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Indicatrix of TE and TM- polarized wave velocities in crystal of classes 4mm, 3m, 6mm with magneto-electric effect

This work is devoted to the theoretical study of the laws of propagation of electromagnetic waves of TE and TM polarization in anisotropic media belonging to classes 4mm, 3m, 6mm, etc. having a magnetoelectric effect. The directions of the vectors of the phase and group velocities of the polarization waves TE and TM at the boundary of a uniaxial crystal with magnetoelectric properties are considered. In the analytical form, the values of the directions of the phase and group velocities of the TE and TM waves are indicated, depending on the direction of the wave vector of the incident wave. The consequences of the obtained results for uniaxial crystals in the absence of magnetoelectric properties are discussed. The solution of the tasks set in this paper is based on the use of the matrix method. On its basis, various problems of wave processes in an isotropic elastic medium, electromagnetic waves in crystals, the distribution of coupled elastic and electromagnetic waves in piezoelectric and piezomagnetic media with a magnetoelectric effect were previously considered. In the presence of a magnetoelectric effect for electromagnetic waves propagating through uniaxial crystals, the parameters of the wave vector, phase and group velocities are determined. The obtained results are analyzed for electromagnetic waves propagating through uniaxial crystals in the absence of a magnetoelectric effect.

The indicatrices of the wave vectors propagating in the plane and the phase velocities of the TM polarization waves are limited $(x0z)$. Based on the Rayleigh equation, the values of the group velocity are obtained. The density of electromagnetic energy fluxes and their components are determined for TE and TM waves. The energy transfer rate and its direction are determined. It is shown that the group velocities and directions obtained from the Rayleigh equation and the Umov-Poynting vector do not coincide.

Key words: anisotropy, electromagnetic waves, uniaxial crystals, magnetoelectric effect, phase and group velocities, TE and TM polarization waves, density vector.

Introduction

Active theoretical and experimental study of heterostructures and composite materials with piezoelectric, piezomagnetic, magnetostrictive and ferromagnetic properties are currently underway.

The aim of research is to create materials with magnetoelectric properties, for their practical application in instrument engineering, micro and nanoelectronics, information technologies [1-4].

The paper discusses the theoretical study of the patterns of TE and TM- polarized electromagnetic waves propagation in anisotropic medium related to classes 4mm, 3m, 6mm, etc., with magnetoelectric effect.

A tensor describing the magnetoelectric effect is adopted in the form [1]:

$$\hat{\alpha} = \begin{Bmatrix} 0 & \alpha_{xy} & 0 \\ -\alpha_{xy} & 0 & 0 \\ 0 & 0 & \alpha_z \end{Bmatrix} . \quad (1)$$

Wave processes are considered based on Maxwell's equations:

$$rot\vec{E} = -\frac{\partial \vec{B}}{\partial t}; \; rot\vec{H} = \frac{\partial \vec{D}}{\partial t} \quad (2)$$

$$div\vec{D} = 0; \quad div\vec{B} = 0; \quad (3)$$

The material ratio has the following form:

$$D_i = \varepsilon_0 \varepsilon_{ij} E_j - \alpha_{ij} H_j, \quad B_i = \mu_0 \mu_{ij} H_j - \alpha_{ij} E_j . \quad (4)$$

Dielectric and magnetic constant tensors correspond to their type for uniaxial crystals.

The ratios (6) taking into account (1) have the form:

$$\begin{gathered} D_x = \varepsilon_x E_x - \alpha_{xy} H_y, \quad D_y = \varepsilon_y E_y + \alpha_{xy} H_x, \quad D_z = \varepsilon_z E_z \\ B_x = \mu_x H_x - \alpha_{xy} E_y, \quad B_y = \mu_y H_y + \alpha_{xy} E_x, \quad B_z = \mu_z H_z \end{gathered} \quad (5)$$

For uniaxial crystals $\varepsilon_x = \varepsilon_y$; $\mu_x = \mu_y$ absolute permeability to vacuum ε_0 *u* μ_0 is contained in ε_{ij} *u* μ_{ij}.

