ЖЫЛУФИЗИКАСЫ ЖӘНЕ ТЕОРИЯЛЫҚ ЖЫЛУТЕХНИКАСЫ ТЕПЛОФИЗИКА И ТЕОРЕТИЧЕСКАЯ ТЕПЛОТЕХНИКА THERMOPHYSICS AND THEORETICAL THERMOENGINEERING

Article

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Mathematical model and numerical calculation of the movement of oil products in helicoidal heat exchangers

Heating of oil and oil products is widely used to reduce energy losses during transportation. An approach is developed to determine the effective length of the heat exchanger and the temperature of the cold coolant (oil) at its outlet in the case of a strong dependence of oil viscosity on temperature. The method of the log-mean temperature difference, modified for the case of variable viscosity, and the methods of computational fluid dynamics (CFD) are used for calculations. The results of numerical calculations are compared with the data obtained on the basis of a theoretical approach at a constant viscosity. When using a theoretical approach with a constant or variable viscosity, the heat transfer coefficients to cold and hot coolants are found using criterion dependencies. The Reynolds-averaged Navier-Stokes (RANS) and a turbulence model that takes into account the laminar-turbulent transition are applied. In the case of variable oil viscosity, a transition from the laminar flow regime to the turbulent one is manifested, which has a significant effect on the effective length of the heat exchanger. The obtained results of CFD calculations are of interest for the design of heat exchangers of a new type, for example, helicoidal ones.

Keywords: heat transfer, numerical calculation, helical heat exchanger, oil, hydrodynamics, coolants, heat flow, laminar-turbulent transition, Navier-Stokes equation, flow turbulization.

Introduction

Helicoidal heat exchangers are designed with profiled pipes and fans with a screw profile, which improves the conditions for heat exchange. In works [1-2], modeling and calculation of the hydrodynamics of heat carriers (water, oil) flowing through smooth pipes are given. The obtained results of numerical calculations are used to find optimal ways to intensify the heat transfer process [3–5]. Research shows that the influence of the viscosity of the pumped oil on the hydraulic properties of the pipeline decreases when pumping in a developed turbulent regime.

Heat exchangers are used in many applications, with efficient heat exchangers being a basic requirement of the industry. Efforts to increase heat transfer, increase heat transfer rates, reduce the size of heat exchangers and improve efficiency since the beginning of global industrialization. The higher heat transfer potential of spiral coils is of interest to many researchers who study the fluid dynamics inside the spiral tubes of the heat exchanger they serve. The bending of the pipe causes the application of centrifugal force, which leads to the formation of a secondary flow due to the curvature of the pipe. The centrifugal force is controlled by the centrifugal force, which is determined by the curvature of the twisting caused by the liquid is affected by the pitch or angle of the coil. Liquids arising from the outside of the pipe move at a higher speed than those flowing inside the pipe, which is caused by the curvature affecting the speed of movement [6]. In work [7], spiral coils in heat exchangers, and their performance and

efficiency were analyzed by studying factors affecting the performance and efficiency of a spiral heat exchanger, such as the coefficient of curvature and other factors. The helical coil in heat exchangers (HCHE) provides higher heat transfer speeds and efficiency than straight pipes and other heat exchangers due to the development of secondary flow inside the spiral tube, while the heat transfer coefficient increases with increasing curvature coefficient (HCHE) at the same flow rates.

This article discusses the numerical calculation of a helicoidal heat exchanger. Exactly, the results of calculating the oil temperature and heat flow at the outlet of the pipe winding on the surface are presented. When solving this problem, the number of twists N, the heat transfer coefficient, the flow velocity and the temperature of the oil at the pipe inlet varied as input parameters.

Calculation method

The problem is solved in the Ansys Fluent software package, which uses stationary Navier-Stokes equations averaged by Reynolds (Navier–Stokes equations averaged by Reynolds).

The numerical calculation was carried out by the finite Volume method FVM (Finite Volume Method) using an uneven grid in the computational domain. Schematically, the calculated pipe with windings and the calculated grid on it is shown in Figure 1.





Figure 1. Calculation grid of the pipe with windings

Figure 2. Calculation grid for a quarter of a pipe

To save computing resources (time and memory), the calculation was carried out on the fourth part of the pipe, since the pipe is axisymmetric. Figure 2 shows a quarter of the calculated grid, which shows the thickening of the grid at the pipe boundary, where there are the greatest gradients of flow parameters. More details about unstructured grids are described in [8].

