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Finite element modeling of heat propagation of a complete rod of constant cross-section

In this paper, the definition of the temperature distribution field for a rod made of heat-resistant alloy EI48 is introduced. The authors consider for the study a complete rod of circular cross-section of radius R, of limited length L. Studied body is under the influence of a heat flow q from the surface over the entire cross-sectional area of the left end, and heat exchange with the environment occurs on the cross-sectional area of the right end. The rod is thermally insulated along the side surface. The authors consider two cases: the first is the heat flow with intensity q can be set on the area of a small circle with radius r <R, the second is the heat flow can be set on its part, that is, on the area $\pi\left(\frac{R}{2}\right)^2$. During the study, the authors showed that during the thermomechanical process, the strength of each section of the load-bearing structural elements is significantly influenced by the temperature distribution field. The influence of high temperature on the morphology of heat-resistant alloys is also shown. This leads to the fact that in some parts of the structural elements the temperature will be acceptable, and in some — critical. As a result, rapid wear of structural elements and loss of their physical qualities occur. Therefore, mathematical modeling of temperature distribution field for a body of various configurations is an urgent problem. The article presents a method for constructing a mathematical model and a corresponding computational algorithm that allows solving a class of problems to determine the regularities of the temperature distribution field in the elements of rod-shaped structures. To do this, the authors used the energy-variation principle in combination with the finite element method.

Keywords: mathematical model, complete rod, heat flow, cross-section, functional, heat exchange, thermal insulation, temperature distribution field.

Introduction

The methodology for building a mathematical model and the problem of developing a heat propagation process in one-dimensional and multi-dimensional structural elements of complex configuration, made of heat-resistant alloys is important and relevant.

There are many works devoted to the problem of the effect of thermomechanical process on changing the structure and composition of the material of any technical unit or structure. From this we can distinguish the following authors: Segerlind L., Nozdrev V.F., Kudaykulov A.K., Pisarenko G.S., Birger I.A., Panovko Y.G., Khimushin F.F., Zenkevich O., Critch F., Federov Y.A., Bakulin V.N., Afanasyeva V.V., Oleynikov A.I., Jordar A., Yakobi A.I. and others. Analyzing the above-mentioned works we encounter some shortcomings. These works take into account influence on body temperature distribution of separate external factors: either heat insulation, or heat exchange with environment, or heat flow or temperature, etc. Here we developed a mathematical model of an insulated rod of constant cross-section under the influence of heat flow and heat exchange with the environment.

A mathematical model of the temperature distribution field of a rod of different configuration in the simultaneous presence of heat flow, thermal insulation and heat exchange using minimization of the total heat energy functional can be successfully applied to solve many scientific and applied problems. Basically, such problems are encountered in the intensive development of modern technological processes in the field of metal science. The obtained scientific results are confirmed by solving real test problems, which confirms the high degree of theoretical and practical importance of the topic.

The objects of the research are load-bearing structural elements in the form of a complete rod made of heat-resistant alloys.

Subject. A full rod of limited length, constant cross-section, completely insulated along the side surface, a heat flow is set on the small circle cross-sectional area of the left end, and heat exchange to the environment takes place through the cross-sectional area of the right end.

The aim and objectives of this paper are to investigate the temperature field distribution based on the application of the energy principle using finite elements.

The paper presents a method of heat transfer in one-dimensional bodies, with the problem of temperature distribution over the volume of bodies of various configurations made of heat-resistant alloys formalized and solved based on minimization of the total heat energy. Numerical solutions of test problems of steady-state thermal conductivity for one-dimensional structural elements are new approaches for establishing a pattern of temperature field distribution.

By changing some parameters of the structure of structural elements, such as the radius of the rod cross section, it will be possible to identify all vulnerable places in the structural elements and protect them from deformation or fracture. Such predictions and hypotheses greatly reduce the detection of critical temperatures throughout the body. Therefore, theoretical mathematical modeling of temperature distribution over the volume of bodies of different configuration can be implemented to solve problems of optimization of operation modes of main technological units, turbine units and internal combustion engines.

