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Excitation of transverse and longitudinal thermomagnetic waves in anisotropic conducting media in the presence of a temperature gradient $\vec{\nabla}T$ without an external magnetic field H

In anisotropic conducting media, the excited thermomagnetic waves at different orientations of the magnetic field and temperature gradient significantly depend on the direction of the anisotropic medium. Theoretical calculations of the electrical conductivity tensor σ_{ik} depending on the frequency of the thermomagnetic wave are of scientific interest. In this theoretical work, the frequencies of the thermomagnetic wave at $\vec{k} \parallel \vec{\nabla}T$ and at $\vec{k} \perp \vec{\nabla}T$ were found, and it was proven that at longitudinal ($\vec{k} \parallel \vec{\nabla}T$) and at transverse ($\vec{k} \perp \vec{\nabla}T$) the direction of the frequency and growth rate of these waves depends differently on the external magnetic field. The work theoretically studies the conditions for the excitation of thermomagnetic waves. It is indicated that the directions of external fields play a significant role in the appearance of growing waves in the sample. It is shown that, depending on the value of the electrical conductivity tensor σ_{ik}, thermomagnetic waves are excited in the longitudinal (i.e. $\vec{k} \parallel \vec{\nabla}T$) and transverse (i.e. $\vec{k} \perp \vec{\nabla}T$) directions. The frequencies of these thermomagnetic waves in both the longitudinal and transverse directions have been calculated. The growth rates of these waves are determined by the values of the inverse electrical conductivity tensor σ_{ik}. It has been proven that the excited wave is mainly of a thermomagnetic nature. In theory, the dispersion equation obtained is of algebraically high powers relative to the oscillation frequency. The dispersion equation in both cases (longitudinal $\vec{k} \parallel \vec{\nabla}T$ and transverse $\vec{k} \perp \vec{\nabla}T$) contains terms in which there are thermomagnetic frequencies in a low degree of frequency. It has been proven that if the value of the electrical conductivity tensor σ_{ik} is the same, then the propagation frequencies of thermomagnetic waves are different. The theory is constructed without an external magnetic field $H_0 = 0$. In the presence of an external magnetic field, the conditions for the excitation of thermomagnetic waves, and of course the conditions for their growth, will change significantly.

Keywords: frequency, increment, thermomagnetic waves, transverse waves, longitudinal waves, growth, electrical conductivity tensor, inverse tensor.

Introduction

In works [1–4] it was proven that hydrodynamic movements in a plasma in which there is a constant temperature gradient give rise to a magnetic field. In this case, the plasma has oscillatory properties that are noticeably different from ordinary plasma. In such a plasma, thermomagnetic waves are excited, in which only the magnetic field oscillates. In the presence of an external magnetic field, the wave vector of thermomagnetic waves is perpendicular to the magnetic field or lies in the plane $\vec{H},\vec{\nabla}T$. The speed of thermomagnetic waves is comparable to the speed of sound and the speed of the Alfvén wave. The temperature gradient does not depend on time or coordinates. The Larmor frequency of charge carriers is less than the frequency of their collisions, i.e.

$$\Omega\tau << 1,\ \Omega = \frac{eH}{mc}\ . \tag{1}$$

It has been suggested that magnetic fields exist for explaining the production of cosmic rays and cosmic radio waves. Such fields are based on both the Fermi's statistical mechanism for the acceleration of charged particles and bremsstrahlung at radio frequency by relativistic electrons.

However, the mechanism by which sufficiently strong magnetic fields can be created remains unclear. In [1] it is shown that hydrodynamic motion in a nonequilibrium plasma, in which the existence of a temperature gradient leads to the formation of magnetic fields. Under such conditions, ions arise for which the Larmor frequency of ions is comparable to the oscillation frequency. An increase in parametric resonance and, possibly, resonant acceleration of ions is realized. In sufficiently weak acoustic waves, parametric resonance occurs for electrons. In a hot plasma with high radiation pressure, much stronger magnetic fields arise in a turbulent nonequilibrium plasma. In [1] it is shown that plasma with a temperature gradient ∇T has oscillatory properties that differ from normal plasma. In the absence of an external magnetic field and hydrodynamic motion, transverse "thermomagnetic" waves are possible in the plasma. Magnetic field oscillations occur in them. If there is a constant external magnetic field H, then the wave vector of the thermomagnetic wave must be perpendicular to it.

In addition, the Alfvén wave is split up into "hydrothermomagnetie" waves in which the vectors v and H are perpendicular to ∇T. The spectrum of magnetic sound waves can be modified. The speed of propagation of the thermomagnetic waves is comparable to the velocity of sound and velocity of the Alfvén wave.

