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# Evaluation of influence of thermoplastic slurry flow conditions on heat transfer coefficient during beryllium ceramic formation

The article presents the results of calculation of the influence of thermoplastic slurry flow conditions on the total heat transfer coefficient when forming ceramic products. Methods of detailed description of casting processes and thermal calculations have been developed to ensure reliable correspondence of calculated data and experimental results implemented on the basis of calculational experiment. The modeling of physical processes occurring during the formation of products allows you to discover new opportunities for improving the quality of castings by allowing you to more closely track the change in the temperature-phase fields of the process and clearly present the solidification kinetics depending on the casting modes. The speed of heat removal from the casting during the solidification period determines the rate of movement of the slurry, along with the temperature field on which the width of the transition region depends. These factors have a direct influence on the formation of the structure of beryllium ceramics.

The study of the heat exchange process in the formation of ceramic products depending on temperature, heat at phase transition is the main task, since they largely determine the technological and operational characteristics of beryllium ceramics. The design data allows to determine the optimal conditions of the ceramic casting process and to obtain a solidified product with a uniform structure at the outlet.

*Keywords:* thermoplastic slurry, beryllium oxide, hydrodynamics, heat exchange, casting process, formation, solidification process, ceramics.

### Introduction

High-density ceramics made of beryllium oxide are widely used in various fields of modern technology due to a number of valuable properties and, above all, unique thermal conductivity. The slurry used to make MIM (Metal injection molding) ceramics is a dispersion system in which one phase is a solid mineral powder and the other is a thermoplastic binder [1]. The high thermal conductivity of the beryllium oxide powder during the molding step results in increased "rigidity" of the casting systems, which makes it difficult to control structure formation during slurry movement. The complexity of the process under study is that consideration should take into account factors such as the dependence of thermophysical properties on temperature, the phase transformations of liquid suspensions into a solid state, the heat of crystallization, and a sharp change in the temperature boundary conditions on the cooling circuits [2].

Creation of an effective method of controlling the process of forming thermoplastic slurry from beryllium oxide with uniform properties is a fundamental problem of foundry [3, 4]. The relevance of the problem lies in determining the optimal conditions for obtaining high-quality ceramic products in the casting process by mathematical modeling. The use of a mathematical model of physical processes occurring during the formation of products makes it possible to discover a new resource for improving the quality of castings due to the possibility of more detailed tracking of changes in temperature-phase fields during cooling [4] and to clearly present the kinetics of solidification depending on casting modes and casting configuration features.

In this regard, the tasks of evaluating the effect of the thermoplastic slurry flow *conditions* on the total heat transfer coefficient from the hot slurry to the cooling agent have been considered, in particular, heat transfer coefficients of external and internal convective flows have been determined for calculating the total heat transfer coefficient.

## Mathematical description of the concentric cylinder molding process

The flow and heat exchange of beryllium oxide thermoplastic slurry in the space between two concentric cylinders with radii  $r_1$  and  $r_2$  is considered (Fig. 1). Liquid slurry with initial temperature of 80°C flows into forming cavity, moving along annular gap, is cooled by water washing spinneret from outside [5]. Spinneret cooling circuit is divided into two parts, hot circuit temperature is indicated by  $t_1$ , cold circuit  $t_2$ . As it

moves, the slurry mass begins to gradually solidify, and at the exit from the cavity it acquires a structural shape in the form of a tube.



Figure 1. a) diagram of the slurry flow in the annular cavity of the casting unit
b) external diagram of the experimental installation: 1 – solidified slurry; 2 – mandrel;
3 – thermocouples; 4 – cooling circuit; 5 – hot slurry; 6 – solidification zone

The problem is investigated in a cylindrical coordinate system with zandr axes. Axis OZ is directed along axis of cavity, and axis OR is directed radially to it. The casting speed is directed vertically down the OZaxis. The transverse component of the speed arises from the heat exchange of the liquid slurry with the walls of the annular cavity. Heat exchange with the inner wall of the annular cavity is determined by the thermal conductivity of the mandrel material. Heat exchange with external wall of annular cavity is determined by heat transfer between slurry and cooling water in circuits through the spinneret wall [5]. In cooling circuits there is intensive circulation of water at the specified flow rate.