The basic Navier-Stokes equations and the k- ω SST equations were used for the calculation [9-10]. The continuity equation:

$$\frac{\partial v_j}{\partial x_j} = 0. \tag{1}$$

The equation of momentum:

$$v_j \frac{\partial v_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i} \left[(\nu + \nu_t) \frac{\partial v_i}{\partial x_j} \right].$$
(2)

The energy equation:

$$v_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_i} \left[\left(\frac{\nu}{Pr} + \frac{\nu_t}{Pr_t} \right) \frac{\partial T}{\partial x_j} \right].$$
(3)

Here ρ is the density, and v_i is the component of velocity in the direction of coordinates — x_i , p — pressure, T — temperature, v, v_t — molecular and turbulent viscosity of liquid, Pr, Pr_t — Prandtl number for laminar and turbulent, respectively.

The k- ω turbulence model takes into account two more transport equations. The Reynolds number $Re_{\theta t}$ and the momentum of the transition beginning are two equations written for it, and the second one is obtained for the flow regime intermittency γ of the transformation process. Transmission equation for pulse thickness and Reynolds number and intermittency transport equation are as follows:

Transmission equation for pulse thickness and Reynolds number:

$$\nu_j \frac{\partial Re_{\theta t}}{\partial x_j} = P_{\theta t} + \frac{\partial}{\partial x_j} \bigg[\sigma_{\theta t} (\nu + \nu_t) \frac{\partial Re_{\theta t}}{\partial x_j} \bigg].$$
(4)

Intermittency transport equation:

$$v_j \frac{\partial \gamma}{\partial x_j} = P_\gamma - \mathcal{E}_\gamma + \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_\gamma} \right) \frac{\partial \gamma}{\partial x_j} \right].$$
(5)

Here $P_{\theta t}$ — time derivative of the Reynolds number with respect to the momentum loss thickness, P_{γ} and E_{γ} — time-specific formation and dissipation of intermittency conditions, $\sigma_{\theta t}$ and σ_{γ} are constants of the model.

Results of calculations and discussions

Since the system of initial equations is nonlinear, an iterative approach was used for their numerical solution, in which the linearized Navier-Stokes equations were solved. According to the results of calculations, the average mass temperature and heat flow of oil at the outlet of the pipe are obtained. To solve this problem, the following initial parameters were used: the radius of the pipe $R_1 = 0,006 m$, the radius of the groove $R_2 = 0,001 m$, the length of the pipe $L_1 = 1m$, the heat transfer coefficient $\alpha_1 = 1000 W/(m^2 \cdot K)$, the temperature of the washing liquid (water) $T_2 = 423K$, the speed oil flow $v_1 = 4 m/s$, oil temperature at the pipe inlet $T_{in_1} = 313K$.

Boundary conditions of the third type were applied (heat transfer coefficients and the washing liquid's temperature were measured on the pipe's surface). The number of windings on the pipe surface is determined by the number of twists N, which in this problem ranged from 1 to 40 with an interval of 5.

A graph illustrating the relationship between the number of twists N and the average mass temperature of oil at the pipe's exit is presented in Figure 3. The image illustrates how the temperature of the oil at the outlet rises as the number of twists increases, intensifying the heat exchange between the heat carriers.



Figure 3. Dependence of the average mass temperature at the outlet of the heat exchanger on the twist N

A graph illustrating the relationship between the number of twists N and the heat flow Q through the pipe's surface from the side of the washing liquid is displayed in Figure 4. Here, the amount of heat flow also increases with an increase in the number of twists on the pipe.

The increase in the average mass temperature of oil at the outlet of the heat exchanger (Fig. 3) and the increase in the value of the heat flow (Fig. 4) with an increase in the number of twists of the N tube is explained by turbulence of the flow due to its twisting. When the flow is turbulated, the process of diffusion of liquid particles intensifies and the heat exchange between particles intensifies. This process will occur the more intensively, the more the number of twists of the tube.

Further, heat transfer coefficients α were set in the range from 1000 W/($m^2 \cdot K$) to 6000 W/($m^2 \cdot K$) with a constant number of twists equal to 10.