In this work, the magnetic fields are divided as follows $H_\perp = H_{\perp\infty} + H'_\perp$.

$$\begin{gathered}\upsilon_m k^2 \frac{\partial^2 H_{\perp\infty}}{\partial\xi^2} - k\frac{\partial}{\partial\xi}\left[(\upsilon + u_T - u - u_s)H_{\perp\infty}\right] = ck\frac{\partial E'}{\partial\xi} \\ \frac{\partial H'_\perp}{\partial t} - \upsilon_m k^2\frac{\partial^2 H'_\perp}{\partial\xi^2} - k\frac{\partial}{\partial\xi}\left[(\upsilon + u_T - u)H'_\perp\right] = 0.\end{gathered} \tag{2}$$

Here u_s is speed of sound propagation, υ_m is magnetic viscosity, from the solutions of these equations

$$H_\perp = \frac{cT\tilde{\Lambda}[\nabla\ln T, k]}{ke}\frac{kv_0\cos\xi + v_0\nabla\ln T\sin\xi}{\omega(u - u_T - \upsilon_0\cos\xi + u_s)}\ . \tag{3}$$

At condition $\upsilon(\xi) << u - u_T + u_s$,

$$H_\perp = \frac{1}{2}\tilde{\Lambda}\frac{\nabla_z T}{e}\frac{c}{s}\left(\frac{\upsilon_0}{s}\right)^2 \tag{4}$$

was obtained.

Showing estimates relate to the case $\upsilon_0 << s$ and $\lambda << L$ are valid at $\upsilon_0 \approx s$ and $\lambda \approx L$. $\Omega \approx N\omega/n$ under a condition $\Omega \approx S/\lambda \approx \omega$. Oscillation and Lamor frequencies can be equal in sound waves. It can likely produce in strong magnetic fields. For $|v_{\max}| > |u - u_T + u_s|$ $(|u - u_T + u_s| < S)$ $H_\perp$ can be large than Ω_i. The magnetic fields excited by these processes can be amplified further by of the magnetic field lines.

In [1] the damping of the turbulence that arises from the conversion of part of the energy of turbulence into magnetic energy was estimated. Then for frequency

$$\Omega_1 = eH_1 / mc > v \tag{5}$$

was obtained.

In the hydrodynamic movements, in this work the oscillation frequency

$$\omega(k)=\frac{1}{2}\left[\sqrt{4\omega_A^2+\omega_T^2}\pm\omega_T\right] \tag{6}$$

was obtained

$$\left(\omega^2-\omega\omega_T-\omega_A^2\right)\left\{\omega^4-\omega^3\omega_T-\omega^2k^2\left(s^2+\upsilon_A^2\right)+\right.$$
$$\left.+\omega\left[k^2s^2\omega_T+k^2\upsilon_A\left(k\left[u_1v_A\right]\right)\right]+\omega_s^2\omega_A^2\right\}=0. \tag{7}$$

The second factor in (7) is the frequencies of the magnetic sound waves determines at $\omega_T=0$ [2]. At conditions $\omega_A<<\omega_s<<\omega_T$ or $\omega_T<\omega_s,\omega_A<\omega_T^2/\omega_s$ the solution is $\omega=\pm\omega_s$ and the roots are $\omega\approx\omega_T$, $\omega\approx\omega_A^2/\omega_T$. For large radiation pressure

$$\frac{\omega_1}{\omega_s}=\frac{l}{L}\frac{N}{n}\sqrt{\frac{M}{m}}\geq 1, \tag{8}$$

l is the mean free path of the electrons and N is the number of protons in a unit volume.

In rarefied plasma radiation pressure of the gas and the interaction of electrons with photons is much stronger than Coulomb scattering. The Compton scattering predominates over bremsstrahlung and its corresponding inverse process for electron collisions [3]. The velocity u_i in (7) is increased roughly by N/n times. If N/n is so large that the velocities s and u_T are much smaller than $\left(u_1u_A^2\right)^{1/3}$.