The rheological properties of the slurry depend on the temperature. During the solidification of the slurry, the heat of the phase transition is released [6-7]. Cooling of the slurry can lead to unequal temperature profile and rheological properties of the extruded casting. Solidification will begin on the side of the walls of the annular cavity, while in the central part of the cavity the slurry may be in a liquid state. As a result, the liquid slurry may be fed to compensate for the internal shrinkage of the volume in the cooled zone of the forming cavity. The following is a mathematical model of the process of forming the slurry in the annular cavity.

The movement of the slurry in the annular cavity is considered stationary, and a system of equations of hydrodynamics closed by the Bingham model of non-Newtonian fluid in a narrow channel is used to study it [5].

$$\rho u \frac{\partial u}{\partial z} + \rho v \frac{\partial u}{\partial r} = -\frac{dp}{dz} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \frac{\partial u}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} (r \tau_0), \tag{1}$$

$$\frac{\partial \rho u}{\partial z} + \frac{1}{r} \frac{\partial r \rho v}{\partial r} = 0.$$
<sup>(2)</sup>

In the limit of the solid plastic state of the slurry, the equation of motion (1) expresses the extrusion of the casting from the cavity and takes the form:  $-\frac{dp}{dt} - \frac{1}{2}\frac{\partial}{\partial t}(r\tau)$ 

$$-\frac{1}{dz} - \frac{1}{r} \frac{\partial r}{\partial r} (r t_0).$$
In the steady-state process of slurry solidification, the energy equation is as follows:  

$$\rho u c_p \frac{\partial t}{\partial z} + \rho v c_p \frac{\partial t}{\partial r} = \frac{\partial}{\partial z} \left( \lambda \frac{\partial t}{\partial z} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda \frac{\partial t}{\partial r} \right)$$
(3)

In equations (1)–(3), the following designations are taken: z, r — axial and radial coordinates; u, u are speed vector components; p,  $\rho$ , t,  $\tau_0$ ,  $c_p$ ,  $\mu$ ,  $\lambda$  — pressure, density, temperature, shear stress, apparent

heat capacity, viscosity and thermal conductivity coefficients of the slurry, respectively [6]. The condition for maintaining the mass flow rate determines the pressure gradient for extruding the slurry from the annular cavity [7]:

$$2\pi \int_{r_1}^{r_2} \rho u r dr = \pi (r_2^2 - r_1^2) \rho_0 u_0, \tag{4}$$

where  $r_1, r_2$  — radii of mandrel and spinneret, respectively.

Distribution of speed and temperature at the inlet of the annular cavity are taken as constant over the section, accordingly, all thermophysical properties of the slurry will be constant [5].

at 
$$z = 0$$
:  $u = u_0$ ,  $v = 0$ ,  $t = t_0$ . (5)

On the walls of the cavity in the area of the liquid state of the slurry for speed, the sticking conditions are set:

at 
$$z > 0, r = r_i: u_i = v_1 = 0, \ i = 1,2$$
 (6)

and in the area of the solid plastic state — conditions of non-flow and sliding:

at 
$$z > 0, r = r_i$$
:  $v_1 = 0, \left(\frac{\partial u}{\partial r}\right)_{r_i} = 0.$  (7)

It is considered that heat from the hot slurry is transferred to the walls of the spinneret and mandrel. Then the adiabatic condition can be set on the wall of the mandrel:

at 
$$z > 0$$
,  $r = r_1$ ,  $\frac{\partial t}{\partial r} = 0$ . (8)

Indicating the temperature of water in the hot and cold circuits through the  $t_1$ ,  $t_2$  boundary conditions can be set on the wall of the spinneret in the form of:

at 
$$z > 0$$
,  $r = r_2$ ,  $-\lambda \frac{\partial t}{\partial r} = k(t - t_i)$ ,  $i = 1,2$ , (9)

where k — coefficient of heat transfer on the spinneret wall [8–11].