1. Propagation of electromagnetic waves in the plane **(** $x0z$ **),** $k_y = 0$.

Solution and study in the form of flat waves. Presenting these solutions for electrical and magnetic field components as [5], [7-10]:

$$f(x, y, z, t) = f(x)e^{i\omega t - ik_z z}, \quad (1.1)$$

taking into account the absence of dependence, in this case, on the coordinate *y*, a system of equations was obtained from equations (2), (3) and relations (7):

$$\frac{dE_y}{dx} = -i\omega\mu_z E_z \quad (1.2)$$

$$\frac{dH_z}{dx} = i(\omega\varepsilon_y - \frac{k_z^2 - \omega^2\alpha_{xy}^2}{\omega\mu_x})H_z \quad (1.3)$$

$$\frac{dH_y}{dx} = i\omega\varepsilon_z E_z \quad (1.4)$$

$$\frac{dE_z}{dx} = i(\omega\mu_y - \frac{k_z^2 - \omega^2\alpha_{xy}^2}{\omega\varepsilon_x})H_y . \quad (1.5)$$

The systems of equations (1.2), (1.3) and (1.4), (1.5) are independent. Equations (1.2), (1.3) describe the propagation of TE-polarized waves. Equations (1.4), (1.5) — TM-polarized waves.

1.1 The equation of indicatrices of the TE- wave vector follows from the condition:

Within the framework of the matrix method of the matricant for homogeneous media, the wave vector indicatrix equation can be determined from the condition [12-14]:

$$\det\left[B^2 + k_z^2 I\right] = 0, \tag{1.6}$$

$\det\left[T - Ie^{-i\tilde{k}h}\right] = 0$, $\left(\tilde{k} = k_z\right)$. This follows from the relations $\vec{W}(h) = T(h)\vec{W}_0$; $\vec{W}(h) = \mathrm{e}^{-ik_z h}\vec{W}_0$ and is a consequence of the Floquet-Bloch theorem [13].

With periodic changes in parameters along the z axis:

$$\varepsilon_{ij}(z+h) = \varepsilon_{ij}(z), \quad \mu_{ij}(z+h) = \mu_{ij}(z), \quad \alpha_{ij}(z+h) = \alpha_{ij}(z).$$

Matrix $T(nh) = T^n(h)$; $T(h)$ - the monodromy matrix. For $T^n(h)$ the representation based on Chebyshev-Gegenbauer polynomials is valid. The matrix is a monodromy matrix. A representation based on Chebyshev-Gegenbauer polynomials is valid for.

$$T^n(h) = P_n(\hat{p})T(h) - P_{n-1}(\hat{p}).$$

Calculation of matrix polynomials $P_n(\hat{p})$ is shown in the paper [14]. The dispersion equation, based on knowledge of the matrix structure, can be written in two equivalent forms:

$$\det\left[T(h) - Ie^{-ik_z h}\right] = 0, \ \det\left[T^{-1}(h) - Ie^{ik_z h}\right] = 0.$$

In view of their equivalence, a modified condition follows from them:

$$\det\left[\hat{p} - I\cos k_z h\right] = o. \tag{1.6*}$$

Under the condition $\lambda \succ\succ h$ (λ is the wavelength, h is the period of in homogeneity), analytical representations of matrices for homogeneous media are obtained from (1.6*). In particular, this is due to

where $p = \frac{1}{2}(T + T^{-1})$; $\cos k_z h = \frac{e^{-ik_z h} + e^{ik_z h}}{2}$

$\frac{d\vec{W}}{dz} = B(z)\vec{W}$; $B(z) = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix}$. The matrix T has the form: $T^{\pm 1}(z) = I\cos k_z z \pm \frac{1}{k_z} B\sin k_z z$,

$B = \frac{1}{h}\int_0^h B(z)dz$. When decomposed by $k_z \sim \frac{1}{\lambda}$, $\frac{h}{\lambda} \prec\prec 1$, h not enough because: $\hat{p} = \frac{1}{2}(T + T^{-1})$; T and T^{-1} direct and inverse monodromy matrices.

Have representations [8]: $T = I + \int_0^h B(z)dz + \int_0^h\int_0^z B(z)B(z_1)dzdz_1 + \ldots..$

Similarly: $T^{-1} = I - \int_0^h B(z)dz + \int_0^h\int_0^z B(z_1)B(z)dz_1dz + \ldots..$

then, provided $k_z h \prec\prec 1$ and small h, while preserving the summands up to quadratic terms, we have, in the case of homogeneous media:

$$T = I + Bh + \frac{B^2 h}{2}; \quad T^{-1} = I - Bh + \frac{B^2 h}{2}; \quad B = \frac{1}{h}\int_0^h B(z)dz$$