In the presence of an electric field E, a temperature gradient $\nabla T=const$, a gradient of charge carrier concentrations $\vec{\nabla}n$ and hydrodynamic movements with speed $\vec{\vartheta}(\vec{r},t)$, the electric current density has the form

$$\vec{j}=\sigma\vec{E}^*+\sigma'\left[\vec{E}^*\vec{H}\right]-\alpha\nabla T-\alpha'\left[\nabla TH\right]$$
$$E^*=\vec{E}+\frac{\left[\vec{\vartheta}\vec{H}\right]}{c}+\frac{T}{e}\frac{\nabla n}{n},e>0. \tag{9}$$

The definition E from the vector equation (9) is reduced to solving the vector equation

$$\vec{x}=\vec{a}+\left[\vec{b}\vec{x}\right]. \tag{10}$$

From (10)

$$\left(\vec{b}\vec{x}\right)=\left(\vec{b}\vec{a}\right),\vec{x}=\vec{a}+\left[\vec{b}\vec{a}\right]+\left[\vec{b}\left[\vec{b}\vec{x}\right]\right].$$

At $b^2<<1$,

$$\vec{E}=-\frac{\left[\vec{\vartheta}\vec{H}\right]}{c}-\Lambda'\left[\vec{\nabla}T\vec{H}\right]+\frac{c}{4\pi\sigma}rot\vec{H}-$$
$$-\frac{c\sigma'}{4\pi\sigma^2}\left[rot\vec{H},\vec{H}\right]+\frac{T}{c}\frac{\nabla\rho}{\rho}+\Lambda\nabla T \tag{11}$$

was obtained.

To obtain expression (11), Maxwell's equation $rot\vec{H}=\frac{4\pi}{c}\vec{j}$ was used. Here $\Lambda=\frac{\alpha}{\sigma}$, $\Lambda'=\frac{\alpha'\sigma-\alpha\sigma'}{\sigma^2}$, σ is the electrical conductivity coefficient, Λ is the differential thermopower, Λ' is the Nerst-Ettinizhausen effect coefficient. Substituting (10) into the equation $\frac{\partial\vec{H}}{\partial e}=-crot\vec{E}$, we obtain an equation containing $\vec{H}$ and $\vec{\nabla}T$. It was proven in [1-2] that at $\vec{k}\perp\vec{H}'$ a thermomagnetic wave with a frequency is excited

$$\omega_T=-c\Lambda'\vec{k}\vec{\nabla}T. \tag{12}$$

In anisotropic media with an electronic type of charge carriers, an increasing thermomagnetic wave is excited under certain conditions. Analytical formulas are found for the frequency and for the increment of this wave. The analytical formulas for the tensor of electrical conductivity of the medium are indicated in the form of a table. A formula is found for the ratios of the temperature gradient [5].

In [6] using the kinetic equation for a nonequilibrium process, analytical values of the critical electric field are obtained without an external magnetic field. The estimate of this critical electric field is consistent with Gunn's experiment in which energy is emitted from the sample.

[7] is proved that in anisotropic conducting media of electric type of charge carriers, different waves of a thermomagnetic nature are excited. With the longitudinal $\vec{k} \parallel \vec{\nabla} T$ and transverse $\vec{k} \perp \vec{\nabla} T$ orientation of the wave vector relative to the temperature gradient, waves of a thermomagnetic nature with different frequencies and increments are excited.

From the conclusions [8], it follows that in anisotropic conducting media, it is possible to excite a number of thermomagnetic waves with frequency frequencies. However, at present, there are no experimental data on thermomagnetic waves in the public domain. In [8] frequency is three times less than the frequency of thermomagnetic waves in plasma (i.e., than the frequency ω_T).

In this theoretical work, we will prove

- to determine the frequencies of thermomagnetic waves, when the wave vector of thermomagnetic waves is directed along the temperature gradient $\vec{k} \parallel \vec{\nabla} T$ (longitudinal wave);

- to prove that at $\vec{k} \parallel \vec{\nabla} T$ (longitudinal wave) and at $\vec{k} \perp \vec{\nabla} T$ (transverse wave) thermomagnetic waves can grow (instability).

The growth rate of the thermomagnetic wave differed significantly at $\vec{k} \parallel \vec{\nabla} T$ and at $\vec{k} \perp \vec{\nabla} T$.