At the outlet section of the cavity for temperature the condition is set:

at 
$$z = l$$
,  $\frac{\partial t}{\partial z} = 0$  (10)

The rheological properties of the slurry at the content of the binding  $\omega = 0,117$  depend on temperature, and are expressed by empirical formulas [6-7]. The heat of the phase transition to the energy equation (3) is determined by the method of apparent heat capacity:

$$c_p = c_s \cdot (1 - \alpha(\bar{t})) + c_l \cdot \alpha(\bar{t}) + H_{1 \to 2} \frac{d\alpha}{dt}, \qquad (11)$$

where  $c_s$  — heat capacity of the slurry in solid state,  $c_l$  — heat capacity of the slurry in liquid state,  $\alpha(\bar{t}) = 0$  for the slurry in solid state and  $\alpha(\bar{t}) = 1$  for the slurry in liquid state,  $\bar{t}$  — dimensionless temperature of the slurry [9].

According to the experimental data of a slurry of oxide of beryllium the function  $\alpha(\bar{t})$  has an appearance  $\alpha(\bar{t}) = 5.714 \cdot \bar{t} - 2.857$ . The method of apparent heat capacity allows taking into account the heat of the phase transition, and is convenient for calculations, since the positions of the transition zone are unknown in advance, and are determined as a result of calculations [7, 12].

Equations (1)–(4) were solved by fully coupled finite element method using commercial software COMSOL Multiphysics version 5.6. The area in question is divided into unit cells with  $\Delta z_i$ ,  $\Delta r_j$  sides [13-14]. The pressure gradient is determined by splitting to maintain mass flow (4). To obtain a finite difference analogue of the system of equations of transition (1) and energy (3), an implicit Crank-Nicolson difference scheme has been used, approximating differential equations with a second order of accuracy [14]. Difference

analogue of continuity equation (2) is solved by two-layer scheme of the second order of accuracy. When switching to differential analogues for a circuit channel, the following designations are used:

$$z = z_{n+1} - z_n$$
,  $\Delta r = r_{j+1} - r_{j-1}$ ,  $\Delta r_+ = r_{j+1} - r_j$ ,  $\Delta r_- = r_j - r_{j-1}$ ,

where  $\Delta z$ ,  $\Delta r$  change within the  $0 \le \Delta \le 1$  interval. System of linear equations is presented in vectormatrix form and is solved by run-through method [13–15].

**Results and discussion**: One of the important goals of numerical modeling of the process under consideration is to determine the optimal conditions for creating beryllium oxide products by hot casting. In practice, this process is mainly controlled by the casting speed and hot slurry temperature [6]. The results of numerical calculations of the solidification zone of the slurry and the flow velocity profile at different casting speeds are given in Figure 2. The process of molding beryllium ceramics itself occurs in the temperature range of  $80 \div 20^{\circ}$ C, i.e., a thermoplastic slurry — a suspension with an initial temperature of  $80^{\circ}$ C — flows in and moves into the forming cavity. As it moves, the slurry mass cools and hardens, acquiring a structural shape at the outlet of the pipe. Generalization of the experimental results take into account the dependence of the rheological and thermophysical properties of the thermoplastic slip on temperature in the required range of the hot casting method from  $80^{\circ}$ C to  $20^{\circ}$ C.

As can be seen from Figure 2, the slurry flow is fully developed in the hot circuit, in which the slurry is practically not cooled, and the speed has a parabolic profile. As the slurry cools intensively in the cold path of the spinneret, the speed profile becomes rectangular in shape corresponding to the sliding speed on the wall, and the flow of the slurry becomes a solid mass motion at a constant speed.



Figure 2. Slurry solidification zone and flow rate profile at different casting rates: a) u=20 mm/min; b) u=40 mm/min; c) u=60 mm/min; d) u=80 mm/min; e) u=100 mm/min.

The graphs show the solidification zone of the hot slurry as a function of the casting speed. The zone of solidification of the slurry is defined as the zone between two isochores — the lower (40°C) and upper (54°C) boundaries of crystallization of the slurry [6, 7]. The slurry is still liquid at temperatures above 54°C, and becomes solid at temperatures below 40°C, so more accurate determination of the solidification zone depending on the casting conditions is an important task. As can be seen from Figure 2, the position of the

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solidification zone is transferred (extended) by the flow of the slurry. It becomes wider, especially near the inner wall of the spinneret with an increase in casting speed (Fig. 2, a-e). The results of the calculations show that an increase in the casting speed of more than 100 mm/min is not optimal, since part of the solidification zone can leave the spinneret, which leads to incomplete solidification of the slurry [16-17].

