To plasma electrons motion theory in high-frequency fields

The article is dedicated to the kinetic theory of plasma, where the problem of interaction of high-frequency electric fields with weakly non-uniform plasma is investigated using kinetic equations with binary collision integrals for plasma particles. A new methodology for determining the expression of the averaged high-frequency pressure force is proposed based on solving the kinetic equation and using the method of successive approximations (separation of slow motions and fast oscillations) under the limiting conditions. An expression for the averaged quasi-potential force is derived based on the kinetic equation for the electron distribution function in weakly inhomogeneous magnetically active plasma, taking into account electrons collisions with fixed ions and the presence of a longitudinally inhomogeneous high-frequency electric field using well-known methods of theoretical and mathematical physics, such as successive approximations, averaging over the effective field oscillation period, and integration over trajectories. The amplitude of the field is a slowly varying function in both time and space coordinates. The obtained expression allows us to estimate the influence of collisions of plasma particles on the Miller force, and under limiting conditions it coincides with the known expressions for the high-frequency pressure force derived from the equation of plasma electrons motion in high-frequency fields. In all calculations, the contribution of the magnetic component of the electromagnetic field is neglected, which is quite justified for the longitudinal electric field.

Keywords: weakly inhomogeneous plasma, plasma electrons, kinetic equation, averaged force, electric field, particle collisions, fixed ions, high frequency.

Introduction

The heightened interest shown by scientists in plasma physics over the last quarter-century is primarily due to space exploration and the prospect of obtaining controlled thermonuclear reactions. Among other fields of application, it is worth mentioning plasma accelerators and generators, gas-discharge devices, plasma electronics, etc. Just from this list, it is evident that the plasma under scrutiny by researchers exhibits a tremendous diversity of numerical parameters, sometimes differing by many orders of magnitude.

It is difficult to imagine laboratory plasma without external fields. The existence of powerful sources of monochromatic electromagnetic radiation in various frequency ranges has stimulated the publication of a large number of studies on the interaction of such radiation with matter. In particular, the interaction of intense electromagnetic waves with magnetically active plasma is of interest due to solving a number of issues arising in the fusion programme development process, the study of nonlinear properties of the ionosphere with both given and artificially created inhomogeneity. The peculiar properties plasma appears very clearly when its behaviour under the influence of an electric field of high frequency is examined. Studies of the interaction of charged plasma particles with high-frequency fields have recently gained particular relevance addressing issues related to plasma heating and confinement, nonlinear wave evolution in laboratory and space plasmas, the development of charged particle acceleration methods, exploration of new wave generation and transformation techniques, and more. The initial studies in the late 1950s on the interaction of charged plasma particles with inhomogeneous high-frequency fields revealed the existence of the Miller quasi-potential force and led to ideas for high-frequency plasma confinement and acceleration [1–7]. The averaged effect of high-frequency fields on the plasma has been studied by many authors. At the same time, the theoretical description of the particle motion was based on simple physical grounds on the possibility of separation of slow motions and fast oscillations under compliance with a number of limiting conditions (non-relativistic approximation stationary and homogeneous external magnetic field, homogeneous plasma, high-frequency fields in the quasi-stationary approximation or in the form of plane waves, etc.). Along with this, there is a significant interest in moving beyond the dipole approximation and studying a spatially inhomoge-
neous system of charged particles in inhomogeneous alternating fields. All this indicates the relevance of the study of plasma behaviour in high-frequency fields.

In the works of a number of authors [2–4] devoted to the acceleration of plasma particles and the retention of high-temperature plasma by high-frequency electromagnetic fields, an expression for the force in question was determined based on the equation of electron motion. In this paper, the expression for the Miller force is derived from the kinetic equation for the electron distribution function, which takes into account collisions of electrons with ions and the influence of a longitudinal, high-frequency and inhomogeneous electric field on a weakly inhomogeneous magnetically active plasma, using the method of successive approximations (separation of slow movements and fast oscillations).