Figure 4. The dependence of the heat flow Q on the twist N

Consequently, Figure 5 displays a graph that illustrates how the heat transfer coefficient α and the average mass temperature of oil at the heat exchanger's outlet rely on each other. The graphic illustrates how the average mass temperature at the outlet climbs dramatically as the heat transfer coefficient rises. This is explained by the natural process of increasing the temperature of the liquid (oil) with increasing heat transfer from the external washing liquid (water) to the surface of the inner tube, and from it to the inner liquid, which increases its average mass temperature.



Figure 5. Dependence of the average mass temperature at the outlet of the heat exchanger on the heat transfer coefficient α

The influence of the heat flow through the pipe surface on the heat transfer α is shown in Figure 6. A rise in the heat transfer coefficient causes the heat flow to increase three times from its starting value. The intensification of heat transfer between heat carriers is well described by this process. This happens in a similar way to the previous case, i.e. with an increase in heat transfer from the external washing fluid (water) to the surface of the inner tube, the heat flow towards the oil increases.



Figure 6. The dependence of the heat flow on the heat transfer coefficient



Figure 7. Dependence of the average mass temperature at the outlet of the heat exchanger on the oil flow rate at the inlet

Figures 7 and 8 show graphs of the dependences of the average mass temperature and heat flow through the surface on the oil flow velocity v at the inlet. With an increase in the oil flow rate at the inlet to the pipe, the diffusion process decreases, and, as a result, the main flow rate will increase compared to the fluctuation component at a given number of turns N. This in turn leads to a decrease (slightly) in the flow velocity along the tube, including at the outlet of the tube (Fig. 7). Based on the above, the heat flow will, on the contrary, increase depending on the increase in the flow rate of oil at the inlet (Fig. 8). Both figures show that at approximately a speed of v = 6 m/s, there is a sharp change in the graph. This is due to the fact that at a velocity of v = 6 m/s, the Reynolds number *Re* reaches 2880, where a laminar-turbulent transition mode occurs.

The oil temperature at the heat exchanger's outlet increases when the temperature at the pipe's input is changed from 313 K to 363 K. Figure 9 displays the dependence diagram for this change. The graph indicates that this dependence is linear. This is physically explained very simply — with a higher oil temperature at the inlet, we get a higher oil temperature at the outlet.



Figure 8. Dependence of the heat flow on the oil flow velocity at the inlet



Figure 9. Dependence of the average mass temperature at the outlet of the heat exchanger on the temperature at the inlet

Figure 10 illustrates the opposite situation by demonstrating how the inlet temperature affects the heat flow on the pipe surface. It is evident that when the oil inlet temperature rises, the heat flows the difference between the oil's temperature and the washing liquid's temperature decreases. Since the heat flow is determined by the temperature difference of the media, with an increase in the temperature of the oil at the inlet, the temperature difference between the two liquids decreases (at a constant water temperature), which leads to a decrease in the heat flow on the surface of the tube.

Figure 11 shows comparative graphs of the dependence of the mass-average temperature of oil at the outlet for a tube with a smooth [11] and helicoidal surface with a number of twists N from 10 to 40 with a step of 10. It can be seen from the figure that the temperature of oil in a tube with coils is higher along its entire length compared to a tube with a smooth surface, while reaching a maximum temperature difference of up to 9.5%, which is explained by turbulence of the flow, as well as with an increase in heat flow through the surface due to a slight increase in the surface of the tube due to recesses.



Figure 10. Dependence of the heat flow on the pipe surface on the inlet temperature



Figure 11. Comparison of the dependence of the average mass temperature of oil at the outlet for a tube with a smooth and helicoid surface with a number of twists N = 10-40.

Conclusion

After using a coiled tube as an example, we can thus deduce from the foregoing facts that heat exchangers with helicoid geometries have superior heat transfer capabilities than heat exchangers with smooth tubes. This is because the turbulence caused by twisted windings in the oil flow increases the heat exchange between liquid layers as a result of the diffusion process. Moreover, the oil temperature may rise as a result of a tiny rise in the tube's surface brought on by recesses, which enhances heat transfer through the surface. It can also be noted here that with variable viscosity of oil, the transition from laminar to turbulent mode is manifested, while this effect is not taken into account when calculating for constant viscosity.