Research methodology

It is known from the general course of thermophysics that the established process of heat distribution in one-dimensional structural elements is described by the differential equation of the quasi-harmonic form of the parabolic type [1]:

$$\frac{\partial}{\partial x}\left(K_{xx}\frac{\partial T}{\partial x}\right)+Q=0, \tag{1}$$

where the following boundary conditions take place:

$h = 6\ \left[\frac{\text{Вт}}{(cm^2\ \ ^\circ\text{C})}\right]$, on S_1,

$\mathrm{T} = \mathrm{T_s}(2)$

$$h = 6\ \left[\frac{\text{Вт}}{(cm^2\ \ ^\circ\text{C})}\right], \text{ on } S_2. \tag{3}$$

Here is K_{xx} – is the heat transfer coefficient of the rod material, the dimension of which is $\left[\frac{W}{cm\cdot^\circ C}\right]$ $\left[\frac{\text{Вт}}{(\text{cm}\cdot\ ^\circ\text{C})}\right]$; Q — internal heat source, the dimension of which is $\left[\frac{W}{cm\cdot^\circ C}\right]\left[\frac{\text{Вт}}{(\text{cm}\cdot\ ^\circ\text{C})}\right]$; T_{at} – ambient surface temperature S_2, the dimensionality of which is $[^\circ C]$;T_s — surface temperature S_1, which is considered to be a given and the dimension of which$[^\circ C]$; ℓ_x – the guide cosines of the considered cross-sectional surface of the rod; *q*– a given heat flux on a certain surface of the rod, the dimensionality of which is $\left[\frac{W}{cm^2}\right]\left[\frac{\text{Вт}}{\text{cm}^2}\right]$. In addition, if the heat flux is brought to some surface of the rod, it is taken with a minus sign, and if it is removed from the rod, it is taken with a plus sign; *h*– is the value of the coefficient of heat exchange of the rod with its environment, the dimensionality of which is $\left[\frac{W}{cm^2\cdot^\circ C}\right]$.$\left[\frac{\text{Вт}}{\text{cm}^2}\right]$ Certain parts of the rod can be surrounded by water, soil, sand, ice, etc. In each case, the values of the heat transfer coefficient of the rod with its environment will be different.

Here it should be noted that the boundary condition (3) cannot simultaneously set *q* and *h*. If *q* is given on some surface of the rod, then on that surface the value of h will be zero, and vice versa, i.e. where h is given, then there the value of *q*=0.

It is known from the course of calculus of variations that the solution of equation (1), which satisfies boundary conditions (2) and (3) gives a minimum of the following functional:

$$I = \int\limits_V \frac{1}{2}\left[K_{xx}\left(\frac{\partial T}{\partial x}\right)^2 - 2QT\right]dV + \int\limits_S\left[qT + \frac{h}{2}\left(T - T_{at}\right)^2\right]dS. \tag{4}$$

Then if we find a function $T = T(x)$, which will give a minimum to the functional (4), then it is a solution of equation (1) and will simultaneously satisfy boundary conditions (2) and (3).

This classical theory is applied to a particular applied problem. Consider a complete rod of bounded length L and divide it into $n-1$ parts. In this case we have n nodes. Next, for each individual finite element, taking into account the real conditions, let us write down the functional expressions for each finite element in detail I_1, I_2, ..., I_{n-1} [2-6]. Let's compose the sum of the functionals

$$I = \sum_{k=1}^{n-1} I_k . \tag{5}$$

Minimizing the I- functional by the temperature nodal values, we obtain the following solving system of algebraic equations:

$$\begin{cases} \dfrac{\partial I}{\partial T_1} = 0 \\ \dfrac{\partial I}{\partial T_2} = 0 \\ \dots \\ \dfrac{\partial I}{\partial T_n} = 0 \end{cases} \tag{6}$$

Here it should be noted that when integrating over the volume and surface integrals in the expression of the functionals I_1, I_2, ..., I_{n-1} there are a lot of peculiar problems connected with specificity of structural elements such as full and gentle rods, variable cross-section, presence of internal cavities in some elements, presence of heat flows on local surface of elements of internal sources, and also given temperature values in some nodes. Depending on these data, the number of algebraic equations in system (6) may be less than n. Therefore, we will show these features on each specific example with corresponding physical and mathematical comments. In doing so, we will proceed from the real formulation of problems regardless of their complexity in the sense of physical and mathematical formulation.

Results and Discussion

In order to test this theory, consider a complete rod of limited length L, the cross section of which is a circle with radius R. This rod is completely insulated along its lateral surface. On the full cross-sectional area of the left end a heat flux of intensity q is given (though the heat flux can be given on the area of a small circle of radius $r < R$). On the cross-sectional area of the right end there is a heat transfer to the environment (heat transfer can take place on a small area). In this case, the values of the heat transfer coefficient denote by h, and the values of the ambient temperature denote by T_{at} (Fig. 1) [1, 7-11].