$p = \frac{1}{2}(T + T^{-1}) = I + \frac{B^2 h}{2}$. For $\cos k_z h = 1 - \frac{k_z^2 h^2}{2}$. Then from (1.6*) should: $\det\left[B^2 + Ik_z^2\right] = 0$

$$B = \begin{pmatrix} 0 & b_{12} \\ b_{21} & 0 \end{pmatrix}; \ b_{12} = -i\omega\mu_z, \ b_{21} = i\omega\varepsilon_y - i\frac{k_z^2 - \omega^2\alpha_{xy}^2}{\omega\mu_x}, \tag{1.7}$$

from (1.6) follows: ($k_y = k\cos\theta$, $k_z = k\sin\theta$)

$$k_{TE}^2 = \frac{\omega^2\mu_z(\varepsilon_y\mu_x + \alpha_{xy}^2)}{\mu_x\cos^2\theta + \mu_z\sin^2\theta}, \tag{1.8}$$

θ-angle between axis x and wave vector $\vec{k}$. The TE-wave phase velocity indicatrix based on (1.8) has the form:

$$\nu_{f_{TE}}^2 = \frac{\omega^2}{k^2} = \frac{\mu_x\cos^2\theta + \mu_z\sin^2\theta}{\mu_z(\varepsilon_y\mu_x + \alpha_{xy}^2)}, \tag{1.9}$$

the group velocity is from the Rayleigh equation [5-7]:

$$\vec{v}_g = \vec{n}v_f + \vec{n}_\theta\frac{\partial v_f}{\partial\theta}; \qquad \nu_\theta = \frac{\partial\nu_f}{\partial\theta}, \tag{1.10}$$

unit vector $\vec{n}$ determines direction of phase velocity, $\vec{n}_\theta$ - unit vector perpendicular to $\vec{n}$ vector $\vec{n}$

$$\nu_\theta = \frac{(\mu_z - \mu_x)\sin\theta\cos\theta}{a\nu_f}; \qquad a = \mu_z(\varepsilon_y\mu_x + \alpha_{xy}^2), \tag{1.11}$$

formula (1.9) — (1.11) for group velocity value:

$$v_g^2 = v_f^2 + v_\theta^2, \tag{1.12}$$

angle γ between phase group velocity vectors determines the relation:

$$\tan\gamma = \frac{v_\theta}{v_f} = \frac{(\mu_z - \mu_x)\sin\theta\cos\theta}{\mu_x^2\cos^2\theta + \mu_z\sin^2\theta}, \tag{1.13}$$

from (1.9) — (1.13) follows:

$$v_{g_{TE}}^2 = \frac{1}{a}\,\frac{\mu_x^2\cos^2\theta + \mu_z^2\sin^2\theta}{\mu_x\cos^2\theta + \mu_z\sin^2\theta}. \tag{1.14}$$

1.2. TM wave indicatrices

In this case:

$$b_{12} = i\omega\varepsilon_z, \quad b_{21} = i(\omega\mu_y - \frac{k_z^2 - \omega^2\alpha_{xy}^2}{\omega\varepsilon_x}), \tag{1.15}$$

condition (1.6) gives the indicatrix of the TM wave vector:

$$k_{TM}^2 = \frac{\omega^2\varepsilon_z(\mu_y\varepsilon_x + \alpha_{xy}^2)}{\varepsilon_x\cos^2\theta + \varepsilon_z\sin^2\theta}. \tag{1.16}$$

Calculations similar to those in paragraph 1.1 result in the following formulas:

$$\nu_{f_{TM}}^2 = \frac{\varepsilon_x\cos^2\theta + \varepsilon_z\sin^2\theta}{\varepsilon_z(\varepsilon_x\mu_y + \alpha_{xy}^2)} \tag{1.17}$$

$$\nu_\theta = \frac{(\varepsilon_z - \varepsilon_x)\sin\theta\cos\theta}{\nu_{f_{TM}}\varepsilon_z(\varepsilon_x\mu_y + \alpha_{xy}^2)} \tag{1.18}$$

$$v_{g_{TM}}^2 = \frac{1}{a_{TM}} \frac{\varepsilon_x^2 \cos^2\theta + \varepsilon_z^2 \sin^2\theta}{\varepsilon_x \cos^2\theta + \varepsilon_z \sin^2\theta}, \quad (1.19)$$

angle β, defining direction v_g is from expression $\tan\beta = \frac{\mu_z}{\mu_x}\tan\theta$ (1.20).