Analysis of influence of thermoplastic slurry flow conditions on total heat transfer coefficient from hot slurry to cooling agent has been performed. An important step in hot casting beryllium ceramics is to cool the hot slurry with a layer of circulating water [18-19]. When simulating the hot slurry flow, cooling can be mathematically described by the following equation of the heat flow from the slurry to the cooling agent (water)

$$q = kA_i(T_h - T_c)$$

where  $A_i$  — the area of the inner wall of the spinneret with a length of L,  $T_h \bowtie T_c$  — the temperature of the hot slurry and cooling water, respectively, k — the total heat transfer coefficient from the hot slurry to cooling water, which according to Figure 1 is calculated by the following formula [8–11, 20]

$$k = \frac{1}{\frac{1}{h_i + \frac{r_i}{\lambda} ln \frac{r_o}{r_i} + \frac{r_i}{r_o h_o}}},$$
(12)

Where  $h_i$  and  $h_o$  — internal and external coefficients of convective heat exchange, respectively,  $r_i$ ,  $r_o$  — internal and external radii of the spinneret,  $\lambda$  — thermal conductivity of the material of the spinneret wall. The  $h_i$  and  $h_o$  coefficients are usually determined experimentally for a particular system and are calculated using different empirical formulas for internal and/or external fluid flow conditions. The thermal conductivity of the wall material of the spinneret  $\lambda$ , made of steel, grade 12X18H10T, is 15  $\frac{BT}{M*K}$  at 20°C. Like other oxide materials, the thermal conductivity of beryllium oxide decreases sharply with increasing temperature. The thermal conductivity of thermoplastic beryllium oxide slurry at temperatures of 40 and 60 °C are 254.3 and 181.5 W/(m·K), respectively.

Thus, in order to calculate the total heat transfer coefficient k, it is necessary to determine the convective heat exchange coefficients. To this end, consider the inner and external flows for the circular spinneret — the cooling water flow (external) in the shell by the inner and external radii  $r_o r_w$  and  $r_i$ , respectively, and the hot slurry flow of beryllium oxide in the spinneret by the radius of  $r_i$ . Heat is first transferred from the hot slurry to the inner wall of the spinneret due to internal convection, then it is transferred to the external wall of the spinneret, only then enters the cooling water (Fig. 3). It is believed that there is no thermal contact layer with a certain thermal resistance between the slurry and the inner wall, external wall and water.



Figure 3. Schematic representation of heat flow from hot slurry to cooling water

The coefficient of external convective heat exchange can be calculated from the condition of cooling water flow in the external casing of the spinneret, which is the external flow for the spinneret. To do this, we will use the known empirical formulas for the Nusselt number  $Nu_o = \frac{h_o D_o}{\lambda}$  when cooling the cylinder with an external stream of water, where the external diameter of the spinneret  $D_o$  Empirical formulas for calculating the Nusselt number are usually searched as dependencies on the Reynolds and Prandtl numbers [18].

In most cases in hot casting, all parameters except the temperature and flow rate of the circulating water are constant values. The temperature and flow rate of the circulating water (Reynolds and Prandtl numbers, respectively) may vary from series to series of experiments, but they remain constant during each series of experiments. Figures 4, 5 show the dependence of the average coefficient of external convective heat exchange on the Reynolds and Prandtl numbers. Design and physical data from experiments given in [1] have been taken as data for calculation of average coefficient of external convective heat exchange. This paper

examines an experiment to study the solidification of a hot slurry in a concentric cylinder cooled by the circulation of water in two circuits with temperatures of 20 and 80°C, respectively (Fig. 1). These temperatures are maintained by changing the hot and flow of coldwater. The height of the circuits is 8 and 20 mm, respectively, and the slurry casting speed varied from 20 to 100 mm/min, and the cold circuit water flow rate varied from 250 to 1500 l/hr., while the hot circuit water flow rate was maintained at a constant value of 500 l/hr.

