелі процестерді талдау

Авторлар теориялық электротехниканың кейбір тривиальды емес мәселелерін шешу үшін талдаудың стандартты емес математикалық аппаратын пайдалануды алғаш ұсынып отыр. Стандартты емес талдау аксиоматикасы ауысу заңдылықтары бұзылған кезде жоғары ретті электр тізбектеріндегі өтпелі процестерді талдауды жеңілдетуге мүмкіндік беретіні анықталды. Теориялық электротехникада стандартты емес талдау әдістерін қолдану ауысу заңдылықтарын бұзатын тізбектердің өтпелі процестерін талдаудың дәстүрлі классикалық әдісін қолдануға мүмкіндік беретіні көрсетілген. Стандартты емес талдау әдістерін қолдану арқылы ғана мұндай схемаларда энергияның сақталу заңының орындалуын дәлелдеуге болады. Сонымен қатар, мұндай есептерді шешу мысалдары келтірілген. Дифференциалдық есептеулер мен шекаралық өтулерді қолданатын және стандартты тәсілдермен шешімі шектелетін немесе мүмкін болмайтын ғылым мен техниканың әртүрлі салаларындағы есептерде стандартты емес талдау әдістерін қолдану аясын кеңейту ұсынылады.

Кілт сөздер: шексіз аз сан, шексіз үлкен сан, гипернақты сан, стандартты емес сан, стандартты сан, екінші ретті өтпелі процесі.

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Нестандартный анализ в электротехнике. Анализ переходных процессов в электрических цепях второго порядка с нарушением законов переключения

Авторы впервые предложили использовать математический аппарат нестандартного анализа для решения некоторых нетривиальных задач теоретической электротехники. Установлено, что аксиоматика нестандартного анализа позволяет упростить анализ переходных процессов в электрических цепях высокого порядка при нарушении законов переключения. Показано, что применение нестандартных методов анализа в теоретической электротехнике дает возможность использовать традиционный классический метод анализа переходных процессов цепей с нарушением законов переключения. Только с помощью методов нестандартного анализа можно строго доказать выполнение закона сохранения энергии в таких схемах. Также приведены примеры решения таких задач. Рекомендовано расширить область применения нестандартных методов анализа в задачах различных отраслей науки и техники, использующих дифференциальное исчисление и граничные переходы, решение которых ограничено или невозможно стандартными подходами.

Ключевые слова: бесконечно малое число, бесконечно большое число, гипердействительное число, нестандартное число, стандартное число, переходный процесс второго порядка.

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