2. Energy-flux density [10-12].

2.1. In the case of TE-polarized waves in (1.7), (1.8) as opposed to zero components:

$$E_y,\ H_z,\ H_x. \quad (2.1)$$

From Maxwell's equations for plane waves we have the following:

$$H_z = \frac{k_x}{\omega\mu_z} E_y \quad (2.2)$$

$$H_x = -\frac{k_z - \omega\alpha_{xy}}{\omega\mu_x} E_y. \quad (2.3)$$

The flux density of the electromagnetic energy of the wave is determined by the Umov-Poynting formula:

$$\vec{S} = \left[\vec{E} \times \vec{H}\right]. \quad (2.4)$$

Based on (2.4) we get the following:

$$S_x = \frac{k_x}{\omega\mu_z} E_y^2 = \frac{k_x}{\omega\mu_z\varepsilon_y} \varepsilon_y E_y^2 \quad (2.5)$$

$$S_z = \frac{k_z - \omega\alpha_{xy}}{\omega\mu_x} E_y^2 = \frac{k_z - \omega\alpha_{xy}}{\omega\mu_x\varepsilon_y} \varepsilon_y E_y^2. \quad (2.6)$$

From the relation S_z / S_x the direction of the energy flux density vector follows:

$$\tan\beta_e = \frac{S_z}{S_x} = \frac{\mu_z}{\mu_x} \frac{k_z - \omega\alpha_{xy}}{k_x}, \quad (2.7)$$

when $\alpha_{xy} = 0$, we get the $\tan\beta_e = \frac{k_z}{k_x}\frac{\mu_z}{\mu_x} = \frac{\mu_z}{\mu_x}\tan\theta$ transition rate, the group velocity is determined by the ratio:

$$v_{g_e}^2 = \frac{S^2}{\varepsilon_y^2 E_y^4} = \frac{S_x^2 + S_z^2}{\varepsilon_y^2 E_y^4} = \frac{\mu_x^2 k_x^2 + (k_z - \omega\alpha_{xy})^2 \mu_z^2}{\omega^2 \mu_x^2 \varepsilon_y^2 \mu_z^2}. \quad (2.8)$$

The components of the wave vector k_x *u* k_z are determined on the basis of (1.8) for TE waves and on the basis of formula (1.16) for TM waves.

2.2. When the propagation of TM- polarized waves, the wave field has components:

$$H_y,\ E_z,\ E_x. \quad (2.9)$$

Dependencies are fair:

$$E_z = -\frac{k_x}{\omega\varepsilon_z} H_y \quad (2.10)$$

$$E_x = \frac{k_z + \omega\alpha_{xy}}{\omega\varepsilon_x} H_y \quad (2.11)$$

Based on the Umov-Poynting formula, the S_x *и* S_z components are:

$$S_x = -E_z H_y = \frac{k_x}{\omega\varepsilon_z\mu_y}\mu_y H_y^2 \quad (2.12)$$

$$S_z = E_x H_y = \frac{k_z + \omega\alpha_{xy}}{\omega\varepsilon_x\mu_y}\mu_y H_y^2 \quad (2.13)$$

Direction of energy flow: $\tan\beta_{TM} = \frac{S_z}{S_x} = \frac{\varepsilon_z}{\varepsilon_x}\frac{k_z + \omega\alpha_{xy}}{k_x}$, (2.14)

from (2.12), (2.13) follows the energy transfer rate

$$v_{g_{TM}}^2 = \frac{S_x^2 + S_z^2}{\mu_y^2 H_y^4} = \frac{\varepsilon_x^2 k_x^2 + (k_z + \omega\alpha_{xy})^2 \varepsilon_z^2}{\omega^2\varepsilon_x^2\varepsilon_y^2\mu_y^2} \quad (2.15)$$

Results and discussion

The propagation of TE and TM-polarized electromagnetic waves in uniaxial crystals (classes 4 mm, 3m, 6mm) in the presence of the magneto-electric effect has been discussed. The indicatrices of the wave vectors and phase velocities of TE and TM waves propagating in the plane ($x0z$) have been determined. On the basis of the Rayleigh equation, the magnitude and direction of the group velocity and its indicatrix have been determined. The flux density and electromagnetic waves components transferred by TE and TM waves, their directions and transfer rates have been defined. Experience showed that the group velocity and direction obtained on the basis of the Rayleigh equations and angles transfer rate, following from the Umov-Poynting vector disagreed. The research is based on the matrix method [8-9].