Materials and Methods

In the presence of a temperature gradient and an external magnetic field in an isotropic solid, the total electric field [4] has the form:

$$\vec{E} = \varsigma \vec{j} + \varsigma' \left[j \vec{H} \right] + \varsigma'' \left(\vec{j} \vec{H} \right) \vec{H} + \Lambda \frac{\partial T}{\partial x} + \Lambda' \left[\vec{\nabla} T \vec{H} \right] + \Lambda'' \left(\vec{\nabla} T \vec{H} \right) \vec{H}.$$

In an anisotropic solid, all coefficients in equation (13) are tensors. Then for an anisotropic solid body (13) will have the form:

$$E_i = \varsigma_{ik} j'_k + \left[j \vec{H} \right]_k j'_{ik} + \varsigma''_{ik} \left(\vec{j} \vec{H} \right) \vec{H}_k + \Lambda_{ik} \frac{\partial T}{\partial x_k} + \Lambda'_{ik} \left[\vec{\nabla} T \vec{H} \right]_k + \Lambda''_{ik} \left(\vec{\nabla} T \vec{H} \right) \vec{H}_k. \qquad (14)$$

Here j_{ik} is the tensor of the reciprocal value of the ohmic resistance $\varsigma_{ik} = \frac{1}{\sigma_{ik}}$, Λ_{ik} is the differential thermopower, Λ'_{ik} is the Nerist-Ettinizhausen coefficient [8-9]. We will consider an external magnetic field $\vec{H}_0 = 0$ in an anisotropic solid. Then in equation (14) the terms containing $\varsigma'_{ik}, \varsigma''_{ik}, \Lambda''_{ik}$ equal to zero. Then for our problem the system of equations has the form

$$\begin{aligned} E'_i &= \varsigma_{ik} j'_k + \Lambda'_{ik} \left[\vec{\nabla} T H \right]_k \\ rot \vec{E}' &= -\frac{1}{c} \frac{\partial \vec{H}'}{\partial t} \\ rot \vec{H}' &= \frac{4\pi}{c} \vec{j}' + \frac{1}{c} \frac{\partial \vec{E}'}{\partial t}. \end{aligned} \qquad (15)$$

Let us assume that all variables have the character of a monochromatic wave. Then from (15)

$$\begin{aligned} E'_i &= \varsigma_{ik} j'_k + \Lambda'_{ik} \left[\vec{\nabla} T H \right]_k \\ j' &= \frac{ic^2}{4\pi\omega} \left[\vec{k} \left[\vec{k} \vec{E}' \right] \right] + \frac{i\omega}{4\pi} E'_{ik} \end{aligned} \qquad (16)$$

was obtained.

From joint solution (16) for the electric field tensor we obtain the equation

$$E_i' = \frac{ic^2}{4\pi\omega}\varsigma_{ik}\left(\vec{k}\vec{E}'\right)K_k + \frac{i\left(\omega^2 - c^2k^2\right)}{4\pi\omega}\varsigma_{ik}E_k' + \\ + \frac{c\Lambda_{ik}'}{\omega}\left(\vec{\nabla}T\vec{E}'\right)K_k - \frac{c\Lambda_{ik}'}{\omega}\left(\vec{k}\vec{\nabla}T\right)E_k' \tag{17}$$

was obtained.

The solution of equation (17) is generally impossible, and therefore we will consider excitations of thermomagnetic waves in both the transverse and longitudinal directions. To determine the direction of the wave vector $\vec{k}$, you need to choose a coordinate system.

Results and Discussion

Transverse thermomagnetic waves $\vec{k} \perp \vec{\nabla}T$

At $\vec{k} \perp \vec{\nabla}T$ coordinate system

$$k_1 \neq 0,\ k_2 = k_3 = 0,\ k_1\frac{\partial T}{\partial x_1} = \left(\vec{k}\vec{\nabla}T\right) = 0,\ \frac{\partial T}{\partial x_2} \neq 0,\ \frac{\partial T}{\partial x_3} = 0\ . \tag{18}$$

With this choice from (17)

$$E_i' = \left(A\varsigma_{il}k_lk_k + B\varsigma_{ik} + \frac{c\Lambda_{ik}'}{\omega}k_l\frac{\partial T}{\partial x_k}\right)E_k'$$
$$E_i' = \delta_{ik}E_k', \delta_{ik} = \begin{cases}1, at\ i = k\\ 0, at\ i \neq k\end{cases} \tag{19}$$
$$A = \frac{ic^2}{4\pi\omega}, B = \frac{i\left(\omega^2 - c^2k^2\right)}{4\pi\omega}$$

was obtained.

Let's denote

$$\varphi_{ik} = A\varsigma_{il}k_lk_k + B\varsigma_{ik} + \frac{c\Lambda_{il}'}{\omega}k_l\frac{\partial T}{\partial x_k}. \tag{20}$$

From (19)

$$\varphi_{11} = \frac{i\omega}{4\pi}\varsigma_{11}, \varphi_{12} = i\Omega\varsigma_{12} + \frac{\omega_{12}}{\omega}, \varphi_{13} = i\Omega\varsigma_{13}, \Omega = \frac{\omega^2 - c^2k^2}{4\pi\omega}$$
$$\varphi_{21} = \frac{i\omega}{4\pi}\varsigma_{21}, \varphi_{22} = i\Omega\varsigma_{22} + \frac{\omega_{22}}{\omega}, \varphi_{23} = i\Omega\varsigma_{23} \tag{21}$$
$$\varphi_{31} = \frac{i\omega}{4\pi}\varsigma_{31}, \varphi_{32} = i\Omega\varsigma_{32} + \frac{\omega_{32}}{\omega}, \varphi_{33} = i\Omega\varsigma_{33}$$

was obtained.