In Figures 4 and 5, the coefficients of external convective heat exchange for cold and hot circuits depending on the Reynolds number have been calculated using the first and third formulas of the Nusselt number at  $Pr(20^{\circ}C) = 7.02$  and  $Pr(80^{\circ}C) = 2.22$ . Note that the Reynolds number corresponds to the flow rate of cold water of 1000 to 1900 ml/min, and hot water from 100 to 1000 ml/min. As shown in Figures 4 and 5, the coefficients of external convective heat exchange calculated by both formulas increase with the Reynolds number, that is, the flow rate of water in two circuits.

Three empirical formulas are considered — Hilpert's  $(Nu_{o1})$ , Zukauskas  $(Nu_{o2})$  and Churchill-Bernstein  $(Nu_{o3})$  formulas Table 1 [18]. As can be seen from Table 1, the first two formulas work in limited intervals of Reynolds and Prandtl numbers, while the third formula works in a wide range of these numbers. The coefficients  $C_1$ ,  $C_2$ , and the indicators m, n are functions of the number $Re_D$ , and are shown in Tables 2 and 3.



Figure 4. Dependence of external convective heat exchange coefficient  $h_o$  for a) cold circuit ( $Pr(20^\circ C) = 7.02$ ) and b) hot circuit  $Pr(80^\circ C) = 2.22$  on Reynolds number calculated from formulas 1 and 3 of Nusselt number

Т	а	b	1	e	1
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	Note					
	$0.4 \lesssim Re_D \lesssim 4 \cdot 10^5, Pr \gtrsim 0.7$					
Nu	$1 \lesssim Re_D \lesssim 10^6, 0.7 \lesssim Pr \lesssim 500$					
$Nu_{o3} = 0.3 + \frac{0.1}{1}$	$Re_D Pr \gtrsim 0.2$					
Re <sub>D</sub>	C1	m	Re <sub>D</sub>		C <sub>2</sub>	n
0.4-4	0.989	0.330	1-40		0.750	0.4
4-40	0.911	0.385	40-1000		0.510	0.5
40-4000	0.683	0.466	1000-200000		0.260	0.6
4000-40000	0.193	0.618	200000-1000000		0.076	0.7
40000-400000	0.027	0.805				1

Empirical correlations of Nusselt number for cylinder

In addition, the difference in the coefficients of external convective heat exchange increases with an increase in the Reynolds number, and the maximum difference is 10 and 25% at 20 and 80°C, respectively. This shows that both formulas (the Hilpert and Churchill-Bernstein formulas) are suitable for calculating the coefficient of external convective heat exchange, but it is preferable to use the third formula (Churchill-Bernstein) as it operates over a wide range of changes in Reynolds and Prandtl numbers.

In the case of internal flow, the Reynolds, Prandtl and Nusselt numbers are determined by the following formulas

$$Re_D = \frac{U_{cast}D_o}{v_{sl}}; Pr = \frac{\mu_{sl}c_{psl}}{\lambda_{sl}}; Nu_l = \frac{h_lD_l}{\lambda_{sl}}$$

 $U_{cast}$  — casting speed,  $D_i$  — inner diameter of the spinneret,  $v_{sl}$ ,  $\mu_{sl}$  — kinematic and dynamic viscosity of slurry,  $c_{psl}$ ,  $\lambda_{sl}$  — heat capacity and thermal conductivity of slurry.

In the internal flow of the fluid, correlation dependencies of the Nusselt number are used, as in the case of external flow. There are various dependencies of the Nusselt number on the Reynolds and Prandtl number for laminar and turbulent currents. Works [18–21] give empirical formulas for calculation of Nusselt number at flow of various liquids between concentric cylinders (coaxial tube). In this case, we will confine ourselves to calculating the coefficient of internal convective heat exchange based on the Nusselt number on the flow of the thermoplastic slurry in the cylindrical tube.