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Магнитэлектрлік әсері бар 4mm, 3m, 6mm кристалдық кластардағы TE және TM поляризация толқындарының жылдамдық индикатрисалары

Мақала магнитэлектрлік әсері бар 4mm, 3m, 6mm және т.б. кластарға жататын анизотропты орталарда электрмагниттік толқындардың TM және TM поляризациясының таралу заңдылықтарын теориялық зерттеуге арналған. Магнитэлектрлік қасиеттері бар біросьті кристалдың шекарасындағы TE және TM поляризациялық толқындарының фазалық және топтық жылдамдық векторларының бағыттары қарастырылған. Аналитикалық формада түскен толқынның толқындық векторының бағытына байланысты TE және TM толқындарының фазалық және топтық жылдамдықтарының бағыттарының мәндері көрсетілген. Алынған нәтижелердің магнитэлектрлік қасиеттері болмаған кезде біросьті кристалдар үшін салдары талқыланған. Бұл жұмыс қойылған есептерді шешуде матрицалық әдісті қолдануға негізделген. Оның негізінде бұрын изотропты серпімді ортадағы толқындық процестердің, кристалдардағы электрмагниттік толқындардың, магнитэлектрлік әсері бар пьезоэлектрлік және пьезомагниттік орталарда байланысқан серпімді және электрмагниттік толқындардың таралуының әртүрлі мәселелері қарастырылған. Біросьті кристалдар арқылы таралатын электрмагниттік толқындар үшін магнитэлектрлік әсер болған кезде толқындық вектордың, фазалық және топтық жылдамдықтардың параметрлері анықталды. Нәтижелер магнитэлектрлік әсер болмаған кезде біросьті кристалдар арқылы таралатын электрмагниттік толқындар үшін талданған. Жазықтықта таралатын толқындық векторлардың индикаторлары TE және TM поляризация толқындарының фазалық жылдамдығы шектеулі $(x0z)$. Рэлей теңдеуі негізінде топтық жылдамдық мәндері алынды. Электрмагниттік энергия ағындарының тығыздығы және олардың құрамдас бөліктері TE және TM толқындары үшін анықталған. Энергияның берілу жылдамдығы және оның бағыты айқындалған. Рэлей теңдеуі мен Умов-Пойнтинг векторынан алынған топтық жылдамдықтар мен бағыттар сәйкес келмейтіні көрсетілген.

Кілт сөздер: анизотропия, электрмагниттік толқындар, біросьті кристалдар, магнитэлектрлік әсер, фазалық және топтық жылдамдықтар, TE және TM поляризация толқындары, тығыздық векторы.

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Индикатрисы скоростей волн TE и TM поляризации в кристаллических классах 4mm, 3m, 6mm с магнитоэлектрическим эффектом

Статья посвящена теоретическому изучению законов распространения электромагнитных волн TE и TM поляризации в анизотропных средах, относящихся к классам 4mm, 3m, 6mm и другим, обладающим магнитоэлектрическим эффектом. Рассмотрены направления векторов фазовой и групповой скоростей поляризационных волн TE и TM на границе одноосного кристалла с магнитоэлектрическими свойствами. В аналитической форме указаны значения направлений фазовой и групповой скоростей волн TE и TM в зависимости от направления волнового вектора падающей волны. Обсуждены последствия полученных результатов для одноосных кристаллов при отсутствии магнитоэлектрических свойств. Решение задач, поставленных в настоящей работе, основано на использовании матричного метода. На его основе ранее рассматривались различные проблемы волновых процессов в изотропной упругой среде, электромагнитных волн в кристаллах, распространения связанных упругих и электромагнитных волн в пьезоэлектрических и пьезомагнитных средах с магнитоэлектрическим эффектом. При наличии магнитоэлектрического эффекта для электромагнитных волн, распространяющихся через одноосные кристаллы, определяются параметры волнового вектора, фазовой и групповой скоростей. Полученные результаты проанализированы для электромагнитных волн, распространяющихся через одноосные кристаллы в отсутствие магнитоэлектрического эффекта. Индикатрисы волновых векторов, распространяющихся в плоскости, и фазовые скорости волн ТЕ и ТМ поляризации ограничены $(x0z)$. На основе уравнения Рэлея получены значения групповой скорости. Плотность потоков электромагнитной энергии и их составляющие определены для волн ТЕ и ТМ. Найдены скорость передачи энергии и ее направление. Показано, что групповые скорости и направления, полученные из уравнения Рэлея и вектора Умова-Пойнтинга, не совпадают.

Ключевые слова: анизотропия, электромагнитные волны, одноосные кристаллы, магнитоэлектрический эффект, фазовая и групповая скорости, волны поляризации ТЕ и ТМ, вектор плотности.

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