Substituting (21) into (19), we got follow dispersion equation in tensor form

$$\left|\left(\varphi_{ik} - \delta_{ik}\right)\right| = 0 \tag{22}$$

or

$$\varphi_{31}\varphi_{12}\varphi_{23} + \varphi_{21}\varphi_{32}\varphi_{13} + \\ + \left(\varphi_{11} - 1\right)\left(\varphi_{22} - 1\right)\left(\varphi_{33} - 1\right) - \\ - \varphi_{31}\varphi_{13}\left(\varphi_{22} - 1\right) - \\ - \varphi_{32}\varphi_{23}\left(\varphi_{11} - 1\right) - \varphi_{21}\varphi_{12}\left(\varphi_{33} - 1\right) = 0 \tag{23}$$

was obtained.

Dispersion equation (23) taking into account (21) has the form

$$\sum_{i=1}^{5} u_i\omega_i + u_0 = 0. \tag{24}$$

The fifth degree relative to the purity of vibrations of equation (24) has a very complex form. Simplifying equation (24) requires a lot of mathematical approximations. However, equation (24) is easily simplified depending on the tensor ς_{ik}.

If

$$\varsigma_{12}=\varsigma_{13}=\varsigma_{22}=\varsigma_{23}=\varsigma_{32}=\varsigma_{33} \\ \varsigma_{11}=\varsigma_{21}=\varsigma_{31}. \tag{25}$$

The dispersion equation has the form:

$$\frac{1}{2\pi}\left(\frac{i\varsigma_{11}}{2}+\varsigma_{22}\right)\omega^2+\left[\frac{i\varsigma_{11}}{4\pi}(\omega_{13}+\omega_{12}-\omega_{22})-1\right]\omega+ \\ +\omega_{22}+\omega_{33}-\frac{ic^2k^2}{2\pi}\varsigma_{22}=0. \tag{26}$$

Substituting for frequency

$$\omega=\omega_0+i\gamma,\ \gamma<<\omega_0.$$

We can get from (26) next two equation for definition ω_0 and γ

$$\frac{1}{2\pi}\varsigma\omega_0^2-\frac{1}{4\pi}\varsigma\omega_0\gamma-\frac{\varsigma}{4\pi}(\omega_{13}+\omega_{12}-\omega_{22})\gamma- \\ -\omega_0+\omega_{22}+\omega_{33}=0 \tag{27}$$

$$\frac{1}{4\pi}\varsigma\omega_0^2+\frac{1}{2\pi}\varsigma\omega_0\gamma+\frac{\varsigma}{4\pi}(\omega_{13}+\omega_{12}-\omega_{22})\omega_0- \\ -\gamma-\frac{c^2k^2\varsigma}{2\pi}=0. \tag{28}$$

From (28)

$$\gamma=\frac{1}{4\pi}\varsigma\omega_0^2+\frac{\varsigma}{4\pi}(\omega_{13}+\omega_{12}-\omega_{22})\omega_0-\frac{c^2k^2\varsigma}{2\pi}. \tag{29}$$

Substituting (29) into (27)

$$\frac{1}{2\pi}\varsigma\omega_0^2-\frac{\varsigma}{4\pi}(\omega_{13}+\omega_{12}-\omega_{22})\left[\frac{1}{4\pi}\varsigma\omega_0^2+\right. \\ \left.+\frac{\varsigma}{4\pi}(\omega_{13}+\omega_{12}-\omega_{22})\omega_0-\frac{c^2k^2\varsigma}{2\pi}\right]- \\ -\omega_0+\omega_{22}+\omega_{33}=0. \tag{30}$$

From (30) is shown, at $\omega_{22}=\omega_{13}+\omega_{12}$, $\varsigma=\dfrac{\pi}{2(\omega_{22}+\omega_{33})}$.