As it is known, in the case of a fully developed laminar flow with a uniform heat flow on the cylinder wall and a uniform distribution of the wall temperature, the Nusselt number is constant and equals 4.36 and 3.66, respectively [18]. But, when the thermoplastic slurry flows in the cylindrical tube, its intensive cooling takes place, moreover, uneven, through the wall, which leads to a possible change in the speed profiles along the cylinder, which dictates the use of a non-constant Nusselt number, at least along the length of the tube. Since the slurry is a highly viscous liquid, its flow in the cylinder differs with a large Prandtl number, which leads to the choice of empirical formulas for calculating Nusselt, working for such a flow. For the case in question, i.e. for a course with Prandtl number  $Pr \gtrsim 5$  the following Nusselt equation is suitable

$$Nu_{i4} = 3.66 + \frac{0.0668 \frac{D_i}{L} Re_D Pr}{1 + 0.04 \left(\frac{D_i}{L} Re_D Pr\right)^{2/3}}.$$
(13)

Slurry casting occurs when the temperature changes from 80 to 20°C, which corresponds to the Prandtl number of 1300–22000. Casting speed varies from 20 to 100 mm/min [1], which corresponds to Reynolds number on the order of ~10<sup>-4</sup>-10<sup>-3</sup>. Figure 6 shows the change in the internal convective heat exchange coefficient of  $h_i$ , calculated from the Nusselt number  $Nu_{i4}$ , according to the Reynolds number at $Pr(50^{\circ}C) = 5700$  and  $Pr(60^{\circ}C) = 3584$ . The coefficient of internal convective heat exchange increases with an increase in Reynolds number and a decrease in Prandtl number, but it is significantly lower than the coefficient of external convective heat exchange (Fig. 4 a-b). Figure 7 shows the dependence of the Nusselt number for the inner current on the Reynolds number for  $Pr(50^{\circ}C) = 5700$  and  $Pr(60^{\circ}C) = 3584$ . The Nusselt number for  $Pr(50^{\circ}C) = 5700$  and  $Pr(60^{\circ}C) = 3584$ . The Nusselt number for the inner current on the Reynolds number for  $Pr(50^{\circ}C) = 5700$  and  $Pr(60^{\circ}C) = 3584$ . The Nusselt number for the inner current on the Reynolds number for  $Pr(50^{\circ}C) = 5700$  and  $Pr(60^{\circ}C) = 3584$ . The Nusselt number for the inner current on the Reynolds number for  $Pr(50^{\circ}C) = 5700$  and  $Pr(60^{\circ}C) = 3584$ . The Nusselt number calculated using the last formula (13) varies around the Nusselt number values for a fully developed laminar flow with uniform heat flow on the cylinder wall and uniform wall temperature distribution.



Figure 5. Dependence of the coefficient of internal convective heat exchange  $h_0$  on the Reynolds number for  $Pr(50^{\circ}\text{C}) = 5700 \text{ and} Pr(60^{\circ}\text{C}) = 3584$ 



Figure 6. Dependence of the Nusselt number for the inner flow on the Reynolds number for

 $Pr(50^{\circ}C) = 5700$  and  $Pr(60^{\circ}C) = 3584$ .

The dependence of the total heat transfer coefficient k on the hot slurry to cold water on the Reynolds number of the internal flow at different Reynolds numbers of the water flow in the cold circuit, calculated using the formula (13), is presented. This factor is used in modeling the flow of a hot slurry that cools through the walls when the temperature changes in the spinneret wall material is neglected, which is not always a correct assumption. This means that in some cases, especially when the heat transfer rate through the wall material is of the same order with the heat transfer rate in the layers near the wall, or when the spinneret wall is not thin.



Figure 7. The dependence of the total coefficient on hot slurry to cold water on the Reynolds number of the internal flow at different Reynolds numbers of the water flow in the cold circuit

As shown in Figure 6, the heat transfer coefficient k increases as the Reynolds number of internal and external flows increases. It is also noted that k is mainly controlled by internal convective heat exchange, for which the coefficient of heat exchange is significantly lower than the coefficient of external heat exchange.

### Conclusions

Thermal calculations of the laminar flow of the slurry in the cavity formed by two concentrically arranged pipes during the formation of beryllium ceramics have been carried out. From analysis of experimental data and calculations having obtained during movement of viscoplastic slurry in concentric circular channel, it is established that with increase of cooling water flow rate in two circuits, external convective heat exchange coefficients increase. Heat exchange and heat transfer coefficients on the walls of the annular cavity have been estimated according to special criterion relationships.

Experimental data having obtained from the study of hydrodynamics and heat exchange of the casting process of ceramic products have been analyzed and generalized using a mathematical model. The system of equations of the laws of conservation of impulse, mass and energy of non-Newtonian fluid is considered

together with the Shvedov-Bingham rheological model. Rheological and thermophysical properties of the slurry have been found on the basis of experimental data, and express dependence on temperature.