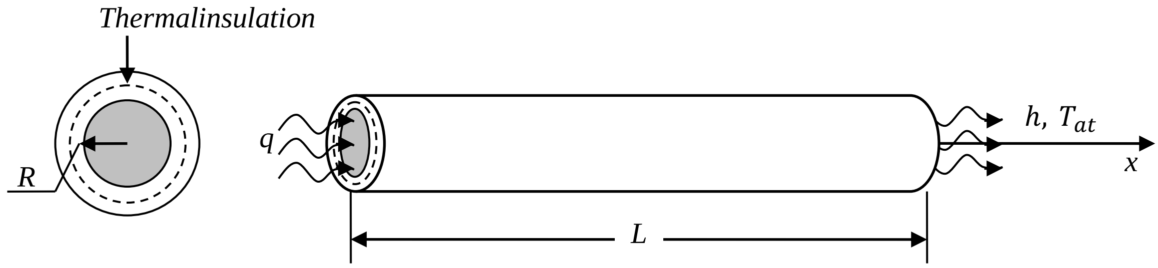

Figure1. Thermally insulated full rod

For convenience, we first discretize the rod in question using three finite elements of equal length $\ell = \dfrac{L}{3}$ (Fig. 2). This will allow us to perform all calculations manually. It should be noted here that it is not necessary to discretize the rod with the same element lengths. The length of each element can be different, i.e. $\ell_1 \neq \ell_2 \neq \ell_3$, etc.

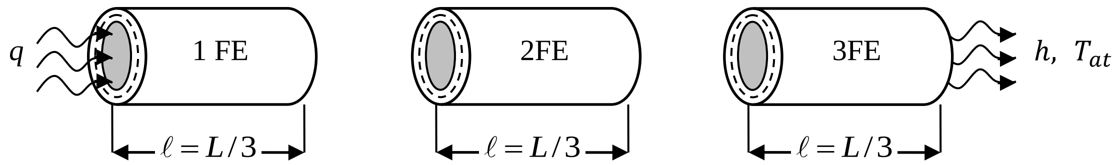

Figure 2 .Three discrete finite elements.

In the considered problem there is no internal heat source, i.e.Q=0. For the 1st finite element the expression of the form function is as follows

$$\left.\begin{aligned}\varphi_1(x) &= \frac{x_2 - x}{\ell_1}\\ \varphi_2(x) &= \frac{x - x_1}{\ell_1}\end{aligned}\right\} \quad \text{when } x_1 \le x \le x_2\,;\; x_2 - x_1 = l_1$$

The temperature values at any point within the length of the first finite element are determined by the temperature values of the nodes T_1 and T_2 according to the formula

$$T = \varphi_1(x)T_1 + \varphi_2(x)T_2 = \frac{x_2 - x}{\ell_1}T_1 + \frac{x - x_1}{\ell_1}T_2 \tag{7}$$

Then the functional expression for the first finite element is as follows [1, 2, 12-16]:

$$I_1 = \int\limits_{V^{(1)}} \frac{1}{2}\left[K_{xx}^{(1)}\left(\frac{\partial T}{\partial x}\right)^2\right]dV + \int\limits_{S_1} qT_1 dS\,,$$

where $K_{xx}^{(1)}$ – is the value of the heat transfer coefficient of the material of the first finite element, S_1 – is the cross-sectional area of the left end of the rod, which corresponds to the first node. This functional takes into account the presence of heat flow with intensity $\boldsymbol{q}$ on the cross-sectional area S_1, which corresponds to the first node. From (7) we define the expression for the temperature gradient within the length of the first finite element

$$\frac{dT}{dx} = \frac{d\varphi_1(x)}{dx}T_1 + \frac{d\varphi_2(x)}{dx}T_2 = \frac{T_2 - T_1}{\ell_1}.$$

Then the expression for I_1 it has the following form

$$I_1 = \int\limits_{V^{(1)}} \frac{1}{2}\left[K_{xx}^{(1)}\left(\frac{T_2 - T_1}{\ell_1}\right)^2\right]dV + \int\limits_{S_1} qT_1 dS = \frac{K_{xx}^{(1)}A^{(1)}}{2\ell_1^2}\int\limits_{x_1}^{x_2}(T_2 - T_1)^2\,dx + qT_1A_1 =$$

$$= \frac{K_{xx}^{(1)}A^{(1)}}{2\ell_1}(T_2 - T_1)^2 + qT_1A_1,$$

where $A_1 = S_1$ is the cross-sectional area of the left end of the rod, where the heat flux with intensity q, $A^{(1)}$ – is applied, are the values of the cross-sectional area of the first finite element. Thus, for the first finite element we have:

$$I_1 = \frac{K_{xx}^{(1)}A^{(1)}}{2\ell_1}(T_2 - T_1)^2 + qT_1A_1, \tag{8}$$

T_1, T_2 – are the nodal temperature values at nodes 1 and 2, respectively. Similarly, let us write the functional expression for the second (inner) finite element:

$$I_2 = \int\limits_{V^{(2)}} \frac{1}{2}\left[K_{xx}^{(2)}\left(\frac{T_3 - T_2}{\ell_2}\right)^2\right]dV = \frac{K_{xx}^{(2)}A^{(2)}}{2\ell_2}(T_3 - T_2)^2 \quad . \tag{9}$$