Thus, real part of frequency of thermomagnetic wave is

$$\omega_0=2(\omega_{22}+\omega_{33}). \tag{31}$$

Substituting (31) into (29)

$$\gamma=\frac{1}{2}(\omega_{22}+\omega_{33})-\frac{1}{2}\frac{c^2k^2}{\omega_{22}+\omega_{33}} \tag{32}$$

was obtained.

From (32) it is shown wave with frequency (31) will rise is

$$\omega_{22}+\omega_{33}>>ck.$$

Longitudinal thermomagnetic wave $\vec{k}\parallel\vec{\nabla}T$

At condition $\vec{k}\parallel\vec{\nabla}T$ axis can be chosen like

$$k_1 = k,\ k_2 = k_3 = 0,\ \frac{\partial T}{\partial x_2} = \frac{\partial T}{\partial x_3} = 0, k_1 \frac{\partial T}{\partial x_1} \neq 0\ \ \frac{\partial T}{\partial x_2} \neq 0,\ \frac{\partial T}{\partial x_3} = 0. \quad (33)$$

At this choose, tensors φ_{ik} are

$$\varphi_{11} = \frac{i\omega}{4\pi}\varsigma_{11}, \varphi_{12} = i\Omega\varsigma_{12} + \frac{\omega_{12}}{\omega}, \varphi_{13} = i\Omega\varsigma_{13} + \frac{\omega_{13}}{\omega}$$
$$\varphi_{21} = \frac{i\omega}{4\pi}\varsigma_{21}, \varphi_{22} = i\Omega\varsigma_{22} + \frac{\omega_{22}}{\omega}, \varphi_{23} = i\Omega\varsigma_{23} + \frac{\omega_{23}}{\omega} \quad (34)$$
$$\varphi_{31} = \frac{i\omega}{4\pi}\varsigma_{31}, \varphi_{32} = i\Omega\varsigma_{32} + \frac{\omega_{32}}{\omega}, \varphi_{33} = i\Omega\varsigma_{33} + \frac{\omega_{33}}{\omega}.$$

Substituting (34) into (23) we got follow dispersion equation

$$-\frac{i}{64\pi^3}\left(\varsigma_{31}\varsigma_{21}\varsigma_{23} + \varsigma_{31}\varsigma_{13}\varsigma_{32}\right)\omega^4 + -\frac{1}{64\pi^2}\left(\varsigma_{11}\varsigma_{22} + \varsigma_{11}\varsigma_{33}\right)\omega^3 +$$
$$+\left[\frac{i}{64\pi^3}\left(\varsigma_{31}\varsigma_{21}\varsigma_{23} + 2\varsigma_{31}\varsigma_{13}\varsigma_{32}\right)c^2k^2 + \frac{i}{4\pi}\left(\varsigma_{11} + \varsigma_{22} + \varsigma_{33}\right) +\right.$$
$$\left.+\frac{\omega_{22}}{16\pi^2}\left(\varsigma_{11}\varsigma_{33} + 2\varsigma_{31}\varsigma_{13}\right)\right]\omega^3 + \left[-\frac{1}{64\pi^3}\left(\varsigma_{11}\varsigma_{22} + \varsigma_{11}\varsigma_{33}\right)c^2k^2 - 1 +\right. \quad (35)$$
$$\left.+ + \frac{i\omega_{22}}{4\pi}\left(\varsigma_{11} - \varsigma_{21}\right)\right]\omega - \frac{ic^2k^2}{4\pi}\left(\frac{1}{64\pi^2}\varsigma_{31}\varsigma_{13}\varsigma_{32}c^2k^2 - \varsigma_{22} - \varsigma_{33}\right) -$$
$$-\frac{1}{64\pi^2}\omega_{22}c^2k^2\left(\varsigma_{11}\varsigma_{33} + \varsigma_{13}\varsigma_{31}\right) - \omega_{22} = 0.$$

Solving (24) in general is difficult and almost impossible, so we choose the following value. The tensors have the same values in all directions ς_{ik}, then from (35)

$$x^4 + 16\pi i x^3 + \left(-48\pi^2 + 12\pi i\omega_{22}\varsigma\right)x^2 +$$
$$+64\pi^3 i\left(-1 + i\frac{\omega_{22}\varsigma}{2\pi}\right)x - \omega_{22}\varsigma = 0 \quad (36)$$
$$\omega = ck\varsigma x$$
$$ck\varsigma << 1$$

was obtained.