**Experimental**

Plasma can be considered as an ideal system where particles interact only in collisions. In this case collisions of plasma particles are considered as a correction leading to slow dissipation of energy, concentrated in collective degrees of freedom. Developing a consistent collision theory in plasma encounters significant difficulties associated with the slow decrease of Coulomb forces with increasing distance between interacting particles. At any given time, each charged plasma particle is exposed to a huge number of surrounding particles, and all of these effects shall be somehow taken into account. Instead of a simple two-body problem, we face the challenging problem of many-body interactions. In a strict formulation such a problem is hardly solvable. To make a solution possible, it is necessary to introduce some simplifications. The simplest is the pair collision approximation, in which the plasma particle interactions are reduced to independent and instantaneous interactions of pairs of particles.

**Results and Discussion**

It is known that in high-frequency electromagnetic fields, particles (specifically, electrons) experience not only the generalized Lorentz force $\vec{F}_0$, but also an additional force $\vec{F}_M$ determined by the high-frequency quasi-potential $U_e$, i.e., $\vec{F}_M = -n_e \text{grad} U_e$ where $n_e$ is the electron concentration. As a result of its action, the plasma electrons tend to move to the field minimum. The physics of such a phenomenon can be explained as follows. The electromagnetic field, causing high-frequency oscillations of electrons with velocity $\vec{v}_e$ creates as if an additional high-frequency pressure $P_{\text{hf}} = n_e m_e v_e^2$ (or $P_{\text{hf}} \sim n_e U_e$, where $m_e$ is the electron mass), as a result of which the plasma electrons are displaced from the areas occupied by the field. In oscillatory motion, the force $\vec{F}_M = -\text{grad} P_{\text{hf}}$ is directed against the displacement of electrons, so when an electron is displaced to the right, a return force of greater magnitude acts on it compared to when it is displaced to the left. The averaged force $\vec{F}_M$, so called the high-frequency pressure force, or sometimes the quasi-potential Miller force, do not depend on the particle charge sign. An expression for the studied force, based on the equation of motion of electrons, was defined in the works of a number of authors [8–12], devoted to the acceleration of plasma particles and the confinement of high-temperature plasma by high-frequency electromagnetic fields. In this article, the expression for the Miller force is derived from the kinetic equation for the electron distribution function, considering electron-ion collisions and the influence of a longitudinal, high-frequency electric field

$$\vec{E} = \vec{E}_0(\epsilon \vec{r}, \epsilon t) \sin \omega_0 t$$

(1)

on weakly inhomogeneous magnetically active plasma, using the method of successive approximations (separation of slow motions and fast oscillations), where $\vec{E}_0$ the amplitude of the field is a slowly varying function in both time $t$ and space coordinates $\vec{r}$. The parameter $\epsilon$, characterising the slowness of change $\vec{E}_0$, fulfils the condition $\epsilon = (V_f / \omega_0 L) << 1$, where $V_f$ — thermal velocity of electrons; $\omega_0$ — frequency, $L$ — typical value of the change in the electron distribution function $F_e$ (or typical size of the area occupied by the plasma). When referring to high field frequencies, we will assume that during the period of field oscilla-
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...tion \( T = 2\pi / \omega_0 \) the electron goes a distance \( l_{eq} \) much shorter than the free path length \( l \), i.e., \( l_{eq} < l < L \) and hence it can be considered as moving in an almost homogeneous field. Here \( l_{eq} = \frac{V_e \omega_0}{e} \) and \( l = \frac{V_e}{v_\text{ei}} \); \( v_\text{ei} \) is the frequency of electron-ion collisions. We will assume that the plasma

- is weakly inhomogeneous, i.e., the functions \( F_e \) and \( \vec{E} \) do not change much at a distance of the Debye radius \( r_d \) or the condition \( \left( L / \nu_0 T \right) >> 1 \) is fulfilled, where \( \nu_0 \) is the average particle velocity;
- fully ionised, i.e., \( \gamma \to \infty \), where \( \gamma \) is the degree of ionisation;
- strongly discharged, i.e.,

\[
\frac{v_e}{\omega_0} < \varepsilon \quad \text{or} \quad \omega_0 >> v_\text{ei}
\]

\( l >> \left( mV_T^2 / eE \right) \).

In addition, we will assume that:

- the activity of the electric field on the length \( l \) is greater than the energy of thermal motion \( kT_e \) (strong field condition), i.e. \( eEl / k_BT_e >> 1 \);
- due to the smallness of the ratio \( \left( m_\text{i} / m_e \right) << 1 \) (or \( m_\text{i} = \infty \) ) we can assume that the ions are fixed, where \( e \) — electron charge; \( m_\text{i} \) — ion mass, \( k \) — Boltzmann constant; \( T_e \) — electron temperature. It should be noted that due to the field's longitudinally \( \vec{E} \), the magnetic component of the electromagnetic field can be neglected, which gives significant simplifications of a computational nature, and the collisions of electrons among themselves are also neglected.