But, for the third finite element, the functional expression I_3 must be written taking into account the heat exchange process of the rod with the environment through the cross-sectional area of the right end of the rod. Where are the values of the heat transfer coefficient *h*, and the ambient temperature T_{at}. Then

$$I_3 = \int\limits_{V^{(3)}} \frac{1}{2}\left[K_{xx}^{(3)}\left(\frac{T_4 - T_3}{l_3}\right)^2\right]dV + \int\limits_{S_4} \frac{h}{2}\left(T - T_{at}\right)^2 dS,$$

where S_4 – s the cross-sectional area of the right end of the rod, which corresponds to the fourth node, and along which there is heat exchange with the environment. After integrating the expression I_3 we have:

$$I_3 = \frac{K_{xx}^{(3)}A^{(3)}}{2l_3}\left(T_4 - T_3\right)^2 + \frac{hA_4}{2}\left(T_4 - T_{at}\right)^2 ,$$

(10)

where $A_4 = S_4$. Thus, for the considered rod, taking into account the specificity and formulation of the problem, the full expression of the functional will be:

$$I = I_1 + I_2 + I_3 =$$

$$\frac{K_{xx}^{(1)}A^{(1)}}{2l_1}\left(T_2 - T_1\right)^2 + qT_1A_1 + \frac{K_{xx}^{(2)}A^{(2)}}{2l_2}\left(T_3 - T_2\right)^2 + \frac{K_{xx}^{(3)}A^{(3)}}{2l_3}\left(T_4 - T_3\right)^2 + \frac{hA_4}{2}\left(T_4 - T_{at}\right)^2 .$$

Then minimizing the functional *I* over the nodal values of temperature T_1, T_2, T_3 и T_4 and obtain the following basic system of solving algebraic equations with respect to the desired nodal values of temperature:

$$\left.\begin{aligned}
&\frac{\partial I}{\partial T_1} = 0 \Rightarrow -\frac{K_{xx}^{(1)}A^{(1)}}{l_1}\left(T_2 - T_1\right) + qA_1 = 0\\
&\frac{\partial I}{\partial T_2} = 0 \Rightarrow \frac{K_{xx}^{(1)}A^{(1)}}{l_1}\left(T_2 - T_1\right) - \frac{K_{xx}^{(2)}A^{(2)}}{l_2}\left(T_3 - T_2\right) = 0\\
&\frac{\partial I}{\partial T_3} = 0 \Rightarrow \frac{K_{xx}^{(2)}A^{(2)}}{l_2}\left(T_3 - T_2\right) - \frac{K_{xx}^{(3)}A^{(3)}}{l_3}\left(T_4 - T_3\right) = 0\\
&\frac{\partial I}{\partial T_4} = 0 \Rightarrow \frac{K_{xx}^{(3)}A^{(3)}}{l_3}\left(T_4 - T_3\right) + hA_4\left(T_4 - T_{at}\right) = 0
\end{aligned}\right\} \tag{11}$$

In order to obtain numerical results, let us set specific application problems for the parameters in the system (15). Assume that the considered rod is homogeneous and of constant cross-section. Then for the numerical solution we assume the following initial data [1, 2, 17-22]:

$$K_{xx}^{(1)} = K_{xx}^{(2)} = K_{xx}^{(3)} = 100\left[\frac{W}{cm^{\circ}C}\right]; \; l_1 = l_2 = l_3 = \frac{L}{3} = \frac{15}{3} = 5[cm]; \; R = 2[cm]$$

$$A_1 = A_4 = A^{(1)} = A^{(2)} = A^{(3)} = \pi R^2 = 4\pi\left[cm^2\right].$$

Values of the coefficient of heat exchange with the environment h=6 $\mathrm{h} = 6\ \left[\frac{\text{Вт}}{(\text{cm}^2\ \ {}^\circ\text{C})}\right]$,of the available heat flow (supply flow) q=-180 $\mathrm{q} = \text{-}180\ \left[\frac{\text{Вт}}{\text{cm}^2}\right]$,ambient temperatures of the cross section of the right end (4th node) $T_{at} = 16\left[{}^\circ C\right]$$\mathrm{T_{at}} = 16\ [\ {}^\circ\mathrm{C}]$. Now using the system of equations (11) we find the temperature values at nodes 1, 2, 3 and 4, i.e. T_1, T_2, T_3 и T_4.