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Н.Ж. Жәйшібеков, Д.Е. Құрманова, А.С. Жұманбаева

Геликоидты жылу алмастырғыштардағы мұнай өнімдері қозғалысының математикалық моделі және сандық есебі

Мақала геликоидты жылу алмастырғыштағы жылу алмасу процестерін сандық есептеуге арналған. Жылу беруді жақсарту үшін геликоидты жылу алмастырғыштар профильді құбырларды және бұрандалы кырлы профильді пайдаланады. Тасымалдау кезінде энергия шығынын азайту үшін мұнай мен мұнай өнімдерін қыздыру кеңінен қолданылады. Мұнай тұтқырлығының температураға қатты тәуелділігі жағдайында жылу алмастырғыштың тиімді ұзындығын және оның шығуындағы суық салқындатқыштың (мұнайдың) температурасын анықтауға арналған тәсіл әзірленді. Есептеулер үшін тұтқырлықтың өзгермелі жағдайына өзгертілген температураның орташа айырмашылығының логарифмдік әдісі және есептеу гидродинамикасы (CFD) әдістері пайдаланылған. Сандық есептеулердің нәтижелері тұрақты тұтқырлық кезінде теориялық тәсіл негізінде алынған мәліметтермен салыстырылған. Тұрақты немесе өзгермелі тұтқырлығы бар теориялық тәсілді қолданған кезде суық және ыстық салқындатқыш сұйықтықтардың жылу беру коэффициенттері критериалды тәуелділіктерді қолдану арқылы анықталады. Рейнольдс бойынша орташа Навье-Стокс әдісі (RANS) және ламинарлытурбулентті ауысуды ескеретін турбуленттілік моделі қолданылған. Майдың өзгермелі тұтқырлығы жағдайында ламинарлы ағын режимінен турбулентті режимге ауысу көрінеді, бұл жылу алмастырғыштың тиімді ұзындығына айтарлықтай әсер етеді. Алынған CFD есептеулерінің нәтижелері жылу алмастырғыштардың жаңа түрін, мысалы, геликоидты жобалауға қызығушылық тудырады. Жұмыста ағынның параметрлеріне, сондай-ақ түтіктің бұралу санына байланысты түтіктің шығуындағы мұнайдың массалық-орташа температурасы, кіріс және шығыс температурасы, жылдамдығы, құбыр бетіндегі жылу ағынының есептеу нәтижелері берілген.

Кілт сөздер: жылу беру, сандық есептеу, геликоидты жылу алмастырғыш, май, гидродинамика, салқындатқыш сұйықтықтар, жылу ағыны, ламинарлы-турбулентті ауысу, Навье-Стокс теңдеуі, ағынның турбулизациясы.

Н.Ж. Джайчибеков, Д.Е. Курманова, А.С. Жуманбаева

Математическая модель и численный расчет движения нефтепродуктов в теплообменных аппаратах геликоидной формы

Статья посвящена численному расчету теплообменных процессов в теплообменнике геликоидной формы. В геликоидных теплообменниках использованы профилированные трубки и ребра винтового профиля, что улучшает теплообмен. Нагрев нефти и нефтепродуктов широко применяется для снижения потерь энергии при транспортировке. Разработан подход для определения эффективной длины теплообменника и температуры холодного теплоносителя (масла) на его выходе в случае сильной зависимости вязкости масла от температуры. Для расчетов используются метод логарифмической средней разности температур, модифицированный для случая переменной вязкости, и методы вычислительной гидродинамики (CFD). Результаты численных расчетов сравниваются с данными, полученными на основе теоретического подхода при постоянной вязкости. При применении теоретического подхода с постоянной или переменной вязкостью коэффициенты теплоотдачи холодным и горячим охлаждающим жидкостям определяются с использованием критериальных зависимостей. Используются усредненный по Рейнольдсу метод Навье-Стокса (RANS) и модель турбулентности, учитывающая ламинарно-турбулентный переход. В случае переменной вязкости масла проявляется переход от ламинарного режима течения к турбулентному, что оказывает существенное влияние на эффективную длину теплообменника. Полученные результаты CFD-расчетов представляют интерес для проектирования теплообменников нового типа, например геликоидальных. В работе приведены результаты расчетов среднемассовой температуры нефти на выходе и теплового потока на поверхности трубки в зависимости от параметров потока, а также от числа закруток трубки.

Ключевые слова: теплопередача, численный расчет, геликоидный теплообменник, масло, гидродинамика, охлаждающие жидкости, тепловой поток, ламинарно-турбулентный переход, уравнение Навье-Стокса, турбулизация потока.

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