The kinetic equation for \( F_e \), taking into account collisions of electrons with ions and fields (1), is written in the form

\[
\hat{L}_0 F_e + \vec{v} \frac{\partial F_e}{\partial \vec{v}} = St_e_i(F_e),
\]

where \( \vec{v} = \vec{p}/m_e \) is the electron velocity;

\[
\hat{L}_0 = \frac{\partial}{\partial t} - e\vec{E} \frac{\partial}{\partial \vec{v}} .
\]

In accordance with (1) the function \( F_e \) depends on both fast and slow variables, i.e. \( F_e = F_e(t, \vec{p}, \varepsilon t, \varepsilon \vec{r}) \). In this connection, using the method of successive approximations, we can write:

\[
\frac{\partial F_e}{\partial t} \to \frac{\partial F_e}{\partial t} + \varepsilon \frac{\partial F_e}{\partial \varepsilon t};
\]

\[
\frac{\partial F_e}{\partial \vec{r}} \to \varepsilon \frac{\partial F_e}{\partial \varepsilon \vec{r}},
\]

and also

\[
F_e = F_e^0 + \varepsilon F_e^1 .
\]

where \( F_e^1 \) is the fast-variable part, \( F_e^0 \) is the stationary (slowly changing) part of the distribution function.

Substituting expressions (3) and (4) into (2), then equating terms at the same degrees of the parameter \( \varepsilon \), we obtain the following two equations of zero and first approximations
\begin{align}
\hat{L}_e F_{e}^0 &= S_{et} \left( F_{e}^0 \right), \\
\hat{L}_o F_{e}^0 &= -\hat{L}_x F_{e}^0,
\end{align}

where
\[
\hat{L}_e = \frac{\partial}{\partial \epsilon t} + \nu_e \frac{\partial}{\partial \epsilon}.
\]

Considering the obtained system of equations, it is seen that the left part of the subsequent equation is determined by the solution of the previous equation and can be considered to be known. The integral of collisions of electrons with ions in (5) is defined by the expression [13-14]:
\[
S_{et} \left( F_{e}^0 \right) = \frac{3\sqrt{\pi} \left( kT_e / m_e \right)^{3/2}}{8\tau_{ei}} \frac{\partial F_{e}^0}{\partial \epsilon_e},
\]

where \( \tau_{ei} \equiv 1 / \nu_{ei}^{-1} \) — characteristic time of collisions of electrons with ions:
\[
\tau_{ei} = \frac{3\sqrt{m_e} \left( kT_e \right)^{3/2}}{4\sqrt{2\pi} \Lambda e^2 \epsilon_e^{-2} n_i}.
\]

\( \Lambda \) Coulomb logarithm; \( e_i \) and \( n_i \) — charge and ion concentration, respectively. It should be noted that in (7) the terms of order \( \left( m_e / m_i \right) \) are neglected. After substituting instead of \( F_{e}^0 \) in expression (7), the solution
\[
F_{e}^{0(0)} = n_e \left( 2\pi m_e kT_e \right)^{-3/2} \exp \left\{ - \left( 2m_e kT_e \right)^{-1} \left( \bar{p} - e\omega_0^{-1} \bar{E}_0 \cos \omega_0 t \right)^2 \right\},
\]

of the homogeneous equation \( \hat{L}_o F_{e}^{0(0)} = 0 \), we have
\[
\hat{L}_o F_{e}^0 = \Gamma = n_e \left( 2\pi m_e kT_e \right)^{-3/2} \frac{4\pi\Lambda e^2 \epsilon_e^{-2} n_i}{\nu_e m_e} \bar{p} \left( \bar{p} - e\omega_0^{-1} \bar{E}_0 \cos \omega_0 t \right) \times
\]
\[
\times \exp \left\{ - \left( 2m_e kT_e \right)^{-1} \left( \bar{p} - e\omega_0^{-1} \bar{E}_0 \cos \omega_0 t \right)^2 \right\}.
\]