Based on the initial data we have:

$$\frac{K_{xx}\cdot A}{\ell} = \frac{100\cdot 4\pi}{5} = 80\pi,\ q\cdot A_1 = -180\cdot 4\pi = -720\pi,\ h\cdot A_4\cdot T_{at} = 6\cdot 4\pi\cdot 16 = 384\pi.$$

The obtained numerical data are substituted into the system of equations (11) and we obtain the following system of algebraic equations, relative to the temperature nodal values:

$$\begin{cases} -80\pi(T_2 - T_1) - 720\pi = 0, \\ 80\pi(T_2 - T_1) - 80\pi(T_3 - T_2) = 0, \\ 80\pi(T_3 - T_2) - 80\pi(T_4 - T_3) = 0, \\ 80\pi(T_4 - T_3) + 24\pi(T_4 - 16) = 0. \end{cases}$$

By opening the brackets and reducing both parts of the equations, after slight simplifications, we obtain the following final solving system of algebraic equations:

$$\begin{cases} 80T_1 - 80T_2 = 720 \\ -80T_1 + 160T_2 - 80T_3 = 0 \\ -80T_2 + 160T_3 - 80T_4 = 0 \\ -80T_3 + 104T_4 = 384 \end{cases} \tag{12}$$

From the first equation of the original system (12) we have:

$$80T_1 = 80T_2 + 720 \Rightarrow T_1 = T_2 + 9. \tag{13}$$

Substituting $80T_1$, into the second equation of the system (12) we find that

$$T_2 = T_3 + 9 \tag{14}$$

Substituting the found value T_2 into the third equation of the system (12) we obtain that

$$T_3 = T_4 + 9 \tag{15}$$

Finally, substituting the found values into the last equation of the system (12) we find that $80T_4 - 80T_4 - 720 + 24T_4 = 384$. From this we get that

$$24T_4 = 384 + 720 \Rightarrow T_4 = \frac{384+720}{24} = 46\ {}^\circ C.$$

Substituting the value T_4 into equation (15) we find the temperature in the inner 3rd node $T_3 = T_4 + 9 = 46 + 9 = 55\ {}^\circ C$.

Substituting T_3 into equation (14) let's find the temperature in the inner second node

$$T_2 = T_3 + 9 = 55 + 9 = 64\ {}^\circ C.$$

Finally, by substituting the value of T_2 into equation (13), we find the temperature value in the 1st node on the cross-sectional area of which the heat flux of intensity is given q=-180 $\mathrm{q} = \text{-}180\ \left[\frac{\text{Вт}}{\text{cm}^2}\right]$,

$$T_1 = T_2 + 9 = 64 + 9 = 73\ {}^\circ C.$$

Here we consider the first case, which heat flux intensity q is given on the area of a small circle of a rod with radius $r<R$.

Now consider the second case, when the heat flux with intensity q is applied not to the full area of the left end, but to a part of it, that is, to the area $\pi\left(\frac{R}{2}\right)^2$ (Fig. 3) [1-6, 22-23]:

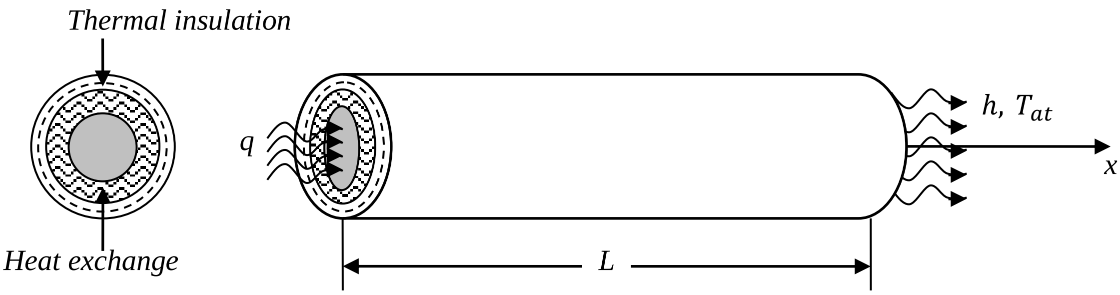

Figure 3. Calculation scheme

At the same time on the remaining cross-sectional area of the left end, i.e. on the $\pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \frac{3\pi R^2}{4} cm^2$ $\pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \frac{3\pi R^2}{4}$ cm^2there is a heat exchange with the environment h_1=6 $h = 6$ $\left[\frac{Вт}{(cm^2\ \ °C)}\right]$,$h_1 = 6$ $\left[\frac{Вт}{(cm^2\cdot\ °C)}\right]$, there is a heat exchange with the environment $T_{at_1} = 20^\circ C$, $T_{at} = 16^0 C$.

The area to which the heat flow is brought

$q = -180$ $\left[\frac{Вт}{cm^2}\right]$ denote by $S_{11} = \pi\left(\frac{R}{2}^{2}\right) = \frac{\pi R^4}{4} cm^2$.