Assuming $x = x_0 + ix_1, x_1 << 0$ and separating real and imaginary parts in (36), we got

$$x_0^4 - 48\pi x_0^2 x_1 - 48\pi^2 x_0^2 - 24\pi\omega_{22}\varsigma x_0 x_1 +$$
$$+64\pi^3\frac{\omega_{22}\varsigma}{2\pi}x_0 + 64\pi^3 x_1 - \omega_{22}\varsigma = 0 \quad (37)$$

$$4x_0^3 x_1 + 16\pi x_0^3 - 96\pi^2 x_0 x_1 + 12\pi\omega_{22}\varsigma x_0^2 -$$
$$-64\pi^3 x_0 - 32\pi^2\omega_{22}\varsigma x_1 = 0. \quad (38)$$

From (37-38) it is shown, at $x_0 >> 1$ thermomagnetic wave doesn't exist. Thermomagnetic waves exist at $x_0 << 1$ (26-27)

$$-24\pi\omega_{22}\varsigma x_0 x_1 + 32\pi^2\omega_{22}\varsigma x_0 + 64\pi^3 x_1 - \omega_{22}\varsigma = 0. \quad (39)$$

Then we got

$$-96\pi^2 x_0 x_1 - 64\pi^3 x_0 - 32\pi^2\omega_{22}\varsigma x_1 = 0. \quad (40)$$

From (40) $x_0 = -\frac{\omega_{22}\varsigma}{2\pi}x_1$, $x_0 < \frac{1}{3\pi}\omega_{22}\varsigma$ was obtained.

Substituting into (39) values of imaginary and real part of the frequency are

$$x_1 = \frac{2\pi}{3}$$
$$\omega_1 = \frac{2\pi}{3} \cdot \frac{1}{\varsigma} \quad (41)$$
$$\omega_0 = -\frac{\omega_{22}}{2\pi} \cdot \frac{2\pi}{3} = -\frac{\omega_{22}}{3}.$$

From (41) it is shown, wave with frequency $\omega_0 = -\frac{\omega_{22}}{3}$ is growing.

$$\frac{\omega_0}{\omega_1} = \frac{\omega_{22}}{3} \cdot \frac{3\varsigma}{2\pi} = \frac{\omega_{22}\varsigma}{2\pi} << 1 \quad (42)$$
$$\omega_{22}\varsigma << 2\pi$$

Conclusion

Thus, thermomagnetic waves with different frequencies are excited in anisotropic conducting media. These waves can be longitudinal $\vec{k} \parallel \vec{\nabla} T$ and transverse $\vec{k} \perp \vec{\nabla} T$. Condition (31) is one of the practically possible conditions for solving systems of equations (27-28) in anisotropic media. The conditions we chose proved that the propagation of thermomagnetic waves significantly depends on the chosen directions. These directions are different for transverse and longitudinal thermomagnetic waves. The characteristic dispersion equations for transverse thermomagnetic waves $\vec{k} \perp \vec{\nabla} T$ (30) and for longitudinal thermomagnetic waves $\vec{k} \parallel \vec{\nabla} T$ (35) under other selected conditions φ and k can change significantly. We studied transverse and longitudinal thermomagnetic waves in specific values of the tensors φ and k. The work theoretically studies the conditions for the excitation of thermomagnetic waves. The excited wave is increasing. The directions of external fields play a significant role in creating growing waves. Thermomagnetic waves are excited in the longitudinal and transverse directions depending on the value of the inverse conductivity tensor and the frequencies of these thermomagnetic waves were calculated in both the longitudinal directions and the transverse directions. The growth increments of these waves are determined. In both cases (longitudinal $\vec{k} \parallel \vec{\nabla} T$ and transverse $\vec{k} \perp \vec{\nabla} T$) the thermomagnetic frequencies are low. With the small value of the electrical conductivity tensor, the frequency of propagation of thermomagnetic waves is different. The theory is constructed at conditions $H_0 = 0$, i.e. without an external magnetic field.

Thus, the excitation of longitudinal $\vec{k} \parallel \vec{\nabla} T$ and transverse $\vec{k} \perp \vec{\nabla} T$ thermomagnetic waves in anisotropic conducting media occurs in selected directions in the media. To experimentally observe thermomagnetic waves, it is sufficient to measure the conductivity of the medium in various directions.