The general solution of this equation can be represented as
\[
F_{e}^0 = F_{e}^{0(0)} + F_{e}^{0(1)},
\]

where \( F_{e}^{0(1)} \) is the solution of the inhomogeneous equation. The first approximation equations (6) can be rewritten in the form
\[
\hat{L}_o F_{e}^{1} = -G(t, \bar{p}, \epsilon t, \epsilon \bar{r}),
\]

where \( G = \hat{L}_x F_{e}^0 \). The solution \( F_{e}^{1(0)} \) of the homogeneous equation \( \hat{L}_o F_{e}^{1(0)} = 0 \) is the expression for \( F_{e}^{0(0)} \), and the general solution of (6A) is of the form:
\[ F_e^{(1)} = F_e^{(0)} + F_e^{(1)}, \] (8)

where \( F_e^{(1)} \) is the solution of the inhomogeneous equation. In order to find the solutions \( F_e^{(1)} \) and \( F_e^{(0)} \) let’s write down the characteristics of the homogeneous equation (2), i.e.

\[ \frac{dp}{dt} = -e\vec{E}; \quad m_e \frac{d\vec{r}}{dt} = \vec{p}, \] (9)

solutions which are the values of impulses \( \vec{p} \) and coordinates \( \vec{r} \) at time instant \( t \); they are related to the values \( \vec{P}_0 \) and \( \vec{R} \) at the initial time instant \( t \) as follows [15–18]:

\[
\vec{P}_0(t', t, \vec{p}, \varepsilon t, \varepsilon \vec{r}) \equiv \vec{p} + \frac{eE_0}{\omega_0} \{ 1 - \cos \omega_0 (t - t') \};
\]

\[
\vec{R}(t', t, \vec{P}_0, \vec{r}, \varepsilon t, \varepsilon \vec{r}) \equiv \vec{r} - \frac{\vec{P}_0}{m_e} (t - t') - \frac{eE_0}{m_e \omega_0} \{ \omega_0^{-1} \sin \omega_0 (t - t') - (t - t') \}.
\]

Moving from \( \vec{p} \) to \( \vec{P}_0 \) and from \( \vec{r} \) to \( \vec{R} \), he required solutions are determined [9; 10]:

\[
F_e^{(1)}(t, \vec{P}_0, \varepsilon t, \varepsilon \vec{R}) = -\int_0^t G(t - t', \varepsilon t, \varepsilon \vec{R}, \vec{P}_0) dt'. \] (10)

\[
F_e^{(0)}(t, \vec{P}_0) = \int_0^t \Gamma(t - t', \vec{P}_0) dt'. \] (10A)

Hence, the general solution of equation (2), according to (4), (5B) and (8) will be as follows

\[
F_e = F_e^{(0)} + \int_0^t \Gamma dt' + \varepsilon \left( F_e^{(1)} - \int_0^t G dt' \right). \] (11)

Note that when integrating the integrand exponential functions \( \Gamma \) and \( G \), in (10) the condition \( \left( eE_0 / \omega_0 P_0 \right) < 1 \) was taken into account, using which these functions were expanded into a series. At the same time, only terms were considered in the expansion \( \left[ E_0^2 \right] \).

Oscillatory components in (11) can be eliminated by averaging this relation over \( T \). The operator of averaging the function \( F_e \) will be denoted by the symbol \( \langle \rangle \):

\[
\langle F_e \rangle = T^{-1} \int_0^T F_e dt. \] (11A)

Representing the solution in this form involves constructing the first approximation, the so-called averaging method, which has found wide application in nonlinear physics tasks [19–21].

Finally, integrating the product of \( T^{-1} \vec{P}_0 \langle F_e \rangle \) over \( \vec{P}_0 \) in the limit from \( -\infty \) to \( +\infty \), we obtain the final expressions for the force \( \vec{f}_M \):
\[ \vec{f}_M = -\frac{3}{2} \frac{n_e e^2 E_0}{m_e \omega_0^2} \frac{\partial E_0}{\partial \vec{R}} \left\{ 1 - 0.5 \left( \frac{v_{ei}}{\omega_0} \right)^2 \right\}, \]  
(12)

or

\[ \vec{f}_M = -\frac{3}{2} n_e \frac{\partial U_e}{\partial \vec{R}} \left\{ 1 - 0.5 \left( \frac{v_{ei}}{\omega_0} \right)^2 \right\}, \]

and

\[ U_e = -\frac{1}{2m_e} \left( \frac{e E_0}{\omega_0} \right)^2. \]