Zones of solidification of hot slurry depending on casting speed have been identified. The zone of solidification of the slurry is defined as the zone between two isochores — the lower (40°C) and upper (54°C) boundaries of crystallization of the slurry. The structure of the slurry crystallization front shows that the crystallization rate depends on the mode parameters and channel design data. The use of constructive and experimental data for calculating the average coefficient of external convective heat exchange and the Nusselt number in dependence on the Reynolds and Prandtl numbers makes it possible to obtain temperature and speed profiles explaining the physical essence of the phenomenon, i.e. the process of solidification of the thermoplastic slurry in the annular cavity.

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# Бериллий керамикасын формалау процесінде термопласт шликерін ағыс шарттарының жылу тасымалдау шамасына әсерін бағалау

Мақалада керамикалық бұйымдарды қалыптау кезінде термопластикалық шликер ағыны жағдайларының жалпы жылу беру коэффициентіне әсерін бағалау нәтижелері келтірілген. Есептеу эксперименті негізінде жүзеге асырылатын есептеу деректері мен эксперименттік нәтижелердің сенімді сәйкестігін қамтамасыз ететін құю процестері мен жылу есептеулерінің егжей-тегжейлі сипаттау әдістері әзірленді. Өнімдерді қалыптау кезінде пайда болатын физикалық процестерді модельдеу процестің температуралық-фазалық өрістерінің өзгеруін егжей-тегжейлі бақылау мүмкіндігі арқылы құйма сапасын жақсартудың жаңа мүмкіндіктерін анықтауға және құю режимдеріне байланысты қатаю кинетикасын көрнекі түрде пайдалануға мүмкіндік береді. Қатаю кезеңінде қалыптаудан жылудың бөліну жылдамдығы суспензия қозғалысының жылдамдығын, сонымен қатар өтпелі аймақтың ені тәуелді болатын температура өрісін анықтайды. Бұл факторлар бериллий керамикасының құрылымын қалыптастыруға тікелей әсер етеді. Фазалық ауысу кезінде температура мен жылуға байланысты керамикалық бұйымдарды қалыптау кезінде жылу алмасу процесін зерттеу негізгі міндет болып табылады, өйткені олар бериллий керамикасының технологиялық және пайдалану сипаттамаларын айтарлықтай анықтайды. Есептік деректер керамиканы қалыптау процесінің оңтайлы шарттарын анықтауға және біртекті құрылымы бар қатайтылған өнімді шығаруға мүмкіндік береді.

*Кілт сөздер:* термопластикалық шликер, бериллий оксиді, гидродинамика, жылу алмасу, құю процесі, қалыптау, қатаю.

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# Оценка влияния условий течения термопластичного шликера на коэффициент теплопередачи при формовании бериллиевой керамики

В статье представлены результаты расчетов оценки влияния условий течения термопластичного шликера на общий коэффициент теплопередачи при формовании керамических изделий. Разработаны методы детального описания процессов литья и тепловых расчетов, обеспечивающие достоверное соответствие расчетных данных и экспериментальных результатов, реализуемых на основе вычислительного эксперимента. Моделирование физических процессов, происходящих при формовании изделий, позволяет обнаружить новые возможности повышения качества отливок за счет возможности более детального отслеживания изменения температурно-фазовых полей процесса и наглядно представить кинетику затвердевания в зависимости от режимов литья. Скорость отвода тепла от формовки в период затвердевания определяет скорость движения суспензии, наряду с этим и температурное поле, от которого зависит ширина переходной области. Эти факторы оказывают непосредственное влияние на формирование структуры бериллиевой керамики. Исследование процесса теплообмена при формовании керамических изделий в зависимости от температуры, теплоты при фазовом переходе является основной задачей, поскольку они во многом определяют технологические и эксплуатационные характеристики бериллиевой керамики. Расчетные данные позволяют определить оптимальные условия процесса формования керамики и получить на выходе затвердевшее изделие с однородной структурой.

*Ключевые слова*: термопластичный шликер, оксид бериллия, гидродинамика, теплообмен, процесс литья, формования, затвердевания.

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