Then we denote the remaining cross-sectional areas of the left end of the rod by $S_{12} = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \frac{3\pi R^2}{4} cm^2$.$S_{12} = \pi R^2$-$\frac{\pi R^2}{2} = \frac{3\pi R^2}{4}$ cm^2 Over the total area of the right end of the rod there is a heat exchange with the environment h=6 $h = 6$ $\left[\frac{Вт}{(cm^2\ \ °C)}\right]$,. As in the previous problem, we discretize the rod in question with length L, using three finite elements lengths of which are respectively $\ell_1 = \ell_2 = \ell_3 = \frac{L}{3}$ (Fig. 2). Then in the considered problem for the first finite element the expression of the functional I_1 instead of (8) will be as follows:

$$I_1 = \int\limits_{V^{(1)}} \frac{1}{2}\left[K_{xx}^{(1)}\left(\frac{T_2 - T_1}{l_1}\right)^2\right]dV + \int\limits_{S_{11}} qT_1 dS_{11} + \int\limits_{S_{12}} \frac{h_1}{2}\left(T - T_{at_1}\right)^2 dS_{12} =$$

$$= \frac{K_{xx}^{(1)} A^{(1)}}{2l_1}\left(T_2 - T_1\right)^2 + qT_1 S_{11} + \frac{h_1 S_{12}}{2}\left(T_1 - T_{at_1}\right)^2 .$$

Thus, for the first finite element we have

$$I_1 = \frac{K_{xx}^{(1)} A^{(1)}}{2l_1}\left(T_2 - T_1\right)^2 + qT_1 S_{11} + \frac{h_1 S_{12}}{2}\left(T_1 - T_{at_1}\right)^2 .$$

We leave the functionals for the remaining elements (2-3) the same as (9) and (10):

$$I_2 = \int\limits_{V^{(2)}} \frac{1}{2}\left[K_{xx}^{(2)}\left(\frac{T_3 - T_2}{l_2}\right)^2\right]dV = \frac{K_{xx}^{(2)} A^{(2)}}{2l_2}\left(T_3 - T_2\right)^2;$$

$$I_3 = \int_{V^{(3)}} \frac{1}{2}\left[K_{xx}^{(3)}\left(\frac{T_4 - T_3}{l_3}\right)^2\right]dV + \int_S \frac{h}{2}\left(T - T_{at}\right)^2 dS = \frac{K_{xx}^{(3)}A^{(3)}}{2l_3}\left(T_4 - T_3\right)^2 + \frac{hA_4}{2}\left(T_4 - T_{at}\right)^2.$$

The general expression of the functional for the full rod is as follows

$$I = I_1 + I_2 + I_3 = \frac{K_{xx}^{(1)}A^1}{2l_1}\left(T_2 - T_1\right)^2 + qT_1S_{11} + \frac{h_1S_{12}}{2}\left(T_1 - T_{at_1}\right)^2 + \frac{K_{xx}^{(2)}A^{(2)}}{2l_2}\left(T_3 - T_2\right)^2 + \frac{K_{xx}^{(3)}A^{(3)}}{2l_3}\left(T_4 - T_3\right)^2 + \frac{hA_4}{2}\left(T_4 - T_{at}\right)^2. \tag{16}$$

Now minimize the obtained functional I (16) by the nodal values of temperature T_1, T_2, T_3 и T_4 and obtain the following solving system of algebraic equations:

$$\left.\begin{aligned}
&\frac{\partial I}{\partial T_1} = 0 \Rightarrow -\frac{K_{xx}^{(1)}A^{(1)}}{l_1}\left(T_2 - T_1\right) + qS_{11} + h_1S_{12}\left(T_1 - T_{at_1}\right) = 0\\
&\frac{\partial I}{\partial T_2} = 0 \Rightarrow \frac{K_{xx}^{(1)}A^{(1)}}{l_1}\left(T_2 - T_1\right) - \frac{K_{xx}^{(2)}A^{(2)}}{l_2}\left(T_3 - T_2\right) = 0\\
&\frac{\partial I}{\partial T_3} = 0 \Rightarrow \frac{K_{xx}^{(2)}A^{(2)}}{l_2}\left(T_3 - T_2\right) - \frac{K_{xx}^{(3)}A^{(3)}}{l_3}\left(T_4 - T_3\right) = 0\\
&\frac{\partial I}{\partial T_4} = 0 \Rightarrow \frac{K_{xx}^{(3)}A^{(3)}}{l_3}\left(T_4 - T_3\right) + hA_4\left(T_4 - T_{at}\right) = 0.
\end{aligned}\right\} \tag{17}$$

Calculating the values at R=2 cm. We have $S_{11} = \frac{4\pi}{4} = \pi$; $S_{12} = \frac{3\pi \cdot 4}{4} = 3\pi$; $h_1 \cdot S_{12} = 6 \cdot 3\pi = 18\pi$; $h \cdot A_4 = 6 \cdot 4\pi = 24\pi$.