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Сыртқы магнит өрісі жоқ температура градиенті болған кезде анизотропты өткізгіш ортада көлденең және бойлық термомагниттік толқындардың қозуы

Анизотропты өткізгіш орталарда магнит өрісі мен температура градиентінің әртүрлі бағыттарындағы қозғалған термомагниттік толқындар анизотропты ортаның бағытына айтарлықтай тәуелді. Термомагниттік толқынның жиілігіне байланысты электр өткізгіштік тензорының теориялық есептеулері ғылыми қызығушылық тудырады. Мақалада толқынның жиіліктері табылған, бойлық ($\vec{k} \parallel \vec{\nabla} T$) және көлденең ($\vec{k} \perp \vec{\nabla} T$) бағыттардағы бұл толқындардың жиіліктері мен өсу жылдамдығы сыртқы магнит өрісіне әртүрлі тәуелді болатыны дәлелденді. Авторлар термомагниттік толқындардың қозу жағдайларын теориялық тұрғыдан зерттеді. Үлгіде өсіп келе жатқан толқындардың пайда болуында сыртқы өрістердің бағыты маңызды рөл атқаратыны көрсетілген. Электр өткізгіштік тензорының мәніне байланысты термомагниттік толқындар бойлық (яғни $\vec{k} \parallel \vec{\nabla} T$) және көлденең (яғни $\vec{k} \perp \vec{\nabla} T$) толқындық бағытта қозғалатыны анықталған. Осы термомагниттік толқындардың жиіліктері бойлық және көлденең бағытта да есептелді. Бұл толқындардың өсу жылдамдығы кері электр өткізгіштік тензорының σ_{ik} мәндерімен анықталады. Қозған толқынның табиғаты бойынша негізінен термомагниттік екені дәлелденді. Теориялық тұрғыдан алынған дисперсия теңдеуі тербеліс жиілігіне қатысты алгебралық үлкен қуаттарға ие. Дисперсиялық теңдеудің екі жағдайында (бойлық $\vec{k} \parallel \vec{\nabla} T$ және көлденең $\vec{k} \perp \vec{\nabla} T$ бағыттар) термомагниттік жиіліктер төмен жиілікте болатын мүшелерді қамтиды. Егер электр өткізгіштік тензорының мәні σ_{ik} бірдей болса, термомагниттік толқындардың таралу жиіліктері әртүрлі болатыны дәлелденді. $H_0 = 0$ теориясы сыртқы магнит өрісінсіз құрастырылған. Сыртқы магнит өрісі болған жағдайда термомагниттік толқындардың қозу шарттары және олардың өсу жағдайлары айтарлықтай өзгереді.

Кілт сөздер: жиілік, өсу, термомагниттік толқындар, көлденең толқындар, бойлық толқындар, өсу, электр өткізгіштік тензоры, кері тензор.

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Возбуждение поперечных и продольных термомагнитных волн в анизотропных проводящих средах при наличии температурного градиента без внешнего магнитного поля

В анизотропных проводящих средах возбуждаемые термомагнитные волны при различных ориентациях магнитного поля и градиента температуры существенно зависят от направления анизотропной среды. Научный интерес представляют теоретические расчеты тензора электропроводности σ_{ik} в зависимости от частоты термомагнитной волны. В настоящей статье были найдены частоты термомагнитной волны и доказано, что при продольном ($\vec{k} \parallel \vec{\nabla} T$) и поперечном ($\vec{k} \perp \vec{\nabla} T$) направлениях частоты и скорость роста этих волн по-разному зависят от внешнего магнитного поля. Авторами теоретически исследованы условия возбуждения термомагнитных волн. Указано, что направление внешних полей играет существенную роль в возникновении растущих волн в образце. Показано, что в зависимости от значения тензора электропроводности термомагнитные волны возбуждаются в продольном (то есть $\vec{k} \parallel \vec{\nabla} T$) и поперечном (то есть $\vec{k} \perp \vec{\nabla} T$) направлениях волны. Рассчитаны частоты этих термомагнитных волн как в продольном, так и в поперечном направлении. Скорость роста этих волн определяется значениями обратного тензора электропроводности σ_{ik}. Доказано, что возбуждаемая волна имеет преимущественно термомагнитную природу. Теоретически полученное дисперсионное уравнение имеет алгебраически большие степени относительно частоты колебания. Дисперсионное уравнение в обоих случаях (продольном $\vec{k} \parallel \vec{\nabla} T$ и поперечном $\vec{k} \perp \vec{\nabla} T$ направлениях) содержит члены, в которых присутствуют термомагнитные частоты в низкой степени частоты. Доказано, что если значение тензора электропроводности σ_{ik} одинаковое, то частоты распространения термомагнитных волн различны. Теория построена без внешнего магнитного поля $H_0 = 0$. При наличии внешнего магнитного поля условия возбуждения термомагнитных волн и, конечно, условия их роста существенно меняются.

Ключевые слова: частота, приращение, термомагнитные волны, поперечные волны, продольные волны, рост, тензор электропроводности, обратный тензор.

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