Under condition (*), from (12) follows the expression [11–13]:

\[ \vec{f}_M = -\frac{3 n_e e^2 E_0}{2 m_e \omega_0^2} \frac{\partial E_0}{\partial \vec{R}}. \]

The obtained result coincides, up to a constant, with known expressions for the Miller force, and is also of theoretical interest and reveals the picture of the interaction of a weakly inhomogeneous plasma with a high-frequency electric field. However, the approximate solution (11) allows estimating the influence of electron-ion collisions on the high-frequency pressure force and can be used in constructing the kinetic theory of inhomogeneous plasma located in high-frequency electromagnetic fields. In all calculations, the contribution of the magnetic component of the electromagnetic field is neglected, which is quite true for the longitudinal electric field, and well-known methods of theoretical and mathematical physics are used, such as averaging over the period of oscillation of the electric field and integration along trajectories.

Conclusions

Therefore, in the article a new methodology has been proposed for determining the expression of the averaged high-frequency pressure force. It is based on solving the kinetic equation with the collision integral of plasma particles and the method of successive approximations, while adhering to a number of limiting conditions (weakly inhomogeneous and fully ionized plasma, longitudinal and inhomogeneous high-frequency electric field, nonrelativistic approximation, and pair collisions between particles). The effects of the weak inhomogeneity of the plasma and the external field on particle collisions and the Miller force are estimated. The motion of charged particles in an electric field is considered on the basis of representations of classical physics and these representations retain their force not only when analyzing the movements of charged particles under the action of macroscopic external fields, but form the foundation necessary for understanding the processes of particle interaction in plasma–processes involving microscopic fields of individual particles.

References

Плазмадагы электрондардың жоғарыжиілікті орістердегі қозғалыс теориясына

Макала плазманың кинетикалық теориясына арналған және жоғарыжиілікті электр орісінің әлсіз біртексі плазмага асері тұралы ық болса, плазма болшектер кинетикасына арналған жоғарыжиілікті қозғалыс теориясына қолданылған. Еруыңдың өрісінің әлсіз, әрі магниттік кинетикасына арналған қозғалыстық теориясына арналған.

Қазіргі жағдайда, плазмадағы амплитудада орістер біртексі плазмадағы орістердің эмиссиясының қозғалыстық теориясына арналған. Математикалық моделдер орістің орістердің қозғалыстық теориясына арналған. Кезектестік жуық барлық электрондардың қозғалысына арналған, орістердің қозғалысының қозғалыстық теориясына арналған.

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К теории движения плазменных электронов в высокочастотных полях

Статья посвящена кинетической теории плазмы, и в ней исследована проблема взаимодействия электрического поля высокой частоты со слабо неоднородной плазмой методом кинетических уравнений с интегралами парных столкновений частиц плазмы. Предложена новая методика определения выражения усредненной силы высокочастотного давления, основанная на решении кинетического уравнения и методе последовательных приближений (разделение медленных движений и быстрых осцилляций) при соблюдении ограничительных условий. Выведено выражение для усредненной квазипотенциальной силы на основе кинетического уравнения для функции распределения электронов слабо неоднородной магноактивной плазмы, учитывающей столкновение электронов с неподвижными ионами и наличие продольно-неоднородного электрического поля высокой частоты с использованием общепринятых методов теоретической и математической физики, таких как последовательные приближения, усреднения по периоду колебания действующего поля и интегрирования по траекториям. Амплитуда поля является медленно меняющейся функцией по времени и координатам. Полученное выражение позволяет оценить влияние столкновений плазменных частиц на силу Миллера, и при ограничительных условиях оно совпадает с точностью до постоянной с известными выражениями для силы высокочастотного давления, рассчитанного на основе уравнения движения электронов плазмы в высокочастотных полях. Во всех вычислениях не учитывается вклад магнитной составляющей электромагнитного поля, что вполне справедливо для продольного электрического поля.

Ключевые слова: слабо неоднородная плазма, плазменные электроны, кинетическое уравнение, усредненная сила, электрическое поле, столкновение частиц, неподвижные ионы, высокая частота.

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