Substituting the coefficients into the system of equations (17) we obtain

$$\begin{cases}
-80\pi(T_2 - T_1) - 180\pi + 18\pi(T_1 - 20) = 0,\\
80\pi(T_2 - T_1) - 80\pi(T_3 - T_2) = 0,\\
80\pi(T_3 - T_2) - 80\pi(T_4 - T_3) = 0,\\
80\pi(T_4 - T_3) + 24\pi(T_4 - 16) = 0.
\end{cases}$$

Reducing all terms of the equation by and after a slight simplification we have:

$$\begin{cases}
98T_1 - 80T_2 = 540,\\
-80T_1 + 160T_2 - 80T_3 = 0,\\
-80T_2 + 160T_3 - 80T_4 = 0,\\
-80T_3 + 104T_4 = 384.
\end{cases} \tag{18}$$

From the first equation of the original system:

$$T_1 = \frac{270}{49} + \frac{40}{49} \cdot T_2, \tag{19}$$

Substituting the found value T_1 into the second equation of the system (18) we obtain that

$$T_2 = \frac{270}{58} + \frac{49}{58} \cdot T_3 . \tag{20}$$

Substituting the found T_2 in the third equation of the system (18) we find

$$T_3 = \frac{270}{67} + \frac{58}{67} \cdot T_4 . \tag{21}$$

Substituting the obtained value T_3 into the last equation of the system (18), we find the temperature value of the 4th node, which corresponds to the cross-sectional surface of the right end of the full rod, where the heat exchange takes place:

$$-80\left(\frac{270}{67} + \frac{58}{67} \cdot T_4\right) + 104T_4 = 384 .$$

From this we have

$$T_4 = \frac{47328}{2328} \approx 20.33\ ^\circ C ,$$

then substituting T_4 in (21) we obtain the temperature values in the 3-node.

$$T_3 = \frac{270}{67} + \frac{58}{67} \cdot 20.33 \approx 21.63\ ^\circ C .$$

From (20) the value T_2 will be:

$$T_2 = \frac{270}{58} + \frac{49}{58} \cdot 21.63 \approx 22.93\ ^\circ C .$$

From (19) we find the temperature value of the first node, which corresponds to the cross section of the left end of the rod, where part of the area is subjected to heat flow and the right part is subjected to heat exchange:

$$T_1 = \frac{270}{49} + \frac{40}{49} \cdot 22.93 \approx 24.23\ ^\circ C .$$

Conclusions

It should be noted that three types of boundary conditions have been set in this problem:

1) The surface of the cross-sectional area of the left end of the full rod is given a heat flux and two cases are considered:

a) a given heat flux of intensity q can be set on the area of a small circle of radius *r*<*R*;

b) the heat flux can be set on its part, i.e. on the area $\pi\left(\frac{R}{2}\right)^2$.

2) The lateral surface area along the entire length of the rod is insulated.

The cross-sectional area of the right end is open. It is surrounded by some medium (water, oil, ground, etc.), temperature of which is $T_4 = 16\ ^\circ C$. Values of the coefficient of heat exchange of the rod with this environment *h*=6 $\mathrm{h} = 6\ \left[\frac{\text{Вт}}{(\text{cm}^2\ \ {}^\circ\text{C})}\right]$,, heat transfer coefficient $\mathrm{h} = 6\ \left[\frac{\text{Вт}}{(\text{cm}^2\ \ {}^\circ\text{C})}\right]$, and the coefficient of heat exchange of the rod with the environment $T_{at} = 16^\circ C$ is determined experimentally and in all problems considered by us are considered to be set. Values of heat flux and internal heat source are also set.

As a result, the initial data for both cases are the same, but for the first case, when the heat flux is set on the area of a small circle of radius *r*<*R* the temperature values at the nodal points are as follows: $T_1 = 73^\circ C$, $T_2 = 64^\circ C$, $T_3 = 55^\circ C$, $T_4 = 46^\circ C$ $\mathrm{T}_1 = 73\,^\circ\mathrm{C}$.

Now for the second case, when the heat flux is set on its part, that is, on the area $\pi\left(\frac{R}{2}\right)^2$ the temperature values at the nodal points are as follows: $T_1 = 24.23^\circ C$, $T_2 = 22.93^\circ C$, $T_3 = 21.63^\circ C$, $T_4 = 20.33^\circ C$ $T_1 = 73^\circ C$.

On the basis of the above-stated, practical significance of the conducted research can be determined. The proposed computational algorithm can be used to determine the regularities of the temperature distribution field in rod-type structural elements. Furthermore, the presented method of transferring heat to one-dimensional bodies based on minimizing integral thermal energy allows formalizing and solving the problems of temperature distribution over the volume of bodies of various configurations made of heat-resistant alloys.

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Көлденең қимасы тұрақты біртұтас сырықтың ұзына бойына жылу таралу өрісін шекті элементтер әдісімен пішіндеу

Мақалада ЭИ48 жылуға төзімді құймадан жасалған сырықтың ұзына бойы жылу таралу өрісінің заңдылығын анықтау негізделген. Авторлар ұзындығы шектеулі *L*-ге тең, ұзына бойы көлденең қималарының радиустары *R*-ге тең дөңгелек біртұтас сырықты қарастырған. Зерттеуге алынған дененің сол жақ көлденең қимасының толық бетінің ауданына *q* жылу ағыны түсірілген, ал оң жақ көлденең қимасының ауданынан қоршаған ортамен жылу алмасу жүріп жатыр. Сырықтың бүйір бетінің ауданы жылудан оқшауланған. Мақала авторлары екі жағдайды қарастырған: біріншісі — жылу ағыны *q* қарқынымен сол жақ ауданының $r<R$ радиусымен берілген кішкентай дөңгелек ауданын; екіншісі — жылу ағыны сол жақ бетінің ауданының белгілі бір бөлігіне берілген, яғни $\pi\left(\frac{R}{2}\right)^2$ ауданын. Зерттеу көрсеткендей, жылумеханикалық процесте негізгі құрылғы элементтерінің әр бөлігіндегі ыстыққа төзімділікке жылу таралу өрісінің әсері маңызды болады. Сонымен қатар, жоғары температура ыстыққа төзімді құймалардың морфологиясына әсер ететіндігі зерттелген. Бұл құрылғы элементтерінің кейбір бөліктерінде жылу өтімді, ал кейбір бөліктерінде критикалық жағдайға жететіндігін көрсетеді. Осындай құбылыстардың нәтижесінде құрылымдық элементтер жарамсыз күйге (қирауға) тез ұшырайды және физикалық қасиеттерін жоғалтады. Сондықтан әр түрлі формадағы дене үшін температураның таралу өрісін математикалық модельдеу (пішіндеу) өзекті мәселе. Мақалада сырық тәріздес құрылымдық элементтердегі жылу таралу өрісінің заңдылықтарын анықтау бағытында есептер шығару үшін математикалық модельді (пішінді) құру әдістемесі және сәйкесті есептеу алгортмдері келтірілген. Бұл үшін авторлар энергетикалық-вариациялық принцип негізінде шекті элементтер әдісін қолданды.

Кілт сөздер: математикалық модель (пішін) біртұтас сырық, жылу ағыны, көлденен қима, функционал, жылу алмасу, жылудан оқшаулау, жылу таралу өрісі.

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Конечно-элементное моделирование распространения тепла полного стержня постоянного поперечного сечения

В статье введено определение поля распределения температуры для стержня, изготовленного из жаропрочного сплава ЭИ48. Авторами для исследования выбран полный стержень кругового поперечного сечения радиуса *R* ограниченной длины *L*. Изучаемое тело находится под воздействием теплового потока *q* со стороны поверхности по всей площади поперечного сечения левого конца, а на площади поперечного сечения правого конца происходит теплообмен с окружающей средой. Стержень теплоизолирован по боковой поверхности. Изучены два случая: первый — тепловой поток интенсивностью *q* может быть задан на площади малого круга радиусом $r<R$; второй — тепловой поток может быть задан на ее части, то есть на площади $\pi\left(\frac{R}{2}\right)^2$. При исследовании авторами показано, что при термомеханическом процессе на прочность каждого участка несущих элементов конструкции существенное влияние оказывает поле распределения температуры. Также отмечено влияние высокой температуры на морфологию жаропрочных сплавов. Это приводит к тому, что на каких-

то участках элементов конструкции температура будет допустимой, а на каких-то — критической. Вследствие этого происходит быстрое изнашивание элементов конструкции и потеря их физических качеств. Поэтому математическое моделирование распределения поля температуры для тела различной конфигурации является актуальной проблемой. В статье приведена методика построения математической модели и соответствующего вычислительного алгоритма, позволяющих решать класс задач по определению закономерностей поля распределения температур в элементах конструкций стержневого вида. Для этого авторы использовали энергетическо-вариационный принцип в сочетании метода конечных элементов.

Ключевые слова: математическая модель, полный стержень, тепловой поток, поперечное сечение, функционал, теплообмен, теплоизоляция, поле распределения температуры.